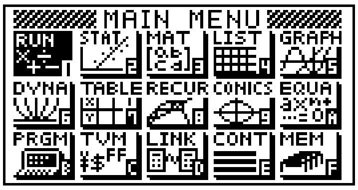
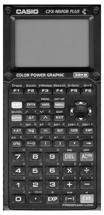
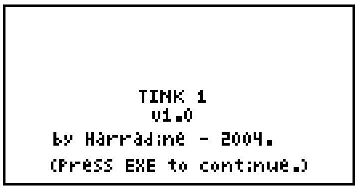
Families – from a mathematical perspective!

A unique learning experience – Student Learning Booklet







A product of the Noel Baker Centre for School Mathematics

WIP (Work in progress)

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Kate O’Donnell (MLC, Sydney) for her recent thinking about the delivery of the approach enclosed.

Marty Schmude (St. Joey’s, NSW) for being a guinea pig.

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Section **Page**

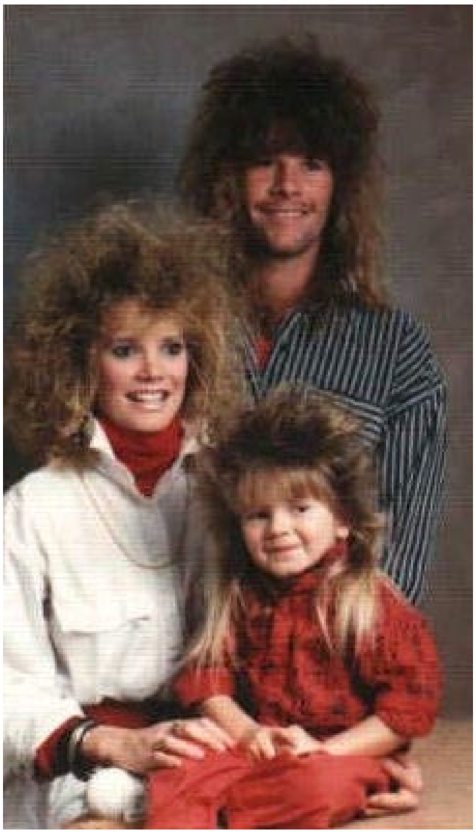
1. A look at some human families (and some non-human families) and their features.

For each of the following families identify the features that set them apart from many other families.

Task 1

1) Suggest a ***surname*** that would suit this family.

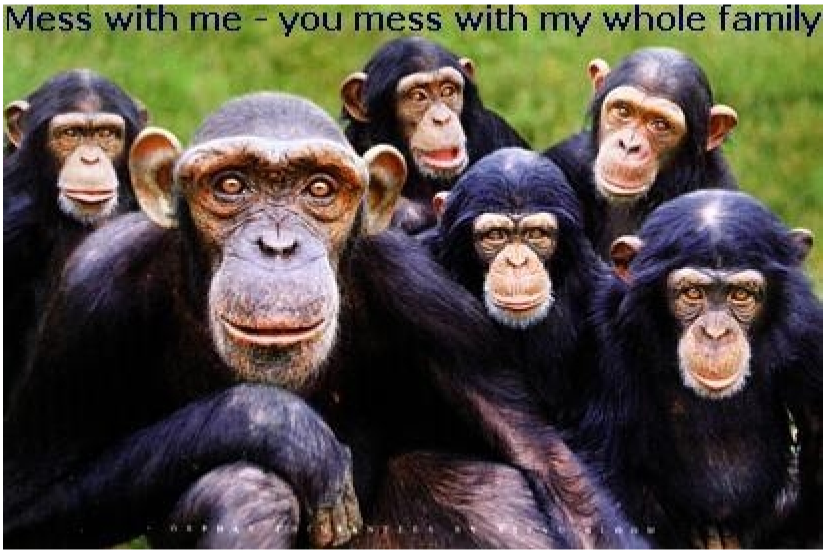
2) Suggest a ***first name*** that would suit each ***individual*** member of this family.



Family One

Family Two

Family Three



Family Four

This poor little fellow has lost his family. What are his ‘special’ features? What might his surname be? What might his first name be?

Summary

So, in our world **families** are a **collection of individuals** that have a **surname** and a **first** name.

2. Representations of human families.

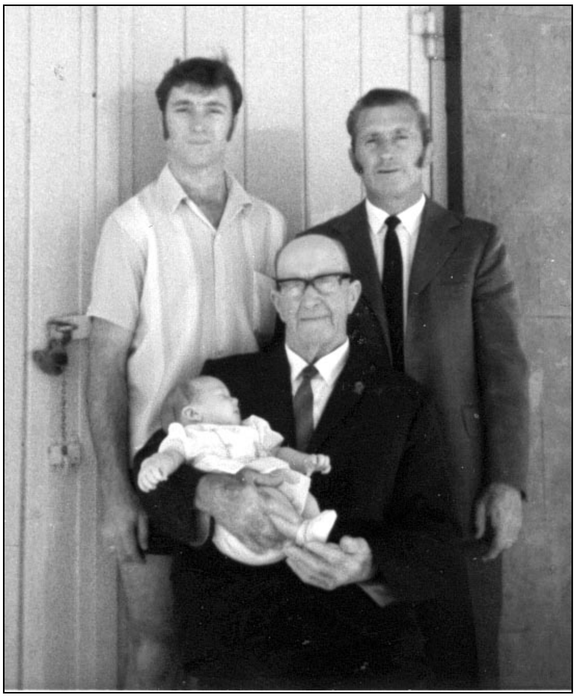
In the previous section we looked at a number of photographs. Each **photograph** was a **representation** of some of the members of each family.

1) Are there other ways of representing families? List as many as you can think of.

2) Does the representation include one, a few or all members of the family?

3) Bring in some different representations of your family and display them in your classroom.

3. Generations



The concept of *generations* in human families can be displayed in a picture like the one seen above.

This picture shows four generations of one family – Great Grandfather, Grandfather, Father and child.

1. If you define yourself as being *generation zero*, List the people in your family in generations …,-3,-2,-1,0,1,2,3,.……

2. How many generations can you go back? When will you be able to move forward?

4. Number families.

The concept of family can be applied to many different situations. In geometry we often talk of *families of shapes*: the family of three sided plane figures, the family of convex polygons and so on.

By applying the concept of family to numbers in a rather creative manner we can start to learn a great deal about the way mathematicians think about mathematics. Consider the following situations.

4.1 Make a rectangle.

Make a rectangle with perimeter 28 cm. Once each student has made one rectangle, place them all on a table, take a photograph of this *family* and display it in your classroom (or better still display all the family members for real).

These are all members of ***one*** family. How could we name each member (What is the same? What is different?)? Apply the concepts of:

- *generation*
- first name and
- surname.

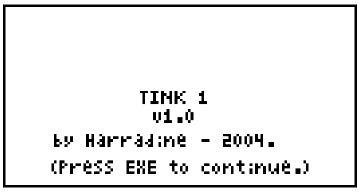
4.2 Make a box.

Make a box from and A-4 piece of paper by cutting *squares* from the corners. Once each student has made one box, place them all on a table, take a photograph of this *family* and display it in your classroom (or better still display all the family members for real).

These are all members of ***one*** family. How could we name each member (What is the same? What is different?)? Apply the concepts of:

- *generation*
- first name and
- surname.

4.3 TINK - I am tinking of a number!



Using your graphic calculator run the ADD called 'TINK1' once. Go to PRGM mode, select TINK1 and press EXE.

Have 7 shots and record your results in the following table.

Number TINK multiplied by.		***Number TINK was thinking of.***	***Operator used by TINK (+ or -)***	***Number TINK added or subtracted.***		***The number TINK 'gets'.***
	×				=	
	×				=	
	×				=	
	×				=	
	×				=	
	×				=	
	×				=	

Every time TINK is run it produces a different ***family*** – you have **some** members of **one** family displayed above. Each member of your class will have produced a **different** family.

1) Display four different families on the whiteboard. How could we name each member in each family? How can we name the families so the name acknowledges the difference in the families, but actually describes **every** member of the family. Apply the concepts of:
   - *generation*
   - first name,
   - surname and
   - finger print.

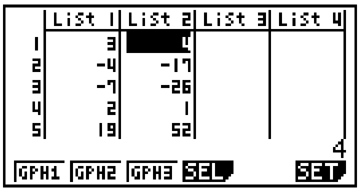
2) For each family state what each family member has in common.

3) How many members does this family have?

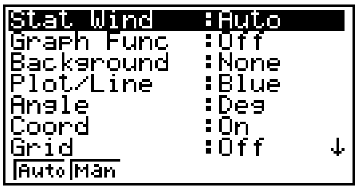
<u>A space for some important notes.</u>

When TINK was running, it stored the number that was being thought of and the 'number got' after the operations had been performed in the STAT mode of your calculator, in Lists 1 and 2 respectively.

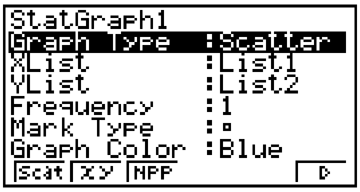
Enter STAT mode and you should find the data. That seen opposite is data from when I played TINK.



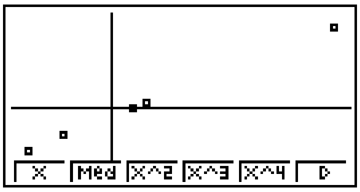
Press SHIFT and then MENU to enter the *brain* of the STAT mode. Here you can ensure that the settings are correct for us to proceed. TINK will have turned of the axes off and so we need to turn them back on. Be sure that the settings are as follows:



Press EXIT and then use SET to set up StatGraph 1 to display this data. Have it set as shown opposite.



Press EXIT and then use GPH 1 (F1) to produce a graph – my graph looks pretty interesting, does yours?



4) Compare your graph to those of others – what do you notice?

 (WIP)

5) Consider the following family of numbers.

Number TINK multiplied by.		*Number TINK was thinking of.*	*Operator used by TINK (+ or -)*	*Number TINK added or subtracted.*		*The number TINK 'gets'.*
5	×	-4	-	2	=	-22
5	×	6	-	2	=	28
5	×	1	-	2	=	3
5	×	9	-	2	=	43
5	×	11	-	2	=	53
5	×	-6	-	2	=	-32
5	×	2	-	2	=	8

a) suggest a surname for this family
b) what generation is the individual with first name –32?
c) Who is generation 12?
d) Who generation is 2?
e) Supposing we can only have integer generations, is 57 a member of this family?

6) Consider the following family of numbers.

Number TINK multiplied by.		*Number TINK was thinking of.*	*Operator used by TINK (+ or -)*	*Number TINK added or subtracted.*		*The number TINK 'gets'.*
6	×	8	+	5	=	53
6	×	-2	+	5	=	-7
6	×	5	+	5	=	
6	×	11	+	5	=	
6	×	-7	+	5	=	-37
6	×	1	+	5	=	
6	×	2	+	5	=	17

a) suggest a surname for this family
b) Complete the table.
c) what generation is the individual with first name 53?
d) Who is generation 12?
e) Who generation is 2?
f) Supposing we can only have integer generations, is 59 a member of this family?
g) Note that 53 is a member of this family and the family from question 5. How does 53 differ in each family?

7) Find two families to which 28 belongs and within which it is the same generation.

5. Correct mathematical terms.

One example of a TINK output can be seen below.

Number TINK multiplied by.		*Number TINK was thinking of.*	*Operator used by TINK (+ or -)*	*Number TINK added or subtracted.*		*The number TINK 'gets'.*
5	×	-4	-	2	=	-22
5	×	6	-	2	=	28
5	×	1	-	2	=	3
5	×	9	-	2	=	43
5	×	11	-	2	=	53
5	×	-6	-	2	=	-32
5	×	2	-	2	=	8

Things in common:

- multiplication by five
- subtraction of 2

Things different:

- the number being multiplied by five
- the result (or the number TINK gets)

We can say that this family has the *surname*

- Five times some number subtract two or
- $5 \times x - 2$ or
- $5x - 2$

where x is a letter that takes the place of the **many numbers** that TINK could have thought of. In the last section we learned we could think of this as the *generation* of the family member.

How many generations are possible?

When a letter takes the place of many different numbers it is called a ***pro-numeral***. (pro– meaning *substituting for*). Pro-numerals are normally presented in *italic* font. A pro-numeral may represent a physical quantity or nothing at all.

The letter chosen as a pro-numeral is often x, but can be anything the user likes. In many cases two or more different pro-numerals are required.

The branch of mathematics that deals with the study and use of pro-numerals is called ***Algebra***.

When a pro-numeral is written next to a number, like $5x$ it is assumed it means $5 \times x$ (just mathematicians being lazy!).

A collection of numbers, operations and pro-numerals, like $5x - 2$ is called an ***algebraic expression***. We learned in the last section we could think of this as the *surname* of the family. It describes **all** members of the family

An algebraic expression can have many **values** (family members). We learned in the last section we could think of this as the *first name* of the family member.

Can you see how an expression, like $5x - 2$, captures all of the members of the family? If not discuss this with others.

In the following exercises, perform all calculations *mentally*.

1) Study the following number family.

5	×	3	-	4
5	×	-7	-	4
5	×	2	-	4
5	×	1	-	4
5	×	-4	-	4
5	×	11	-	4
5	×	7	-	4

a) Write down an algebraic expression that describes each member of this family.
b) Find the value of the expression when the pro-numeral you chose takes the value of 7.
c) Find the value of the expression when the pro-numeral you chose takes the value of 9.
d) Find the value of the expression when the pro-numeral you chose takes the value of –12.
e) Determine the value of the pro-numeral when the algebraic expression has value 19.
f) Determine the value of the pro-numeral when the algebraic expression has value 54.
g) Determine the value of the pro-numeral when the algebraic expression has value 25.
h) How many members are in this family?

2) Study the following number family.

9	×	3	+	7
9	×	-7	+	7
9	×	2	+	7
9	×	1	+	7
9	×	-4	+	7
9	×	11	+	7
9	×	7	+	7

a) Write down an algebraic expression that describes each member of this family.
b) Find the value of the expression when the pro-numeral you chose takes the value of 14.
c) Find the value of the expression when the pro-numeral you chose takes the value of 7.
d) Find the value of the expression when the pro-numeral you chose takes the value of –15.
e) Determine the value of the pro-numeral when the algebraic expression has value 115.
f) Determine the value of the pro-numeral when the algebraic expression has value 61.
g) Determine the value of the pro-numeral when the algebraic expression has value 26.
h) How many members are in this family?

3) Study the following number family.

13	×	3	+	7	+	9	×	3	-	2
13	×	-7	+	7	+	9	×	-7	-	2
13	×	2	+	7	+	9	×	2	-	2
13	×	1	+	7	+	9	×	1	-	2
13	×	-4	+	7	+	9	×	-4	-	2
13	×	11	+	7	+	9	×	11	-	2
13	×	7	+	7	+	9	×	7	-	2

a) Write down an algebraic expression that describes each member of this family.
b) Find the value of the expression when the pro-numeral you chose takes the value of 14.
c) Find the value of the expression when the pro-numeral you chose takes the value of 7.
d) Find the value of the expression when the pro-numeral you chose takes the value of –15.
e) Determine the value of the pro-numeral when the algebraic expression has value 115.
f) Determine the value of the pro-numeral when the algebraic expression has value 61.

g) Determine the value of the pro-numeral when the algebraic expression has value 26.

h) How many members are in this family?

6. Representing families of numbers – in a number of different ways.

Just as we can have multiple representations of human families, we can also have multiple representations of the sorts of number families we have been working with.

Let us consider the following family. Note that we have put some order into the *generations* we are looking at, it makes sense to do this.

2	×	1	-	3
2	×	2	-	3
2	×	3	-	3
2	×	4	-	3
2	×	5	-	3
2	×	6	-	3
2	×	7	-	3

Representation 1
You can see this above. This is not a very common way to represent a family, but a very important one for your understanding.

Representation 2 – An algebraic expression

$$2x - 3$$

Be sure you understand how this *captures* **all** the family members.

Representation 3 – A table of values
But what values? The essence of the family can be captured by table in which the pro-numeral value (generation) and the member's value (first name) are shown. Something like the following:

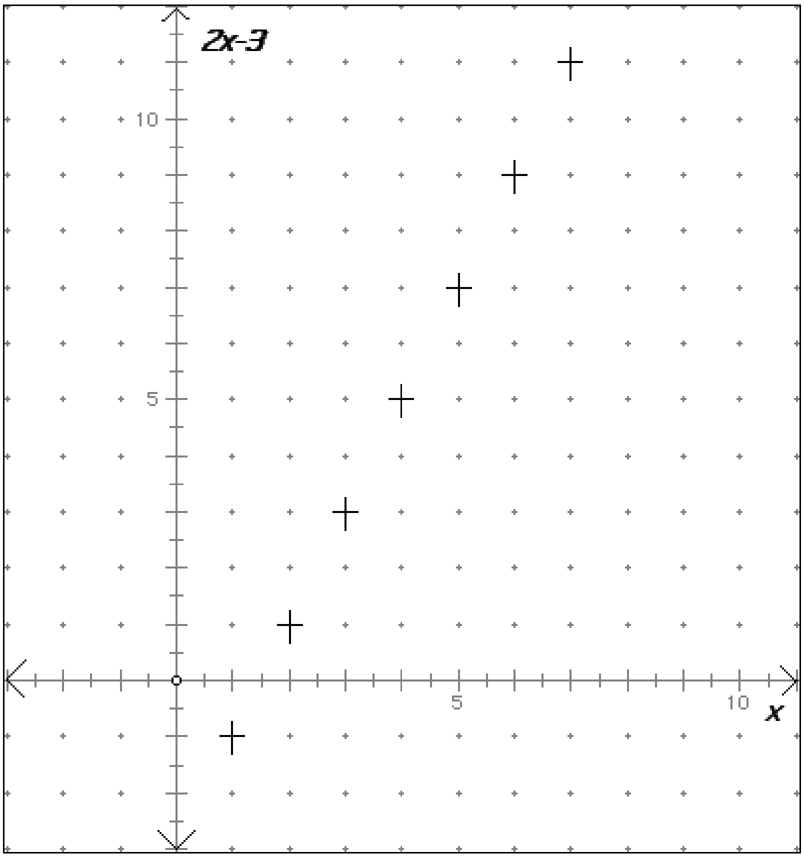
x	$2x$-3
1	-1
2	1
3	3
4	5
5	7
6	9
7	11
8	13

***Representation 4 – A graphical display* (or the Kodak happy snap).**

A family like this can also be represented on a graph where the horizontal axis is a measure of the generation (x) and the vertical axis is a measure of the value of the expression using that particular pro-numeral. The graph is plotted in a similar way that a street directory is used.

The value of the family member is normally assigned the vertical axis as its value might go 'up and down' and hence it is seen to do just that on the graph. The generation concept fits the left to right idea of the graphs horizontal axis. You will see families that go up and down in value as generations pass a little later.

Note that both axes are incremented in the same way – we call this a *square* set up. You will see later that a square set up is important in many cases.



 (WIP)

1) Provide three different representations for each of the following families (**in addition to the one that is given**).

a)

-5	$\times$	1	+	3
-5	$\times$	2	+	3
-5	$\times$	3	+	3
-5	$\times$	4	+	3
-5	$\times$	5	+	3
-5	$\times$	6	+	3
-5	$\times$	7	+	3

b)

5	$\times$	-3	+	4
5	$\times$	-2	+	4
5	$\times$	-1	+	4
5	$\times$	0	+	4
5	$\times$	1	+	4
5	$\times$	2	+	4
5	$\times$	3	+	4

c) $4x - 1$ for x having integer values from –4 to 4.

d)

x	??
1	-3
2	0
3	3
4	6
5	9
6	12
7	15
8	18

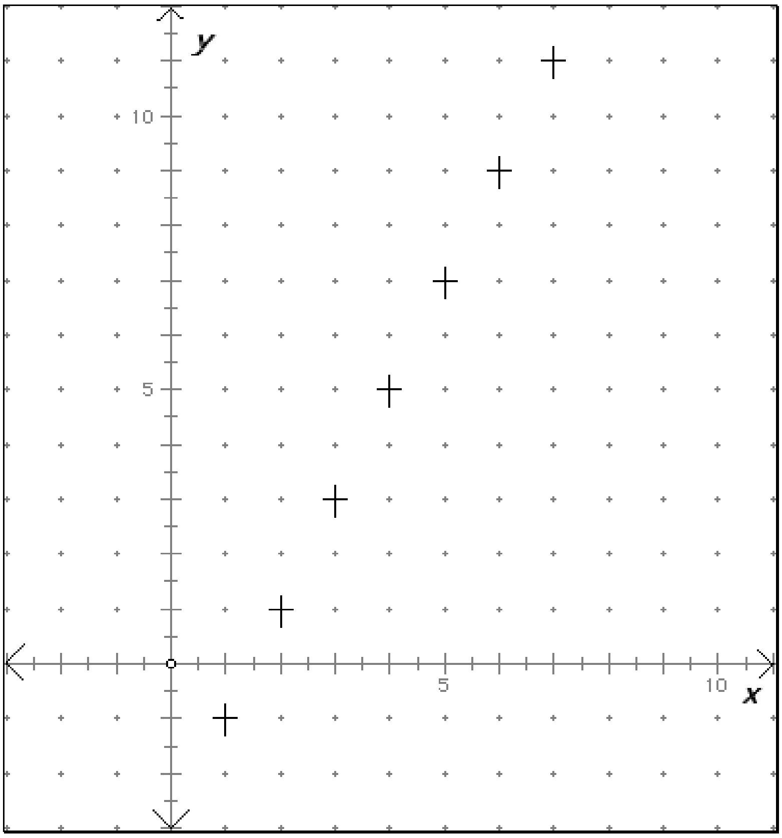
Often, we introduce a **second pro-numeral** to take the place of the value of the family member (first name). In doing this we introduce something called an algebraic equation (because it includes and equal sign). It looks as follows:

$$y = 2x - 3$$

and so the table of values looks as follows (no real difference):

x	y
1	-1
2	1
3	3
4	5
5	7
6	9
7	11
8	13

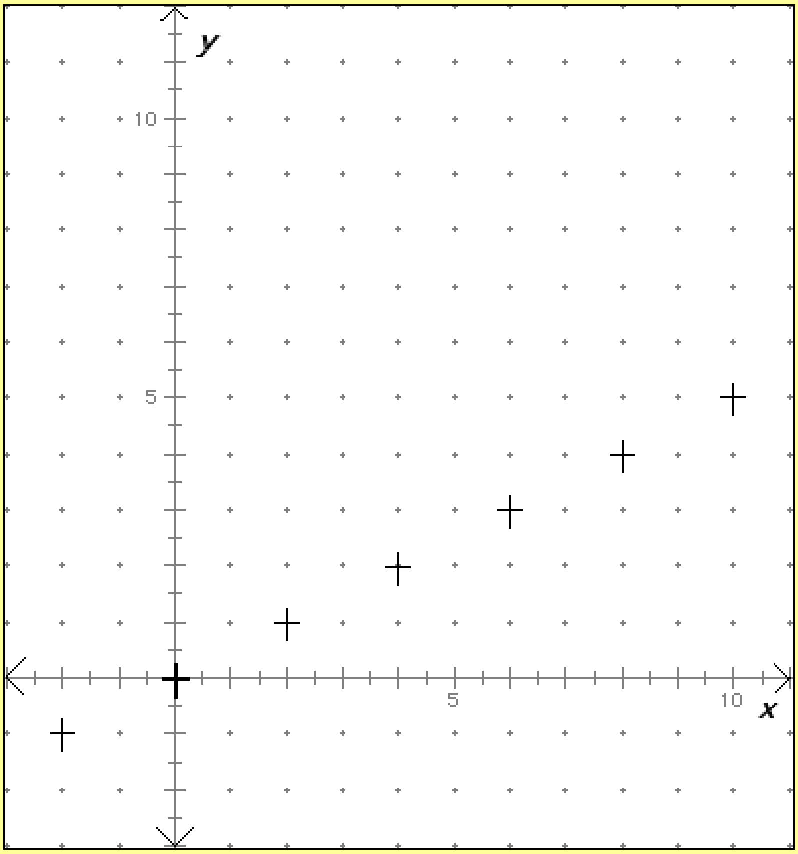
The graph also has only one small change:



2) Provide three different representations for the family known as $y = -3x + 5$

3) Provide three different representations for the family known as $y = 2x - 3$

4) Provide three different representations for the family representing the following graph.

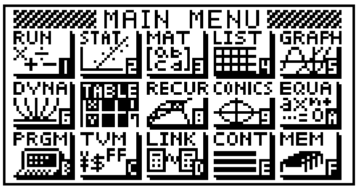


7. Rapid representations – the gcalc!

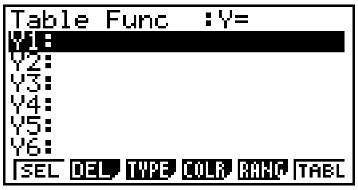
Your graphics calculator has the ability to produce multiple displays of number families very rapidly.

At this level you need to know the **algebraic representation** and it can produce a table and graph (In a later unit you will see it can also determine the algebraic representation!)

With the calculator turned on and the main menu visible, use the arrow key to highlight the TABLE menu (shown opposite).

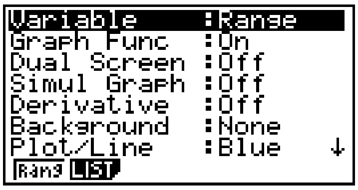


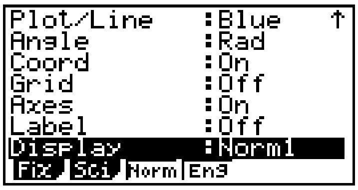
Then press the blue EXE key (alternatively, simply press 7). The following screen will result:



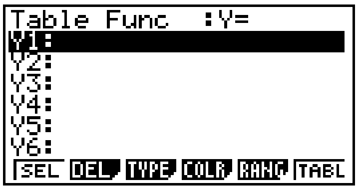
Now press SHIFT and then MENU to reveal the 'setup screen' for this module. Set each option as shown right. Use your arrow keys to scroll down to the 'hidden options'.

You can experiment with different settings later.

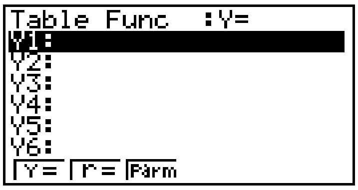




Press the black EXIT key and you will return to this screen:



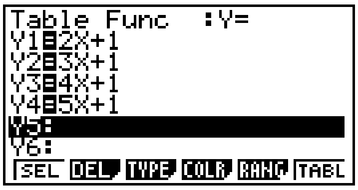
If your screen does not have Y1 and so on, on the left, access TYPE (F3) and access Y= (F1)



We will now enter four different families using their algebraic representation.

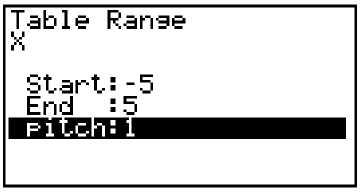
Press `2` followed by the `X, θ, T` key, then `+` then `1`, to enter $y = 2x + 1$

Carry out a similar procedure to enter the others seen opposite.



We now need to tell the calculator the x values (generations) that we wish it to use to calculate y values.

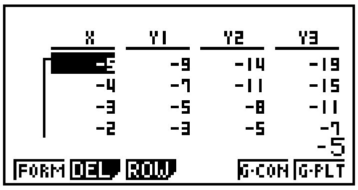
Access `RANGE (F5)` and set the values of start end and pitch to those shown opposite. Pitch is the incremental jumps that you wish to have in x.



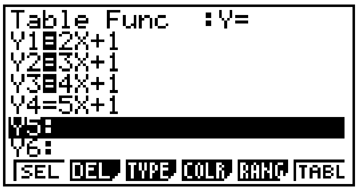
Press the `EXIT` key when you are done.

Access `TABL (F6)` to produce the table.

You can navigate the table using the arrow keys.

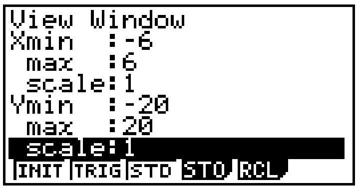


Should you not want to have all the families appear in the graph, press `EXIT`, use the arrow keys to select the rule(s) you do not want (for this exercise select `Y4`) and access `SEL (F1)`. The = sign will no longer be surrounded with a dark rectangle and is said to be NOT SELECTED.



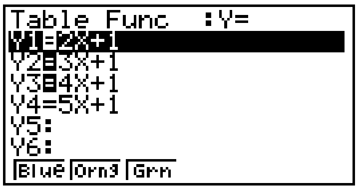
If we wish to draw a graph that illustrates each of these link rules we must first set the scale of the axes.

Press SHIFT and accessV-WIN (F3) and the View Window settings will appear.

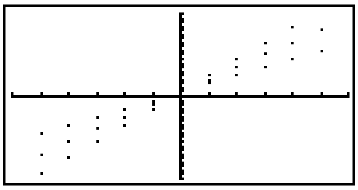


Set the values as shown opposite. Think about why we have set them this way.
Press the `EXIT` key

We can now set each of the link rules to have different coloured graphs. Select the first rule, access `COLR (F4)` and then choose the colour you want by pressing the appropriate key, either `F1`, `F2` or `F3`. Simply arrow down to select the other rules and choose a colour. Press the `EXIT` key.

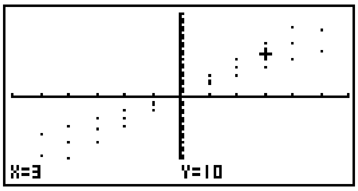


Now access TABL and then G-PLT to produce the required graph.

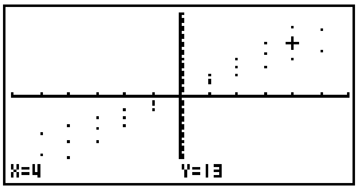


Not seen in colour here unfortunately.

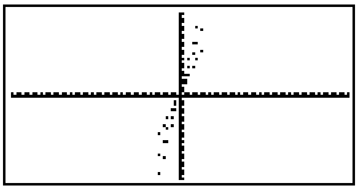
Above the `F1` key you will see an option called `Trace`. Press `SHIFT` and then `Trace` and then use the arrow keys to trace along the points, using the up and down arrow will allow you to change between which family you are tracing along.



Note that the distance moved in the *x direction* is 1 unit, but 3 units are traversed in the *y* direction. This is clearly not the case in 'real' distance terms. This is because we are using a *non-square* set up. Note we have a rectangular screen and the way we set the min and max values ensures it was not square.



The easiest way to get a square set up is to use a Zoom feature. Press SHIFT and then ZOOM, then press DES (F6) and then SQR (F2).



Sometimes the square look is not so nice, but it is important to know when you are and are not set up square.

1) Go back to the previous section and produce displays for each family in that section using your calculator. Check that the displays agree with your 'paper and pen' work.

8. The rectangle and the box revisited – and some special family members.

Recall that in Section 4 you made a family of rectangles and a family of boxes. You should have some photos of these on the wall to remind you.

The rectangles
All the rectangles had the same perimeter – 28cm

If I want you to make a family member with one pair of opposite sides of length 6cm, how long is the other pair?

If I want you to make a family member with one pair of opposite sides of length 7cm, how long is the other pair?

If I want you to make a family member with one pair of opposite sides of length 8cm, how long is the other pair?

How did you determine the other side lengths?

The family of rectangles contains some related families in it. It contains:

- two families of **lengths** and
- one family of **areas**.

Lets look at ***some*** of the members of the family of **one pair** of opposite side lengths.

1,2,3,4,5,6

What is the smallest and largest family member?

If we take a look at the rectangle with one pair of opposite sides 2cm long, the other sides are pre-determined by the fact that the perimeter must be 28 – what must the sides be?

If we take a look at the rectangle with one pair of opposite sides 8cm long, the other sides are pre-determined by the fact that the perimeter must be 28 – what must the sides be?

In table form we could represent the families as follows:

Side lengths – pair 1	Side lengths – pair 2	Area
1	13	13
2	12	24
3	11	33

If we let one family of opposite side lengths be x cm (can you see how this captures all of the possibilities, what would be the family name of the other side lengths?

What would be the name of the family of areas?

Display this family in a number of different ways.

Which of the rectangles has the biggest area in this family?
How do you know?
Do you know beyond absolutely any doubt?

Discuss how this all fits together with your class.

<u>A space for some important notes.</u>

Which of the boxes in the box family has the largest volume?

What are the families within the families? How can they help you to answer the question above?

Aim to make an algebraic expression for the family of volumes.

A space for some important notes.

9. The KISS principle – like and non-like terms.

Before we proceed we need to introduce a new piece of language.

Earlier we defined a collection of numbers, operations and pro-numerals, like $5x - 2$ to be an ***algebraic expression***.

We will now define a pro-numeral when combined with a multiplication operation and a number to be an algebraic ***term***.

Some examples of algebraic terms are:

$7x, 3y, 4a, x, ab.$

9.1 Like terms

With this done let us push on. Recall the family seen in Section 5 Question 3:

13	×	3	+	7	+	9	×	3	-	2
13	×	-7	+	7	+	9	×	-7	-	2
13	×	2	+	7	+	9	×	2	-	2
13	×	1	+	7	+	9	×	1	-	2
13	×	-4	+	7	+	9	×	-4	-	2
13	×	11	+	7	+	9	×	11	-	2
13	×	7	+	7	+	9	×	7	-	2

It is likely that you suggested a suitable name for this family to be:

$$13x + 7 + 9x - 2$$

You would, of course, be correct. But, consider the following logic.

In words, the first member show could be described as:
<u>13 lots of 3</u> add 7 add <u>9 lots of 3</u> subtract 2, which could be considered as

<u>22 lots of 3</u> add 5.

Similar logic could be employed to each member as it is always 'so many lots of the same number' and so we could reach the following:

13	×	3	+	7	+	9	×	3	-	2	=	**22**	×	**3**	**+**	**5**
13	×	-7	+	7	+	9	×	-7	-	2	=	**22**	×	**-7**	**+**	**5**
13	×	2	+	7	+	9	×	2	-	2	=	**22**	×	**2**	**+**	**5**
13	×	1	+	7	+	9	×	1	-	2	=	**22**	×	**1**	**+**	**5**
13	×	-4	+	7	+	9	×	-4	-	2	=	**22**	×	**-4**	**+**	**5**
13	×	11	+	7	+	9	×	11	-	2	=	**22**	×	**11**	**+**	**5**
13	×	7	+	7	+	9	×	7	-	2	=	**22**	×	**7**	**+**	**5**
13	×	*x*	+	7	+	9	×	*x*	-	2	=	**22**	×	***x***	**+**	**5**

In this case, because the 'lots of' are lots of the same number, then we can use the same pro-numeral to describe all of the family members.

In such cases, we can add together the number of 'lots of' and simplify the algebraic expression that is used to represent the whole family. Obviously the numbers (non-algebraic parts) can be added or subtracted as required.

So clearly:

$$13x+7+9x-2 = 22x+5$$

This process is called ***adding like terms***. The word like refers to the fact that the 'lots of' are the same sort of lots!

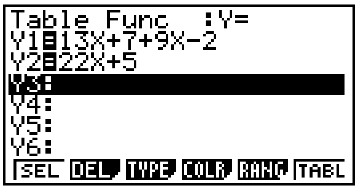
We can illustrate the fact these are the same for some values of x (generation) by checking the left hand side (LHS) of the equals sign has the same value as the RHS when a value is substituted for the pro-numeral. This is illustrated in the table above, or as seen below.

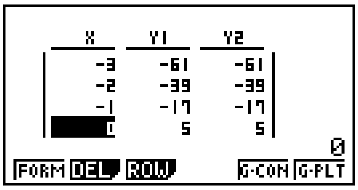
Say we let x be 14, then

$$\begin{aligned} \text{LHS} &= 13x+7+9x-2 \\ &= 13\times14 + 7 + 9\times14 - 2 \\ &= 313 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= 22\times14+5 \\ &= 313 \end{aligned}$$

Alternatively use your graphic calculator as follows:





1) Try this one.

-5	×	3	-	1	+	9	×	3	-	2
-5	×	-7	-	1	+	9	×	-7	-	2
-5	×	2	-	1	+	9	×	2	-	2
-5	×	1	-	1	+	9	×	1	-	2
-5	×	-4	-	1	+	9	×	-4	-	2
-5	×	11	-	1	+	9	×	11	-	2
-5	×	7	-	1	+	9	×	7	-	2

Write down two algebraic expressions that describe all members of the family, one that is simpler than the other.
Use your calculator as a back up to be sure your two expressions are equivalent.

9.2 Non-like terms

Study the family below carefully. What do you notice?

3	×	3	+	7	+	9	×	1	-	2
3	×	-7	+	7	+	9	×	-3	-	2
3	×	2	+	7	+	9	×	3	-	2
3	×	1	+	7	+	9	×	4	-	2
3	×	-4	+	7	+	9	×	-9	-	2
3	×	11	+	7	+	9	×	11	-	2
3	×	7	+	7	+	9	×	12	-	2

Suggest an algebraic expression that describes all members of this family.
Can this be simplified? Explain why or why not.

Some room for some important notes.

Terms that contain different pro-numerals are said to be ***non-like terms***. Such terms cannot be combined (or added)

Finally, consider this family.

3	×	3	×	7	+	5	+	6	×	3	×	7	-	2
3	×	-7	×	2	+	5	+	6	×	-7	×	2	-	2
3	×	2	×	-4	+	5	+	6	×	2	×	-4	-	2
3	×	1	×	5	+	5	+	6	×	1	×	5	-	2
3	×	-4	×	8	+	5	+	6	×	-4	×	8	-	2
3	×	11	×	2	+	5	+	6	×	11	×	2	-	2
3	×	7	×	-3	+	5	+	6	×	7	×	-3	-	2

Suggest two algebraic expressions that describe all members of this family, one simpler than the other. You should be able to check your expressions are correct!

 (WIP)

Some room for some important notes.

10. Running some laps – honing your skills.

Time to find a source of practice questions on:

- Adding and subtracting like terms.
- Multiplying by constants and multiplying algebraic terms.
- Maybe division also.
- Substituting and finding the value of an expression (first name) for a given value of the pro-numeral(s) (generation)
- Also set the task where students write a page about algebra, pro-numerals, what they do and why they are so powerful.
- Some questions which are not likely to be found in a text-book. For example:

1 a) Write down the first 10 consecutive odd positive integers

b) Write down 1 algebraic expression that summarises all of these 10 and all other odd positive integers that exist.

c) Write down a 'different' set of ten consecutive odd positive integers.

d) Write down 1 algebraic expression that summarises all of these 10 and all other odd positive integers that exist.

2 a) Write down the first 10 consecutive even positive integers

b) Write down 1 algebraic expression that summarises all of these 10 and all other even positive integers that exist.

c) Write down a 'different' set of ten consecutive even positive integers.

d) Write down 1 algebraic expression that summarises all of these 10 and all other even positive integers that exist.

3. Write down all pairs of consecutive integers.

4. Write down every 2-digit number, and then every 3 digit number – in less than 30 seconds.

5. What would 2 lots of $7x+4$ be?

6. **What would 7 lots of 8*x*+5 be?**

7. **What would 7 lots of 8*x*-5 be?**

8. **Break 7*x*+14 into 2 equal parts**

9. **Break 15*x*+25 into 5 equal parts.**

10. **Break 15*x*+24 into 5 equal parts.**

Note that in questions 5 – 10, the students can use a g-calc to check their results. This should be encouraged as we are planting some serious seeds here for later work.

11. Using simple algebra to do powerful things – proof, beyond any doubt.

It is hoped that you have a strong feel that algebra (the use of pro-numerals) is all about writing, in a very simple manner, a statement that covers an infinite number of individual cases for some situation.

In fact, algebra is about making what seems impossible, (working with the infinite), possible.

Consider the following:

> Each student in the class is to choose **two consecutive** numbers and then add them together. Compare your results, what do they all have in common? Make a conjecture about the sum of **every pair** of consecutive numbers. Can you prove this conjecture to be true – beyond any doubt? **Can your graphic calculator help?**

Some room for some important notes.

1. Prove that the sum of any two consecutive odd numbers is an even number (try a few individual cases first to check it out.

2. Prove that the sum of any two consecutive even numbers is an even number (try a few individual cases first to check it out.

3. Prove that the sum of a multiples of 5 and the equivalent generation multiple of 3 is a multiple of 8, but that it is not so for the sum of any multiple and 5 and multiple of 3.

4. Go to http://www.flashpsychic.com
   Enjoy the experience – Is it really reading your mind? Or is it just a bit mathematical beauty – can algebra help you explain it?

12. Some more magic?

12.1 MREADER

Using your graphic calculator run the program called MREADER. Enjoy the experience! Compare what happened to you to what happened to your classmates – use a whiteboard to display the occurrences.

Can you explain this – or is it really magic? (Move on if you can't and come back later)

12.2 Some slightly simpler magic.

* Choose a Number
* Add 5
* Double the result
* Subtract 4
* Divide the result by 2
* Subtract the number you started with
* Is the result is 3?

Pick 4 different starting numbers and show that the result is 3 in each case. Does this prove the result will be three no matter what the starting number – i.e. in every possible case?

Some room for some important notes.

12.3 Yet more magic.

Experiment with the following bits of 'magic'. Prove that the resulting number is always the same

Magic 1
Choose a number. Add 3. Multiply by 2. Add 4. Divide by 2. Subtract the number you started with. The result is ??.

Magic2
Choose a number. Double it. Add 9. Add the number you started with. Divide by 3. Add 4. Subtract the number you started with. The result is ??.

12.4 Creating your own magic.

Create a trick of your own. You must prove, using algebra, that your trick will always work – would hate to let the world down!

Program it into your graphic calculator to perform the trick and amuse your friends – how does mine work? Remember some of the 'smarts' that were used in Flash Mind Reader to make it even more magical.

If you need some help programming your calculator, visit,

http://www.casioed.net.au/lmaterials/basicprog.htm (a lovely introductory document by Marty Schmude (ST. Joesph's College – NSW) and

http://www.casioed.net.au/lmaterials/mgems.html (look for Ottoway on programming).

You can also look into the code of the programs you have used in these units for ideas.