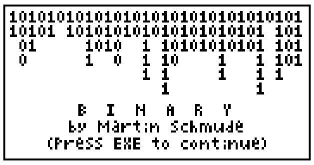
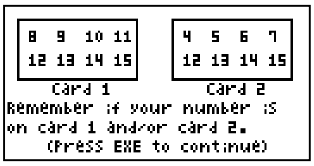
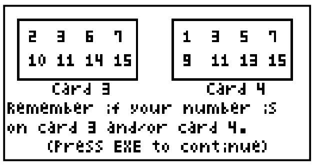
EXTENSION – BINARY NUMBERS

CHALLENGE The start of this unit will be playing the game, BINARY.







Question 1

Write down the patterns that you see for each card.

Question 2

Starting at 1 and moving up to 15, write down the responses to each card (in terms of 1 and 0).

Number	Card 1	Card 2	Card 3	Card 4
1				
2				
3				

Question 3

List the lowest score on each card. See any pattern?

Question 4

Give at least an A4 page explanation of how the trick works. Include a description of the numbers on the card – why are they in that arrangement, can the puzzle be extended to include more numbers (write out what the cards would look like), how is it possible to calculate the chosen number, etc.

THE BEGINNINGS OF BINARY NUMBERS

The binary number system only uses the digits 1 and 0. The first known description of a binary numer system was made somewhere between the 5th and 2nd century BC, but the notion of the number zero had not been discovered. Gottfried Leibiniz (1646-1716) created the first modern version of the binary number system.

Binary digits came into their own with the development of digital technology. In 1975, Bill Gates help found the now massive empire of Microsoft. They were the first to use the binary number system in electronic technology.

In 1998, CEO Bill Gates attempted to patent the numbers one and zero. This meant that Microsoft would have been the only group who could use the digits 1 and 0 for free, and all of Microsoft's rivals would be prohibited from manufacturing or selling products containing zeroes and ones - unless a royalty fee of 10 cents-per-digit-used was paid to the software giant. The cost for a company to pay for every zero and one they use would have sent them bankrupt immediately.

"Because all integers and natural numbers derive from one and zero, Microsoft may, by extension, lay claim to ownership of all mathematics and logic systems, including Euclidean geometry, pulleys and levers, gravity, and the basic Newtonian principles of motion, as well as the concepts of existence and nonexistence," Yale University theoretical mathematics professor J. Edmund Lattimore said. "In other words, pretty much everything."[1]

Fortunately, the patent was denied on the grounds of being anti-competitive and monopolistic. This has allowed other companies to develop new technology and continue to build on others' work.

[1] The Onion, 1998 http://www.theonion.com

KNOW THY OWN SYSTEM

Perhaps the best way to learn the binary number system is to compare it to your own number system and you will notice that the similarities incredible. The number system we use is called the Decimal system. As it suggests with the word 'dec', it uses 10 digits – namely 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Every single number in our system is described using these ten digits. When we write a number, like 3609, the position of the digit determines its value. In this case, we could write 3609 as

$$3609 = (3 \times 1000) + (6 \times 100) + (0 \times 10) + (9 \times 1)$$

$$= (3 \times 10^3) + (6 \times 10^2) + (0 \times 10^1) + (9 \times 10^0)$$

Notice how the value of the numbers goes up powers of ten.

10^3 (1000)	10^2 (100)	10^1 (10)	10^0 (1)
3	6	0	9

Each number simply represents how many groups there are of a certain power of 10. The power is called the index and the number 10 is called the base.

All other number systems work in exactly the same way; it just depends on how many digits they use. The binary system only uses two digits, so every number is expressed using the numerals 0 and 1. For example, the decimal number 19 is represented as 10011 in binary. Once again the position of each number determines its value. Instead of going up in powers of 10's, like in the decimal system, they group in lots of 2's ie. 2^0, 2^1, 2^2, 2^3, … This means that the number 19 must be made up of the numbers 16, 8, 4, 2 and 1.

$$19 = (1 \times 16) + (0 \times 8) + (0 \times 4) + (1 \times 2) + (1 \times 1)$$

$$= (1 \times 2^4) + (0 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$$

This can be shown in a table.

2^4	2^3	2^2	2^1	2^0
(16)	(8)	(4)	(2)	(1)
1	0	0	1	1

From now on, the binary number system can be referred to as base 2, and the decimal number can be referred to as base 10.

THEORY

The number system in base *n*, uses digits 0, 1, 2, …, *n*-1. Any number *a* in base *n* is denoted by a_n.

QUESTION

Explain why it wouldn't make any sense in the binary number system to use digits more than 1, for example, to have 2×2^3?

EXERCISES

1. Write the following numbers in binary.

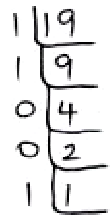
(a) 7_{10} (b) 20_{10} (c) 64_{10} (d) 35_{10} (e) 40_{10}

2. Write the following binary numbers in decimal.

(a) 11_2 (b) 101_2 (c) 1101_2 (d) 100011_2 (e) 101101_2

EASIER WAY PERHAPS?

In the exercises, you probably worked out the answers by working with the powers of two. There is an algorithm that you can use to convert numbers to a certain base. You simply divide the number repeatedly by the base and the quotient goes beneath the number and the remainders make up the equivalent number. Let's see it in an example. We will find the binary equivalent of 19_{10}.



The binary number is then read from the bottom up, so $19_{10} = 10011_2$.

EXERCISES

1. Using the algorithm, convert the following base 10 numbers to binary.

   (a) 11 (b) 26 (c) 35 (d) 40 (e) 55

2. Using the algorithm, convert the following base 2 numbers to decimal.

   (a) 101 (b) 100101 (c) 10111 (d) 111100 (f) 10101

3. Explain why this method works.

Might want to have a look at the CHALLENGE again.

LOGIC GATES & HALL LIGHTS

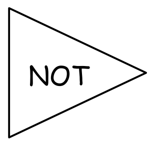
Have you ever wondered how technology does all the amazing things it does? How does my Playstation work? How does my computer know how to process input? We already know that all electronics use binary digits but what does that mean?

The foundation of electronics is remarkably simple. In electronics, the binary digits are actually pulses of electricity. A 1 represents a pulse and a 0 represents no pulse – that is ON or OFF, YES or NO, TRUE or FALSE, OPEN or CLOSED. These pulses are used in three basic pieces of technology. They are called Logic Gates and the most simple gates are call AND, NOT and OR. Every single piece of technology can be broken down into these 3 simple pieces – anything from your digital watch to landing a spacecraft on Mars.

The simplest way to explain how these gates work is to place them in the context of real life examples.

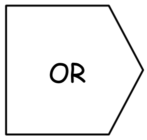
Firstly is the NOT gate. NOT is a word that we learn the meaning of at a very early age because our parents are saying it to us constantly – 'Do NOT touch that', 'do NOT hurt your brother'. It means do the opposite of what you're doing. Imagine your parents say to you, 'if you do NOT make a mess, you can play your computer'. This can be summed up in what is called a Truth Table.

Input (make a mess)	Output (play the computer)
0	1
1	0



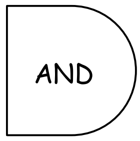
Secondly is the OR gate. 'If you play rugby OR netball, you will become fit', can be summed with a truth table.

Input (play rugby)	Input (play netball)	Output (become fit)
0	0	0
0	1	1
1	0	1
1	1	1

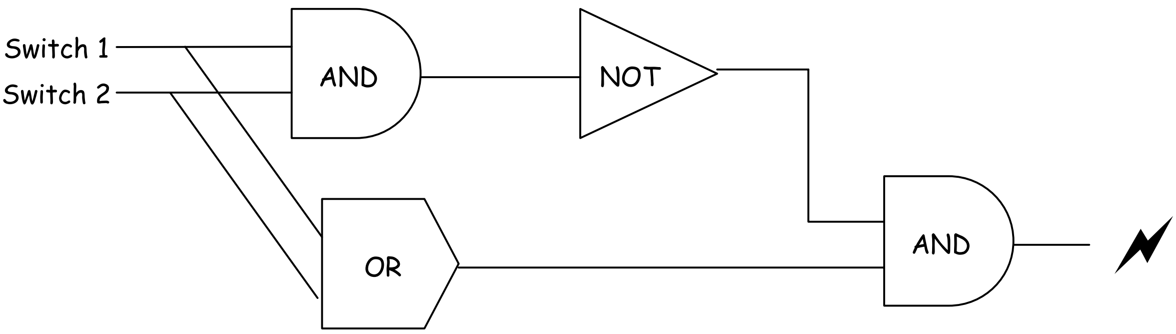


Lastly is the AND gate. 'If you drink AND drive, you're a bloody idiot' can be summed up with the truth table.

Input (drink)	Input (drive)	Output (bloody idiot)
0	0	0
0	1	0
1	0	0
1	1	1



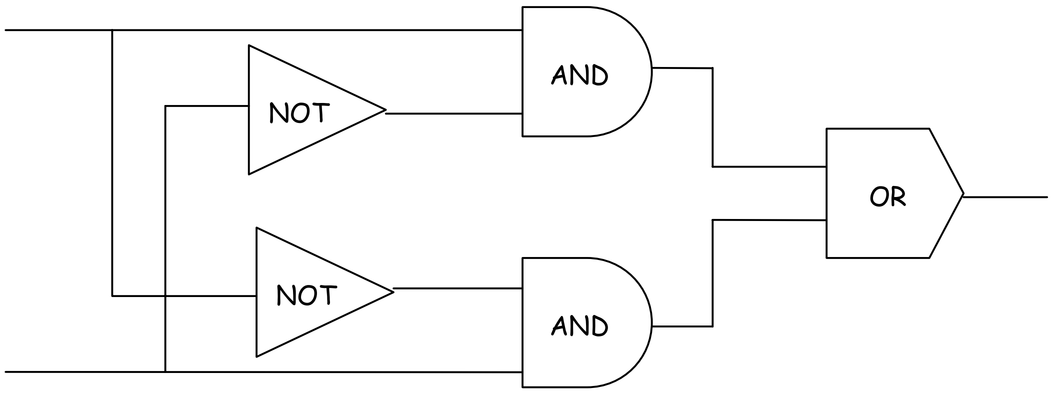
Do you have a light in your home that has a switch at one end of the hall and another at the other end? If you do, you'll have the following circuit in your house. What you'll notice about this circuit is that if both light switches are off, then the light is off; if one switch is off but the other is on, then the light is on; but if both switches are on, then the light is off!



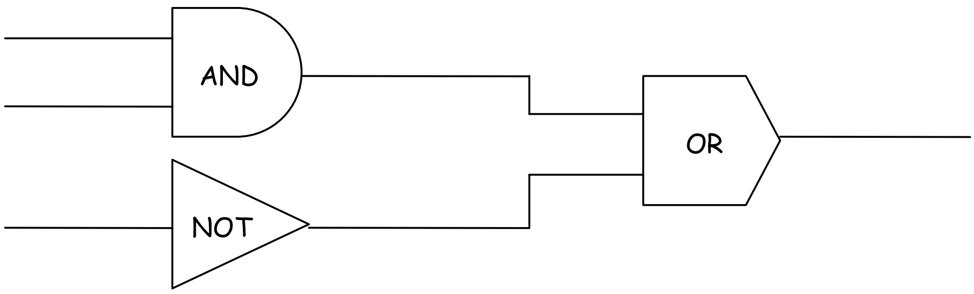
QUESTION The above circuit is called an Exclusive OR gate (XOR gate). Complete a truth table for the circuit above and based on the results, can you think of a reason why it is called this?

EXERCISES Complete a truth table for each circuit below.

1.



2.



3.