

Simple Annuities – Present Value.

OBJECTIVES

- (i) To understand the underlying principle of a present value annuity.
- (ii) To use a CASIO CFX-9850GB PLUS to efficiently compute values associated with present value annuities.

Note that we suggest you complete the activity called 'Understanding and working with compound interest' before starting this set of activities.

EXPLORATORY ACTIVITIES

Activity 1: Paying back a loan.

Suppose I am interesting in buying a new large (45 inch) Sharp LCD TV – RRP \$11 999.

AQUOS



Also, suppose I have to take out a loan for such a large purchase. It is a shame, but many people do not understand the way the repayments are calculated or how to carry out some 'what if' scenarios with loans. This investigation will equip you to be able to do this, with ease.

Suppose you borrow \$12000 from a bank. The bank tells you that you must make regular repayments every month for 3 years until you have paid back the loan.

Unfortunately, for you, it is not as simple as repaying $\$ \frac{1200}{36}$ because the bank has allowed you to use their money and want some money in return for the privilege (this is called interest).

The way the process works can be described as follows:

- You borrow \$12000 (**present value of the loan**) and the bank charges you (adds to the loan) a percentage of this for its use during the first month. At the end of the month you make a repayment (contribution). So at the start of the next month you owe **the starting present value + the interest - your repayment (contribution)**.
- Then the bank charges you a percentage of this month's present value for using it during this month and you make another repayment (contribution) at the end of this month. So at the start of the next month you owe **the previous month's present value + the interest figure - your repayment**.
- This (repetitive) process continues each month until you owe nothing.

Let's experiment. Suppose that the banks states the interest will be 7% per annum compounded monthly and that you have to make monthly repayments of \$40.

To see the amount you would owe at the end of the first month (the present value of the loan at the start of the second month) we could do the following using the 9850GB PLUS:

- Enter RUN mode
- Enter 12000
- Commit it to the calculators answer mode by pressing EXE
- Multiply the Ans(wer) (SHIFT then (-)) by $(1 + \frac{7}{1200})$
- Subtract 40 and calculate the answer by pressing EXE

```
12000
12000
```

```
12000
Ans×(1+7÷1200)-40
12030
```

You should find that present value of the annuity at the start of the next month is \$12030.

Pressing EXE again will repeat the process for the second month. You should find you would owe (the present value at the start of the third month) \$12060.18 (to the nearest cent).

You should realize that something is wrong here. Keep pressing the EXE button to see the amounts you will owe month by month. What do you notice?


```
12000
Ans×(1+7÷1200)-40
12030
12060.175
12090.52602
12121.05409
```



Clearly the amount of your repayment must be greater than the interest charged or you will never pay the loan back.

Lets suppose you pay back \$50 per month. You can investigate this **without** re-entering the whole calculation.

First commit 12000 to the ANS memory (12000 and EXE). You must do this as the machine takes the last number computed and places it in the ANS memory.

```
Ans×(1+7÷1200)-40
12030
12060.175
12090.52602
12121.05409
12000
12000
```

Now press AC/ON to clear the RUN screen and then press the UP ARROW () of the mouse twice to recall the previous calculation we entered (unless you have turned the machine off or changed modes).

Now press  to have the cursor enter the calculation line. You can edit the calculation by pressing  repeatedly until you have the cursor on the 4 in the numeral 40. Then press the DEL(etc) key and then 5.

You should find that you owe \$12020 at the start of the second month. So you are still not making large enough repayments.

Now you can experiment to find the amount you must repay to 'break-even'. Explore this further and share your findings with your colleagues.

Be sure to experiment here with contributions greater than the interest amount.

Can you think of any other situations related to investing where this process is applied?

One such case is a retirement fund where a retiree has, for example, \$100 000 and invests it in an annuity and takes out regular installments (negative contributions if you like).

Such cases as we have seen here are traditionally called '**Present Value**' annuities because they are associated with situations **where you need the money presently – the value of the annuity is in the present**. That is, you **start** with some money in the present and it is eroded to zero eventually.

Activity 2: Generalising the 'Present Value' annuity computation.

A formula exists for computing values associated with Present Value annuities. Its use saves us from repeatedly pressing the equal sign on the calculator.

The derivation of the formula relies on knowledge of geometric series. You might like to research this.

Firstly, define each quantity as follows:

- Let the present value of the annuity, after n compounding periods, be N
- Let the regular contributions (or take outs) be M
- Let the percentage interest rate per compounding period be r (expressed as a decimal)
- Let the number of compounding periods be n

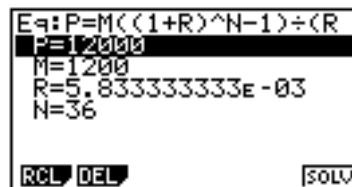
The formula is:

$$N = M \left(\frac{(1+r)^n - 1}{r(1+r)^n} \right)$$

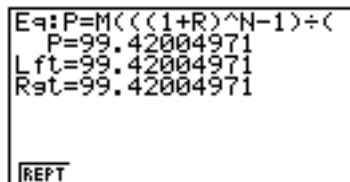
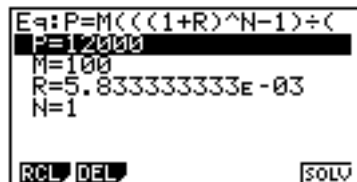
The 9850Gb PLUS can be use to compute the value of N , M , r or n if all but one of the variables is know. In EQUA mode, after choosing SOLV(er) (F3) the formula can be

entered as $P = M \left(\frac{(1+R)^N - 1}{R(1+R)^N} \right)$ using bracket carefully.

Note that N is replaced with P – as sensible change.



So, if $M = 100$, $r = R = \frac{7}{1200}$, and $n = N = 1$, you should find that $P < 100$, can you explain why?



Once the result is given you can continue to use the formula you have entered. Simply press REPT (F1) and you will be prompted for the values of the variables again.

This is a useful feature if you have many of the same computations to do where the values of the variable change often.

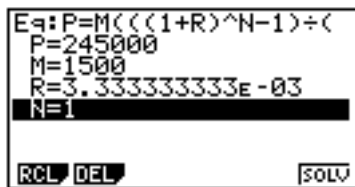
Activity 3: Using the formula more broadly.

Of course we can use the formula for more than just computing the present value of the annuity after n interest periods.

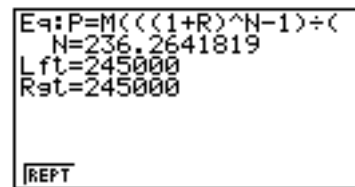
Suppose we wanted to know how long we could remove \$1500 per month from an annuity with present value \$245 000 if the interest paid was 4% compounded monthly.

This would require us to solve the equation $245000 = 1500 \left(\frac{\left(1 + \frac{4}{1200}\right)^n - 1}{\frac{4}{1200} \left(1 + \frac{4}{1200}\right)^n} \right)$ for n .

Proceed as seen below:



and in the blink of an eye



EXERCISES

The purpose of the exercises is to give you an opportunity to do some independent work with simple annuities and to develop fluency with both the mathematical concepts and Casio 9850GB PLUS.

Exercise 1.

Ralph borrows \$15000 to buy a car. The conditions of the loan are that he must make repayments \$350 each month. The interest rate associated with the loan is 12% per annum compounded monthly.

- How long will Ralph take to pay the loan back?
- How much interest does Ralph pay?
- After how many periods will the amount owed first fall below \$10 000

SOLUTIONS

Exercise 1

- After 57 periods (4 years and 9 months) he will owe less than the payment value (\$83.80).
- \$5033.80
- 23 periods.