

Holding Pen Investigation

A note to teachers:

This is a 'quirky algebraic modelling' investigation. That is to say a 'quirky' problem, rather than 'real world' problem, generates the model.

It is important to keep in mind the purposes of running algebraic modelling investigations such as this given the solution has negligible 'real world' importance.

The purposes of this activity include:

- To engage students mathematically
- To give students experience of applying algebra to a problem
- To efficiently generate a table of values:
 - To enable students to 'see the problem' represented in the numbers
 - To expose students to finding a solution within a table
- To efficiently generate graph/s for the problem:
 - To enable students to 'see the problem' represented in the graph/s
 - To find a graphical solution

(And possibly the most important purpose of this activity)

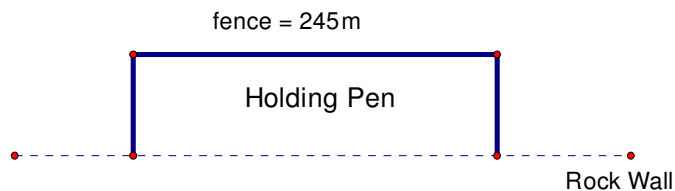
- To create numerous opportunities for the teacher to ask 'Working Mathematically' questions during the activity, thereby giving students many opportunities to think mathematically, make mental links and gain conceptual understanding.

NOTE: If you are new to TABLE and GRAPH modes you may find it beneficial to first work through the worksheet 'Self-Guided_9860_TABLE-GRAPH'.

The Mathematical Farmer and the Holding Pen

An Algebraic Modelling exercise

You are a farmer. You own some land which has a long, straight, 300 metre rock wall running through it. You want to construct a rectangular holding pen for sheep using the rock wall as one side of the holding pen.



You have 245 metres of fencing. You are a farmer who loves mathematical challenges and so you set yourself the task of making the pen so that its area is 90% as large as the largest rectangular pen you could possibly make using the rock wall and the 245m of fencing.

What are the dimensions of the pen that you make?

Submit your findings on paper, explaining how you arrived at your solution.

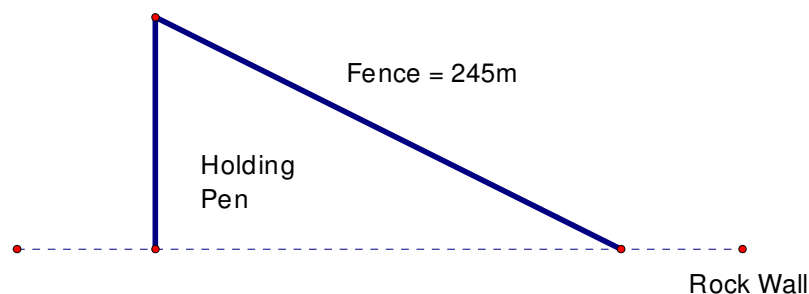
Suggested steps to help solve this problem:

- 1) Draw a diagram of the pen, calling the side adjacent to the wall 'x'
- 2) Make an equation for Area using Y for area.
- 3) Enter the equation into TABLE mode, using X, θ, T for X. Generate a table. (What do the numbers tell you?)
- 4) Refer to the table values to set up the axes in V-Window. (Turn Coordinates and Axes On in SET UP)
- 5) Go to GRAPH mode and sketch the graph
- 6) Find the maximum area possible for the rectangle. Find the area of the holding pen you will construct.
- 7) Use RUN mode to calculate 90% of the maximum area to find the area of the holding pen.
- 8) Use Trace to find the approximate value/s for X for the rectangular holding pen (or pens)
- 9) Use the X-CAL function (G-Solv, F6, X-CAL) to find the 'exact' X value/s for the holding pen area/s found in Q8)
- 10) Calculate the dimensions of the holding pen/s.

Challenge exercise:

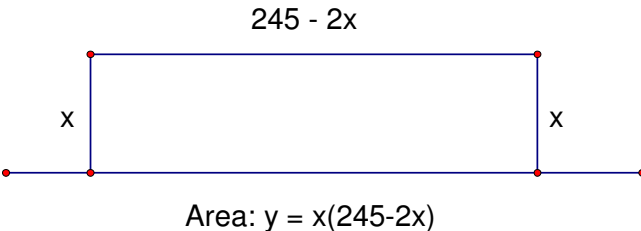

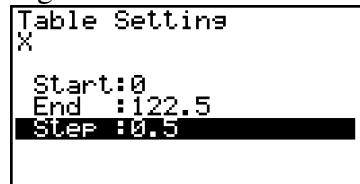
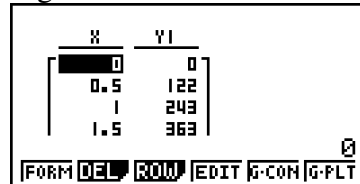
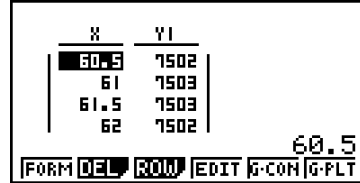
Bill, your neighbour, also a farmer with a mathematical interest, and also, coincidentally, with some land with a long, straight rock wall running through it said he would build a holding pen using the same length of fencing but his would be in the shape of a right angled triangle. He said he would end up with a larger area than you. You replied that he wouldn't even manage an area 90% as big as your area (yours being 90% as big as the maximum possible rectangular area!!)

Who was correct?



Submit your findings on paper, explaining how you arrived at your solution.
See if you can display the solution to this challenge graphically.

Holding Pen Investigation – Solutions and Instructions

Instructions	Screenshots
<p>Q1 & Q2</p> <div style="text-align: center;">  <p style="text-align: center;">Area: $y = x(245-2x)$</p> </div> <p>Q3 Press MENU, go to TABLE and press EXE. Delete any pre-existing formulae using F2 and F1. Enter the equation as per Fig1.</p> <p>Press SET (F5) and enter the settings as per Fig2 (End X value = 122.5 is half the length of the 245m of fencing)</p> <p>Press EXIT then TABL (F6) (Fig3) Use the down arrow to scroll through the table.</p> <p>It appears the maximum area is just over 7500 sq m (Fig4)</p>	<p>Fig1</p>  <p>Fig2</p>  <p>Fig3</p>  <p>Fig4</p> 

Q4 **Press V-Window (SHIFT F3)** and enter the min and max values for X and Y as per Fig5 (ignore scale and dot)

NOTE: The Y min and max values (-2000 rather than zero and 10 000 rather than 7503) enable the entire graph to display when being traced.

Press EXIT. Turn **Coordinates and Axes On** in SET UP (SHIFT MENU), arrow up and use F1 to turn them On as per Fig6

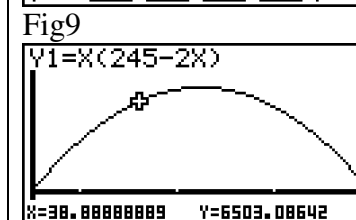
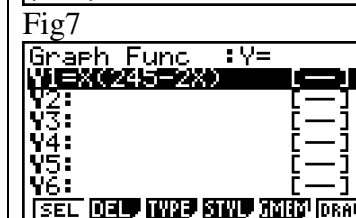
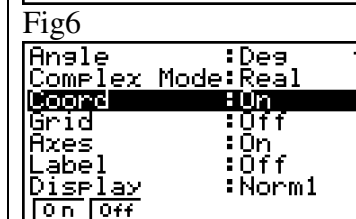
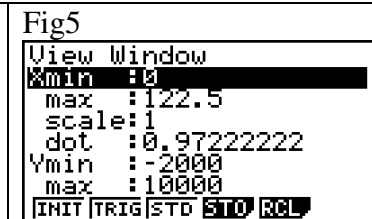
Press EXIT

Q5 **Go to GRAPH (press MENU then 5)** (Fig7)

Note that Y1 is not selected (Fig7). Press F1 to select the graph (Fig8)

Press DRAW (F6). Turn the Trace on with **SHIFT F3** (Fig9)

Use left and right arrow to find an approximate value for the maximum area.



Q6) To find the maximum area go to **G-Solv (SHIFT F5)** (Fig10)

Press MAX (F2) (Fig11)

We can see the maximum value for area is 7503.125 sq m

Q7) However, you are to build a holding pen with an area 90% as big as this.


Go to **RUN mode (MENU, then 1)**

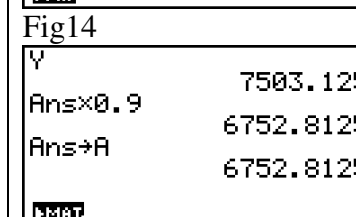
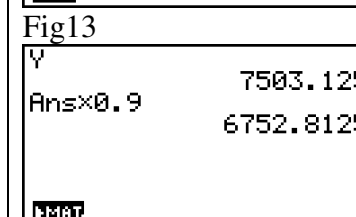
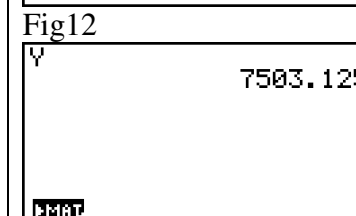
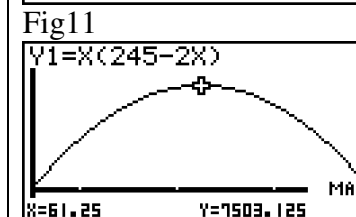
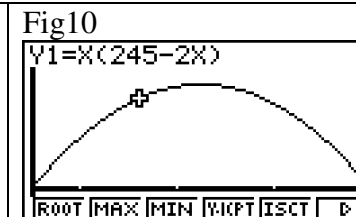
(NOTE: The RUN screenshots display the Linear (not Math) Input Mode. To set RUN to Linear: **Enter RUN (MENU 1), go to SET UP (SHIFT MENU), press Linear (F2).**)

Recall the value for Y (**press ALPHA Y EXE**) (Fig12)

Find 90% of this (**press x 0.9 EXE**) (Fig13)

We can see the area of the holding pen will be 6752.8 sq m (Fig13)

IMPORTANT: We might want to use this number later. To store it in memory **press  then ALPHA A (any letter will do) Press EXE** (Fig14)



To find the dimensions of the holding pen:

Q8) This can be done approximately (but not exactly) by tracing (**SHIFT F1**).
Return to GRAPH mode (**MENU, then 5**). **Press DRAW (F6) then Trace (SHIFT F1)**
Use left and right arrows to find the X value when Y is close to 6752.8 sq m.

We can see in Fig16 that there are 2 solutions for X which are around 42 and 81. This means there are 2 different rectangles which can be used for the holding pen.

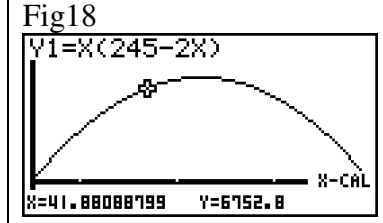
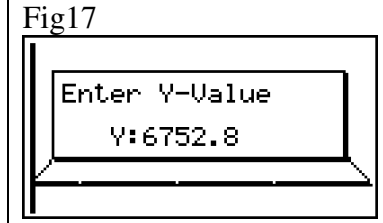
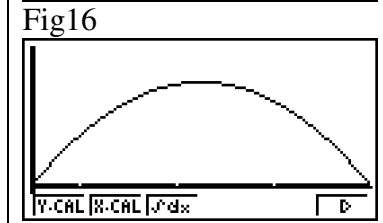
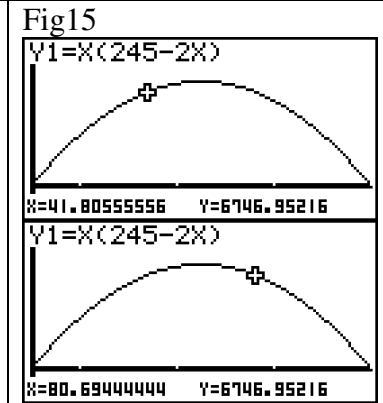
Q9) To find the 'exact' values for X we need to perform a X-Calculation (X-CAL)
To do this go to **G-Solv (SHIFT F5) then press $\boxed{\text{D}}$ (F6)** (Fig16)

Press X-CAL (F2)

A screen will appear for you to enter the Y value as per Fig17. Enter 6752.8

Press EXE (Fig18)

We can see that one value for X is 41.9



To find the other value for X press the right arrow (Fig19)
 We can see the other value for X is 80.6

We can see from the original diagram in Q1 that 1 side of the rectangle is X and the other side is (245-2X)

Therefore to calculate the adjacent side of each triangle we can use RUN mode and substitute each X value into the formula.
 Go to RUN (MENU, then 1) and enter the substitutions as per Fig20.

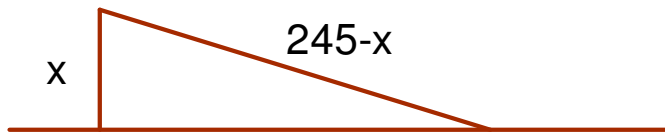
Q10) The dimensions of the 2 possible pens are:

<u>X side</u>	<u>Adjacent side</u>	<u>Rectangular shape</u>
41.9	161.2	a 'long and thin' rectangle
80.6	83.8	almost a square

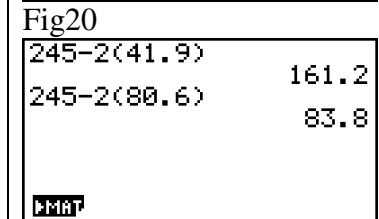
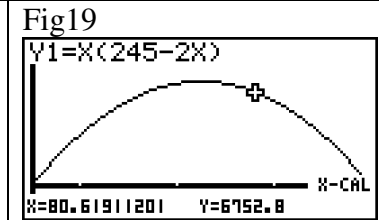
Challenge Exercise

OVERVIEW: What we need to do is find the maximum area of the neighbour's holding pen and see if it is more than 90% of the area of your holding pen.

A diagram of the neighbour's pen is drawn below.



The length of the remaining (base) side is found using pythagoras' theorem: $\text{Base} = \sqrt{(245 - x)^2 - x^2}$



Therefore the formula for the triangle's area is: $Y = 0.5X \times \sqrt{(245 - X)^2 - X^2}$

Go to the Graph Func screen (**MENU, then 5**) and **deselect Y1 (F1)**

Enter the triangle area formula into Y2, using X, θ, T for X, as per Fig21

Press EXE (Fig22)

We can use the same axes settings as for the first part of this investigation (Fig5)

Press DRAW (F6) (Fig23)

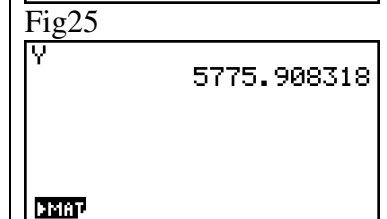
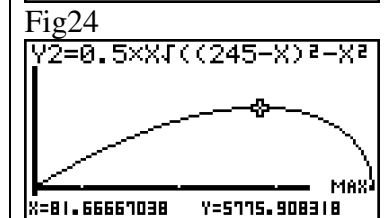
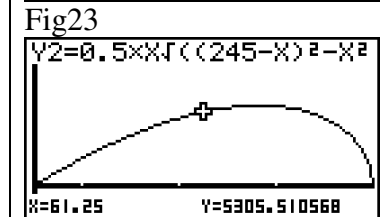
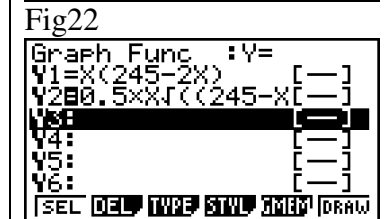
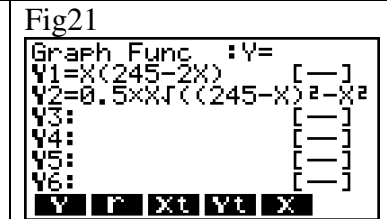
To find the maximum area **go to G-Solv (SHIFT F5)** then **press MAX (F2)** (Fig24)

We can see the maximum area is 5775.9

We will take this value to RUN mode and also recall the value for the area of your holding pen (which was stored on A)

Go to **RUN (MENU, then 1)**

Press **ALPHA Y EXE** (Fig25)



Recall the area of your holding pen (**press ALPHA A EXE**) (Fig26)

We can see that the neighbour's pen (5775.9) is smaller than yours (6752.8), but is it more or less than 90% the size of yours?

Press x 0.9 EXE (Fig27)

We can see that 90% of your holding pen area (6077.5) is still more than the area of your neighbour's (5775.9). Therefore you were correct!

A very neat way of seeing this graphically:

Firstly return to the **Graph Func screen (MENU 5) and select Y1 (F1)** (Fig28)

Press DRAW (Fig29)

We can see graphically that your neighbour's pen is less in area than yours but the graph does not display the winner of the argument because it does not indicate whether your neighbour's is more or less than 90% of your holding pen area.

Press EXIT to return to the Graph Func screen, **enter 6752.8 into Y3 and 6077.5 into Y4** (Fig30)

This will add 2 lines to the graph screen representing '90% of your pen's maximum area' and '90% of your final pen's area'.

Fig26

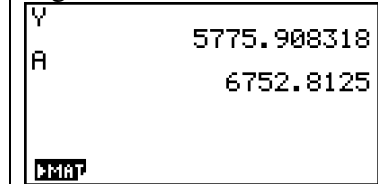


Fig27

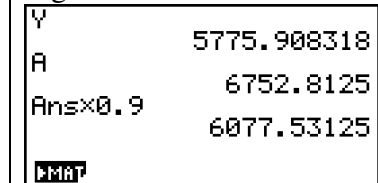


Fig28

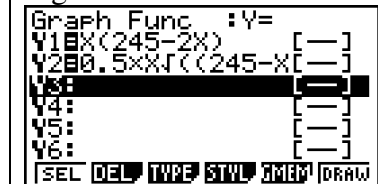


Fig29

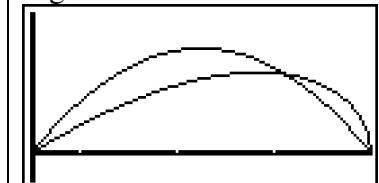
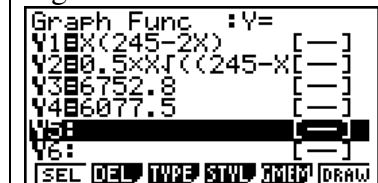


Fig30



Press DRAW (F6) (Fig31)

By **pressing Trace (SHIFT F1)** and arrowing down we can see the '90% of your final pen's area' line has a value of 6077.5 (Fig32)

Graphically we can see that you won the argument!

