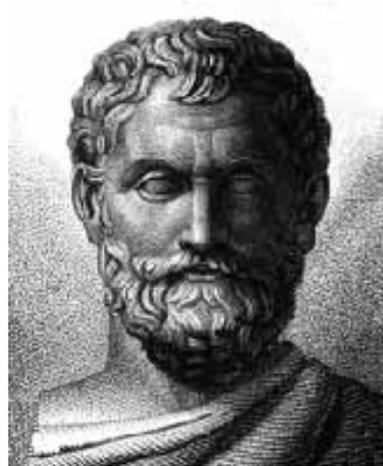


That's a Good Angle

- Circle Theorems 4: The Angle in a Semi-Circle -

A Historic Note

Thales of Miletus 624-547 BCE was one of the first people to study the geometry of circles. The theorem presented here is attributed to him. Thales was also one of the first Greeks to study natural science, philosophy, mathematics and astronomy. He accurately predicted a solar eclipse in the year 585 BCE.



Thales of Miletus

Plato tells us that Thales fell into a well while star-gazing and was rescued by a "clever and pretty maidservant from Thrace".

Prior Learning

Definition: An isosceles triangle is a triangle with at least two equal sides.

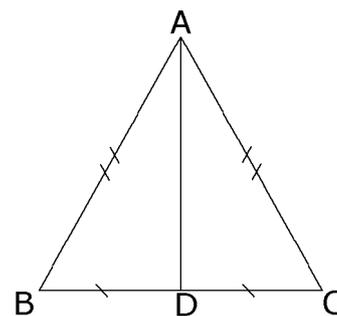
Theorem: The base angles of an isosceles triangle are equal.

Given: any $\triangle ABC$ with $AB = AC$

To Prove: $m\angle ABC = m\angle ACB$

Construction: Find the mid-point D of BC
Join AD

Proof: In $\triangle ADB$ and $\triangle ADC$
 $AB = AC$ (given)
AD is common
 $BD = CD$ (construction)
 $\Rightarrow \triangle ADB$ and $\triangle ADC$ are congruent (SSS)
In particular $m\angle ABC = m\angle ACB$

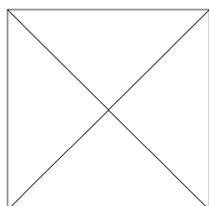


QED

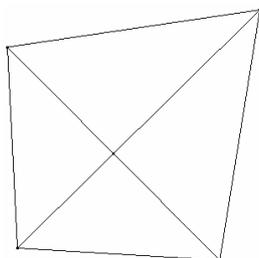
That's a Good Angle

Spot the Right Angle

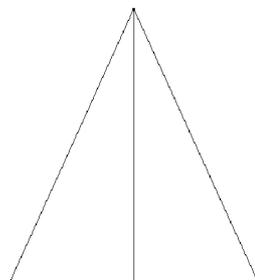
Look at each of the geometric figures below and mark the right angles with a small square.



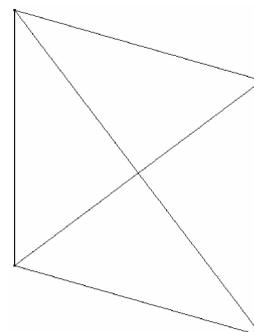
Square



Kite



Isosceles Triangle



Rhombus



Theorem

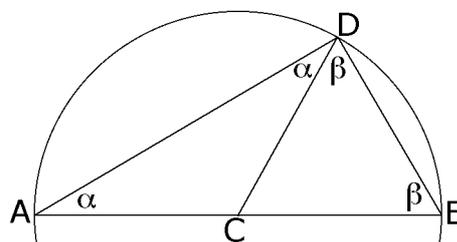
The angle in a semi-circle is a right angle.

Checkpoint

This is a special case of the Angle at the Centre Theorem
from Book III of Euclid's *Elements*

Deductive Proof

Given: AB is a diameter of a circle with centre C
D is any point on the circumference



To Prove: $m\angle ADB = 90^\circ$

Construction: Join CD

Proof: $\triangle ACD$ is isosceles (AC = DC: radii)
 $\Rightarrow m\angle DAC = m\angle ADC = \alpha$ (base \angle s of an isosceles \triangle are equal) :
 $\triangle BCD$ is isosceles (BC = DC: radii)
 $\Rightarrow m\angle DBC = m\angle BDC = \beta$ (base \angle s of an isosceles \triangle are equal)
 However, the sum of the angles of $\triangle ADB$ is 180°
 $\Rightarrow \alpha + \alpha + \beta + \beta = 180^\circ$
 $\Rightarrow \alpha + \beta = 90^\circ$
 $\Rightarrow m\angle ADB = 90^\circ$

QED

That's a Good Angle

Purpose

To demonstrate the Angle in a Semi-Circle Theorem.

from Book III of Euclid's Elements

ClassPad Investigation

Switch on your ClassPad  and tap on the Geometry icon .

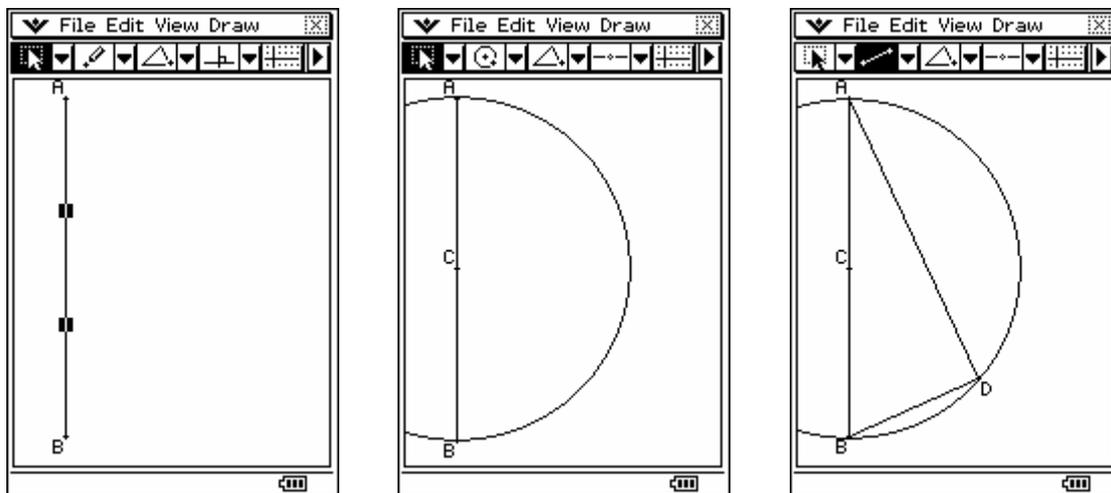
If your spreadsheet has data from a previous investigation, check if you need to save this before you clear it from the screen. To clear the screen, tap on the File menu, select New and if you are sure you then tap OK.

From the Draw Menu tap on Line Segment.

Tap near the top left of your screen and near the bottom left of your screen. This will draw the line segment AB down the left hand side of your screen.

From the left-hand side of the Tool Bar tap on the Selection Arrow . Tap on the line to highlight it. Your screen should look like the first screen below.

From the Draw Menu tap on Construct and then on Midpoint. This locates C the midpoint of AB. Check the second screen.



From the Draw Menu tap on Circle. Tap on the point C and then on the point A. This will draw a circle with centre C and diameter AB.

From the Draw Menu tap on Point. Tap on the circle at a point somewhere away from A and B. This will locate a point D that is constrained to the circle.

Use the Selection Arrow  to tap on the point D, and then try to pull the point D away from the circle. If the circle moves, you know the point D is fastened.

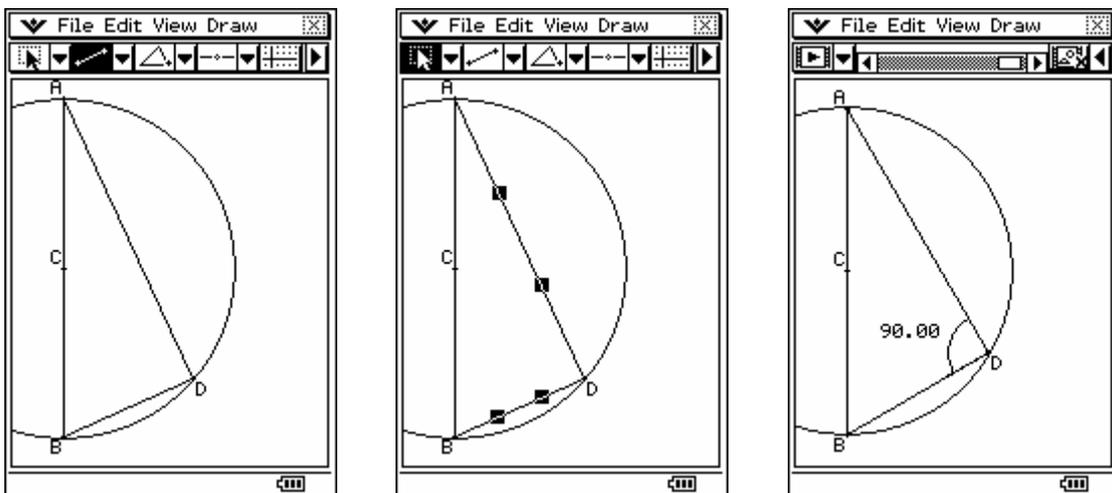
Save your work as SemiC1.

That's a Good Angle

Demonstrating the Theorem

From the Draw Menu tap on Line Segment. Tap on the point B and then on the point D. This will draw the line segment BD. Join AD in the same way.

You will notice that the $\angle ADB$ looks suspiciously like a right angle. To verify this suspicion choose the Selection Arrow  and tap on AD and then on BD. Then tap on the arrow  at the far right of the Tool Bar. This takes you "around the corner" to the Measurement Bar. ClassPad has been programmed to anticipate that you wanted to know the size of the angle between the line segments you have highlighted. You can now see that the angle is actually 90° .



It would be good if you could now move the point D so that you could check if the angle stays at 90° for different positions of D. However, as soon as you tap the point D you lose the highlight on the line segments and the Measurement Bar assumes you want the coordinates of point D.

Alternatively you can highlight the line segments AD and BD and from the Draw Menu tap Attach Angle. Now you can move the point D to different positions around the circle and you can see that $\angle ADB$ stays at 90° .

Animation

To animate the point D, tap on D and tap on the circle.
From the Edit Menu tap Animate and then tap Add Animation.
From the Edit Menu tap Animate and then tap GO (repeat).
You can see that $\angle ADB$ stays at 90° . If you can tap on the line segments as they are moving, you can also see 90° in the Measurement Bar. **Save your work as SemiC2.**



Checkpoint

That's a Good Angle

The Converse Theorem

We can express the Angle in a Semi-Circle Theorem in symbols as:

IF: AB is a diameter

Then: $\angle ADB = 90^\circ$

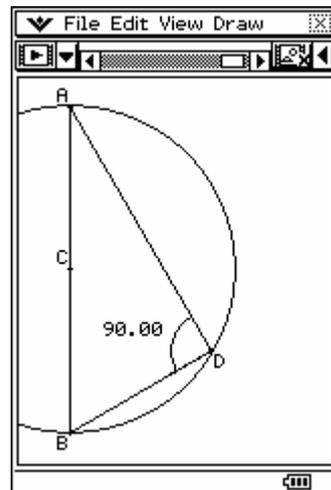
The converse theorem can therefore be expressed as:

IF: $\angle ADB = 90^\circ$

Then: AB is a diameter

The Converse to the Angle in a Semi-Circle Theorem can therefore be written in words as:

A chord that subtends an angle of 90° at the circumference of a circle is a diameter. (The word subtends means holds up or supports.)



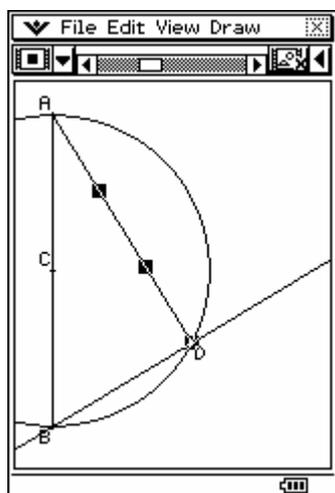
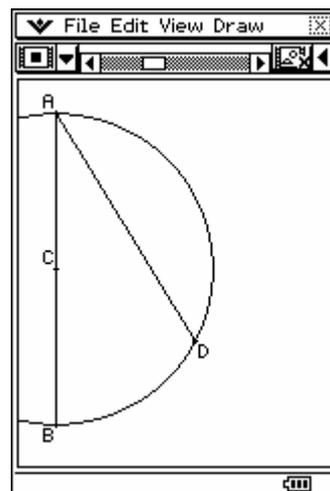
Demonstrating the Converse Theorem

Load the file SemiC1.

You saved this file at a stage where you had drawn a point D on the circumference of a circle centre C having a diameter AB.

From the Draw Menu tap on Line Segment, tap on A and then tap on D. This draws the line segment AD.

From the View Menu tap on Select and then use the Selection Arrow  to highlight the Point D and the Line Segment AD.



From the Draw Menu tap on Construct and then on Perpendicular. This will draw a line that passes through the point D and is perpendicular to the line segment AD.

We know that $\angle ADB = 90^\circ$ because the lines are perpendicular. Wherever we move the point D we find that the new line passes through B and the right angle is therefore subtended by a diameter AB.

Save the file SemiCC

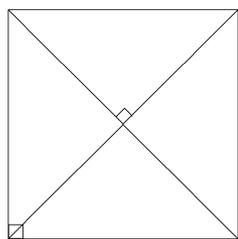
Checkpoint



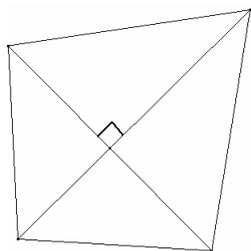
That's a Good Angle

Checkpoints

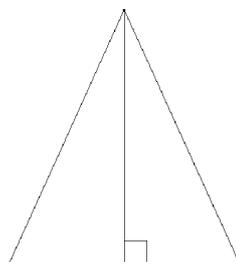
Checkpoint 1



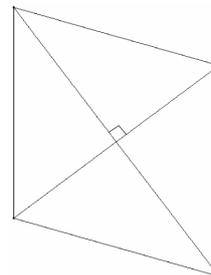
Square



Kite



Isosceles Triangle



Rhombus

Checkpoint 2

Getting the animation to work and display 90° in the Measurement Bar can be very tricky. The only way I could get it to work consistently was to set up the Measurement Bar first, then set the animation going by using the Edit Menu and then try to tap on the lines AD and BD while the animation was working.

Anyway, if a student can show an animation with an attached angle of 90° , I think that is a fair effort.

Checkpoint 3

An animation was not requested here, so those students who produce one have gone a second mile.