NSW GENERAL MATHEMATICS **Preliminary Course**

Mathematical Interactions

Financial Mathematics

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Mathematical Interactions:
Financial Mathematics

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About this book

Calculators are too often regarded as devices to produce answers to numerical questions. However, a graphics calculator like the Casio CFX-9850GB PLUS is much more than a tool for producing answers. It is a tool for exploring mathematical ideas, and we have written this book to offer some suggestions of how to make good use of it when learning about and using the basic mathematical concepts that underpin finance.

We assume that you will read this book with the calculator by your side, and use it as you read. Unlike some mathematics books, in which there are many exercises of various kinds to complete, this one contains only a few 'interactions' and even fewer 'investigations'. The learning journey that we have in mind for this book assumes that you will complete *all* the interactions, rather than only some. The investigations will give you a chance to do some exploring of your own.

We also assume that you will work through this book with a companion: someone to compare your observations and thoughts with; someone to help you if you get stuck; someone to talk to about your mathematical journey. Learning mathematics is not meant to be a lonely affair; we expect you to interact with mathematics, your calculator and other people on your journey.

Throughout the book, there are some calculator instructions, written in a different font (`like this`). These will help you to get started, but we do not regard them as a complete manual, and expect that you will already be a little familiar with the calculator and will also use our *Getting Started* book, the *User's Guide* and other sources to develop your calculator skills.

Financial Mathematics is one of the topics in General Mathematics, mainly because it is an important application of mathematics to the real world and fundamental to all of our lives. Much of our life is centred around controlling, or attempting to control, our own and sometimes others finances. You will learn how to write and use programs as a ready reckoner for every day situations and build on your knowledge of patterning to reinforce your understanding of interest structures.

Our thanks go to Deb Woodard-Knight for trialing and commenting on this book.

We hope that you enjoy your journey!

Barry Kissane

Anthony Harradine

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Checking the pay clerk

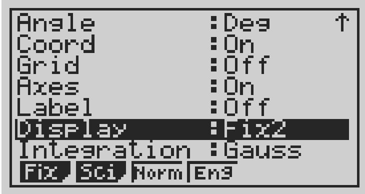
Tamra (pictured opposite) is seventeen years old and has just started to work as a sandwich artist. She is classed as a* weekly employee *(another way of saying permanent) but works part time.

As a part time employee she is paid, by the week, to work less than 38 *ordinary* hours per week. Ordinary hours are defined in a rather complex manner that we have not chosen to include here. Should you have a job, you should study the remuneration and conditions award very carefully. Some weeks she is called on to work *overtime* hours. Overtime hours are any hours not classed as ordinary hours and weekly employees are paid at time and a half for overtime and receive one paid hour off for each hour of overtime worked.

Tamra's *gross wage* per week (before tax) depends on three things, the amount of ordinary hours she works, the number of overtime hours she works and the pay rate for ordinary hours. Ordinary hours are paid at a rate of $8.41. Of course, Tamra has to pay tax on her earnings, so her net wage (after tax) is reduced. The amount of tax paid per week is dependent on the amount earned per week and is considered as an instalment of the tax that will be due at the year's end. The weekly tax instalment is most simply determined using the *Income Tax Instalments Weekly Rates* document produced by the Australian Taxation Office. We acquired our copy from a news agent; it was free. One page of this document together with an example of how to use it have been reproduced on page 30 and 31.

In Tamra's case, checking that the pay clerk has not made any errors is fairly simple. It really only requires four basic calculations and looking up a value from a table. Let us say, for example, that last week Tamra worked 22 ordinary hours and 5 overtime hours. The tax instalment included here assumes that Tamra is claiming the tax free threshold and is not entitled to leave loading (and has supplied a tax file number). Her pay could be calculated as follows (study these calculations carefully and consider the rounding involved).

pay for ordinary hours:	$8.41 × 22	$185.02
pay for overtime hours:	(1.5 × $8.41) × 5	$63.08
gross wage		$248.10
tax	from table on p. 30	$29.60
net wage		**$218.50**



Enter RUN mode then enter SET UP (SHIFT then MENU) to set all calculations to be displayed correct to 2 decimal places as is appropriate when dealing with money. Arrow down to Display and use Fix (F1) and then 2 (F3).

Interaction A

1. Determine Tamra's net wage for a week if she worked 28 ordinary hours and 7 overtime hours.
2. Determine Tamra's net wage for a week if she worked 18 ordinary hours and 5 overtime hours.
3. When Tamra turns eighteen the ordinary hour rate she is paid increases to $9.81. Re-calculate the answers to questions 1 and 2 using this rate of pay.

You can see from **Interaction A** that the calculations involved in determining wages are rather repetitious. We could use our calculator to write a short program to automate these calculations. This would serve the purpose of enabling Tamra to *quickly and accurately* check the pay clerk each week. The process will deepen your understanding of the process as well!

Computer (which is what the calculator is) programs take many forms. Generally a program will ask the user for some information and then carry out some calculations. We have chosen for our program to ask three questions.

Question 1: How many ordinary hours were worked?
Question 2: How many overtime hours were worked?

And once the gross wage has been calculated,

Question 3: What is the tax instalment?

There are to be four calculations in our program. They can be summarised as follows:

pay for ordinary hours:	8.41 × **number of ordinary hours**
pay for overtime hours:	1.5 × 8.41 × **number of overtime hours**
gross wage	**Pay for ordinary hours** + **Pay for overtime hours**
net wage	**gross wage** − **tax**

When writing programs it is more efficient to let a quantity be represented by a letter (a pronumeral as in algebra). It saves typing and it is the way a computer thinks. Let us define our variables (always try to choose letters that make sense – it is hard sometimes, as it is here):

let R = the number of ordinary hours worked
let V = the number of overtime hours worked
let A = pay for ordinary hours
let B = pay for overtime hours
let G = gross wage
let T = amount of tax
let N = the net wage

Hence our calculations become :

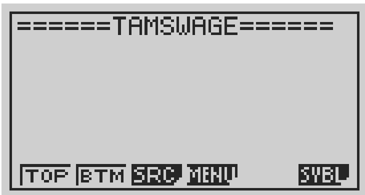
pay for ordinary hours:	**A**	=	$8.41 \times \mathbf{R}$
pay for overtime hours:	**B**	=	$1.5 \times 8.41 \times \mathbf{V}$
gross wage	**G**	=	$\mathbf{A} + \mathbf{B}$
net wage	**N**	=	$\mathbf{G} - \mathbf{T}$

These algebraic formulas use much less space than their worded version above. You will learn more about algebraic formulas in the *Algebraic Modelling* book.

Now let us see how we can form a program using the questions and the calculations. The program code is as follows. The text to the right explains the code.

`"ORD HRS"?→R↵`	[asks user for number of ordinary hours worked]
`"OVER HRS"?→V↵`	[asks user for number of overtime hours worked]
`8.41×R→A↵`	[calculates A]
`1.5×8.41×V→B↵`	[calculates B]
`A+B→G↵`	[calculates G]
`"ORD PAY":A◢`	[displays A on the screen]
`"OVER PAY":B◢`	[displays B on the screen]
`"GROSS WAGE":G◢`	[displays G on the screen]
`"TAX INSTALMENT"?→T↵`	[asks user for the correct tax instalment]
`G−T→N↵`	[calculates N]
`"NET WAGE":N↵`	[displays N on the screen]

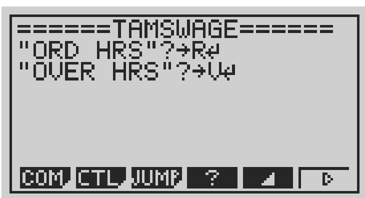
Enter the PRGM module of your calculator. When the Main Menu is visible either use the arrow keys to highlight it and press EXE or simply press the B (log) key.



Use NEW (F3) to start a new program. You must first give the program a name. At this point the keys with pink letters above them act as character keys only – we say the Alpha Lock is on. Type the name TAMSWAGE and then EXE. You can use up to eight characters.

You are now ready to enter the lines of code for the program where the cursor is flashing.

Press SHIFT then ALPHA to turn the alpha lock on. Note the symbols at the base of the screen. Enter " (F2) and then type ORD HRS and enter another " (F2). (The SPACE is achieved using the decimal point key). Then use PRGM (SHIFT then VARS) to access the ? symbol. Enter this symbol (F4)and then press the *arrow* key above the AC/ON key and then enter R (ALPHA then 6) and then press EXE to complete the first line of code. Note that after pressing EXE, a bent arrow (↵) appears at the end of the line to denote a line of code has ended.

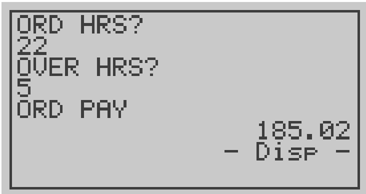


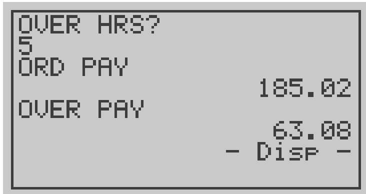
Now we need to access the quotation symbol again. Press EXIT to return to the home screen for the PRGM module. Use SYMBL (F6) to access the quotation symbol (last time we just used Alpha Lock, either works) enter " (F2). Now press SHIFT and then ALPHA to turn the Alpha Lock on and type OVER HRS and then enter another " (F2). Then use PRGM (SHIFT then VARS) to access the ?. Enter the ? (F4) and then press the → key above the AC/ON key and then press ALPHA and 2 to enter V. Press EXE to complete the second line of code.

You should now be able to complete the rest of the code. However, note that the ◢ symbol (the output command) is used at the end of the lines where we want the calculator to tell us the result of a calculation before continuing. It can be accessed in a similar way to the ? symbol (SHIFT then VARS then F5). The : symbol is also accessed in this way, but you will need to use the continuation key (F6) to reveal it. Complete the code entry.

When you are finished, press EXIT twice to return to the screen seen opposite and, with the name of the program highlighted, press EXE to run the program. When a ? is displayed the program is asking you to input some information. Type the information and press EXE.

When – Disp – is displayed on the screen simply press EXE to continue.





Interaction B

1. Use your program to calculate the answers to Interaction A, questions 1 and 2. This will check that your program is working correctly.
2. Determine Tamra's net wage for a week if she worked 21 ordinary hours and 4 overtime hours.
3. Can Tamra use this program when she turns eighteen? Explain why or why not.
4. Enter the PRGM mode and highlight TAMSWAGE. Use EDIT (F2) to display the code. Make the changes to the code so that the program will work when Tamra turns eighteen. Now repeat question 3 from Interaction A to check that your editing has been successful.

It is possible to alter the program to be more flexible. It means asking the user more questions. Look through this program on the next page, enter it into your calculator (or get one person to enter it and then link your calculators with the SB-62 cable and transfer the progam – see our *Getting Started* book for information) as TWAGE2 and try it out. You may like to edit TAMSWAGE rather than enter all this as it is very similar to TAMSWAGE. Use SHIFT then DEL to activate the *insert* facility. Characters are then inserted between others rather than over writing the characters already there. If you write many programs you would be better to use the FA122 or FA123 software available for either Windows or MacIntosh computers. You can then copy and paste code with ease.

```
"ORD HR RATE"?→X↵
"ORD HRS"?→R↵
"OVER HRS"?→V↵
X×R→A↵
1.5×X×V→B↵
A+B→G↵
"ORD PAY":A◢
"OVER PAY":B◢
"GROSS WAGE":G◢
"TAX INSTALMENT"?→T↵
"NET WAGE":N↵
```

Investigation

Determine the wage details of someone in your family. You may be able to find someone with a rather complex pay structure. Some people are paid a commission, double time or may have deductions other than tax (eg. union fees, medical benefits) and so on.

Write a program that will allow this person to check the pay clerk each week.

Describing an additive pattern (linear)

The balance of an investment account grows over time. There is usually a pattern of some kind to such growth. It is important to understand the connection between simple number patterns, their algebraic description and the way an investment account may grow.

A simple type of number pattern is formed by *adding* a constant amount to the previous value in the pattern. We are only going to consider patterns that start with zero. For example:

$$0, \quad 0+5, \quad 0+5+5, \quad 0+5+5+5, \quad 0+5+5+5+5, \quad \ldots$$

or

$$0, \quad 5, \quad 10, \quad 15, \quad 20, \quad \ldots$$

This is an example of a *linear pattern*.

To help describe this pattern we assign a number to each term in the pattern so we know that number's *position* in the pattern. For ease of description we have chosen to start with position zero.

This is best displayed in a table. Let the position number be p and the value of the number in the pattern be v.

position (p)	0	1	2	3	4
value (v)	0	5	10	15	20

Note that as well as the *adding by 5* pattern that can be seen in a horizontal manner, a vertical pattern also exists. The value (v) is simply *5 times* the position number (p). This allows us to very simply describe this pattern with the following rule:

$$v = 5p, \text{ where } p \text{ is an integer}$$

Note that the multiplier, 5 in this case, is the same as the increase in V each time.

Is this a coincidence?

Interaction C

1. Consider the following number pattern

position (p)	0	1	2	3	4
value (v)	0	3	6	9	12

   a) how much is added to each of the previous terms in the pattern?
   b) How many times the position number is the value of each term?
   c) Write down a rule that describes this pattern. Check that your rule works.

2. Consider the following number pattern

position (p)	0	1	2	3	4
value (v)	0	120	240	360	480

   a) how much is added to each of the previous terms in the pattern?
   b) How many times the position number is the value of each term?
   c) Write down a rule that describes this pattern. Check that your rule works.

3. Consider the following number pattern

position (p)	0	1	2	3	4
value (v)	0	-10	-20	-30	-40

   a) how much is added to each of the previous terms in the pattern?
   b) How many times the position number is the value of each term?
   c) Write down a rule that describes this pattern. Check that your rule works.

4. Consider the following number pattern

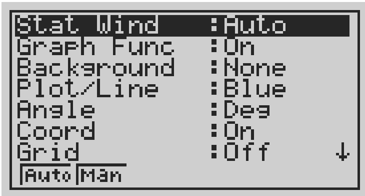
position (p)	0	1	2	3	4
value (v)	0	a	2a	3a	4a

   a) how much is added to each of the previous terms in the pattern?
   b) How many times the position number is the value of each term?
   c) Write down a rule that describes this pattern.

5. Recall we noted in our first example that the number of times the value is of the position number, is the same as the amount that was added to each previous term in the pattern. Did this happen in each of the questions in this interaction? Explain why this is no coincidence and will happen in any *additive* pattern in which the first term has a value of zero.

A graphical display of a pattern can be produced on your calculator and following this we can check that the rule we have generated is correct.

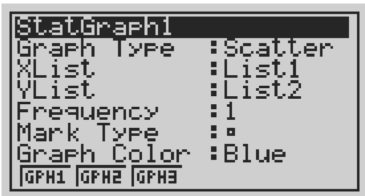
Enter `STAT` mode. Use `SET UP` (`SHIFT` then `MENU`) to ensure that `Stat Wind` is set to `Auto`. This way the calculator will automatically choose appropriate scales for the axes of the graph we will draw.



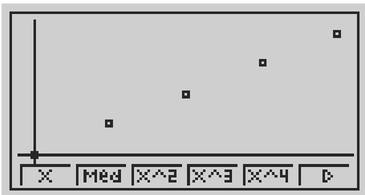
Press `EXIT`.

Lists are available for the entering of data. If data already exits in the lists and you wish to delete them, press `F6` (the continuation key) and then with the cursor in the appropriate column use `DEL·A` (`F4`) to delete all of the data in that list. If you do not want to delete the data, simply move the cursor to an empty list.

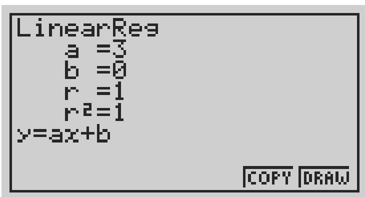
Enter the position and value data from question 1 of Interaction C using the number keys and pressing `EXE` after each number is entered.



Press `F6` (if necessary) and then use `GRPH` (`F1`) and then `SET` (`F6`) to set up `StatGraph1`. Ensure that each setting is as shown opposite. We want to produce a scatterplot of v by p (or `List 2` by `List 1`).



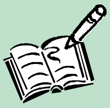
Press `EXIT` and then use `GPH1` (`F1`) to draw the scatterplot.



Note that the data points fall in what appears to be a straight line. We should have been able to predict this from the structure of the table of values. To check that the rule we produced for this data is correct press `X` (`F1`). This will fit a *straight line of best fit* to the data of form $y = ax + b$.

If the data can be modeled *exactly* by a linear rule, then the r^2 value reported by the calculator will be equal to 1, as it is in this case. If it is not 1, then the points do not fall in a perfectly straight line. You will learn more about the r^2 value next year.

The a value is the slope or gradient of the line of best fit – three in this case. This is equivalent to value that is added to form the next number in the pattern. The b value is the vertical intercept and is zero in this case. So the answer we should have arrived at in question 1 of Interaction C is $v = 3p$.



Interaction D

1. Use the technique illustrated above to check you answers to question 2 and 3 in Interaction C.
2. Use the calculator to verify that a linear rule for the following pattern is $v = 7p+5$.

position (p)	0	1	2	3	4
value (v)	5	12	19	26	33

3. Explain why the rule in question 2 does not have zero as the value of b.

Understanding simple interest

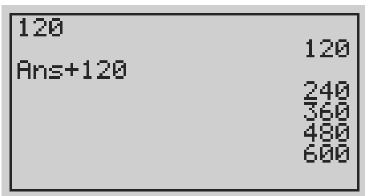
***The concept of* investing *money means that we* lend *some of our money to another person and in return they pay us money for the service of using our money. The money they pay for this service is called* interest.**

It is common for people to quote how much interest they will pay you as a percentage of the original amount loaned (or *principal*).

There are many ways that one may decide interest is to be paid. The most simple is, strangely enough, called *simple interest*.

Simple interest involves a *fixed percentage of the size of the original loan* paid at *regular time periods*. Both the fixed percentage and the time periods are decided upon before a loan agreement is entered into.

Tamra decided to lend her mother Maxine $2400 (some of the hard earned money she had saved from working at Subway). They agreed that Maxine would pay Tamra 5% simple interest per annum. Hence *every year* Maxine paid Tamra 5% of $2400 until she no longer wanted the loan. Then she would have to pay back the $2400. As 5% of $2400 is $120, Maxine would have to pay $120 per year in interest.



To see how much interest Tamra will cumulate as the years pass, enter the RUN mode, enter 120 and press EXE. Then press + and enter 120 and press EXE repeatedly. This method allows you to repeatedly add a constant value efficiently.

Interaction E

1. Check that a 6 year investment of $5600 at 7.2% p.a. simple interest is worth less than a 6 year investment of $7000 at 4% pa simple interest. For how long must each amount be invested in order to have a value of over $20 000?
2. Find the value of an investment of $6400 after 10 years if the interest paid is 5.3% pa simple interest.
3. Jillian invested an inheritance of $4500 at 6.8% pa simple interest. How long will it take for this investment to be worth at least $8000?

To calculate the interest (I), one could simply multiply the principal (P) by the interest rate (r) (as a decimal) and then multiply the result by the number of years (n) for which the loan existed.

So a formula for the amount of simple interest could be,

$$I = Prn.$$

Let us develop this another way. Look at how much interest Tamra would cumulate as the years pass.

After zero years:	interest = \$0
After one year:	interest = \$(2400 × 0.05 × 1) = \$(120 × 1) = \$120
After two years:	interest = \$(2400 × 0.05 × 2) = \$(120 × 2) = \$240
After three years:	interest = \$(2400 × 0.05 × 3) = \$(120 × 3) = \$360
After four years:	interest = \$(2400 × 0.05 × 4) = \$(120 × 4) = \$480

If we let the number of years be n and the amount of interest earned after n years be I then:

position (p)	0	1	2	3	4
value (v)	0	120	240	360	480

This table should be familiar to you from Interaction C. Clearly the pattern exhibited here is additive and the rule for the pattern is:

$$I = 120n$$

What is the significance of the 120? It is the amount of interest paid per year and is the product of the percentage interest rate and the amount loaned or the *principal*.

Hence we could write the rule as:

$$I = 2400 \times 0.05n$$

Should the principal have been \$5000 and the percentage interest rate been 8% , then

$$I = 5000 \times 0.08n$$

Should the principal have been \$12000 and the percentage interest rate been 10%, then

$$I = 12000 \times 0.1n$$

So, should the principal have been $\$P$ and the percentage interest rate been $r\%$ (expressed as a decimal), then

$$I = Prn$$

This is the same simple interest formula you saw earlier. We can use this formula to determine the amount of simple interest earned in any situation, or the value of any of the four variables if we know the other three variable values.

Imagine Bob invests \$500 in an account that pays 6% pa simple interest for 5 years. Determine how much interest he earns in this time.

$$I = Prn$$
$$\Rightarrow \quad I = 500 \times 0.06 \times 5$$
$$\Rightarrow \quad I = \$150$$

Should you have a number of such calculations to do, a *one line program* entered in RUN mode of your calculator would be helpful. It allows you to do repetitive calculations quickly.

Enter RUN mode. Use PRGM (SHIFT then VARS) to reveal the program symbols along the bottom of the screen.

Press F4 to enter the ? symbol and then press the → key (just above the AC/ON key) to enter the arrow. Press ALPHA and then 4 to enter the variable P and then use the continuation key (F6) to reveal further symbols and use (F5) to enter the colon (:). Enter the rest of the commands as seen opposite and then press EXE.

On pressing EXE you will notice that a ? appears. The calculator is requesting the value of P – enter 500 and press EXE. Then respond to the next ? by entering 0.06 and to the final question mark with 5. The result of 150 corresponds to our result above.

By pressing EXE this program begins again. Even if you press AC/ON, or turn the calculator off, you can recall the program by pressing the up arrow key.

Interaction F

1. Use a one line program to find the amount of simple interest earned by investing \$400 at 3% for 7 years.
2. Use a one line program to find the amount of simple interest earned by investing \$800 at 6% for 14 years.
3. Use a one line program to find the amount of simple interest earned by investing \$2000 at $2\frac{1}{3}$% for $6\frac{1}{2}$ years.

How much money would Bob have to invest for five years in an account that pays 6% pa simple interest if he wanted to earn \$300 in interest?

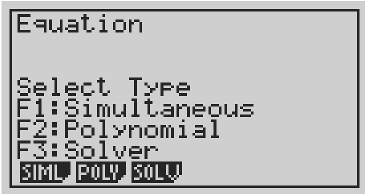
Common sense would tell you that he would need to invest twice the amount that he did before as the interest rate and the term are the same as they were before. Let us check by substituting and rearranging the resulting equation.

$$\begin{aligned} I &= Prn \\ \Rightarrow \quad 300 &= P \times 0.06 \times 5 \\ \Rightarrow \quad 300 &= 0.3P \\ \Rightarrow \quad \frac{300}{0.3} &= \frac{0.3P}{0.3} \\ \Rightarrow \quad P &= 1000 \end{aligned}$$

So \$1000 must be invested.

An efficient way to perform a number of these type of calculations is by using the EQUA mode of the calculator. This mode solves numerous types of equations. For this type we will use what is called the Solver.

Enter the EQUA mode. You will see that you are given three options.

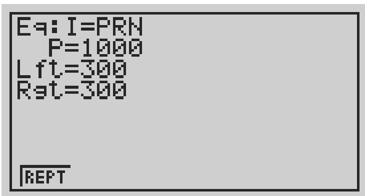


Use SOLV (F3) to enter the solver. If an equation is already present, use DEL (F2) and then YES (F1) to delete it.

Now enter the simple interest formula using ALPHA followed by the appropriate letter. The equals sign is entered by pressing SHIFT and then the decimal point. Press EXE.

Use the arrow keys to highlight each variable in turn and define their value.

Leave *P* as zero, or any value in fact, and *place the cursor on P*. This is how you indicate to the calculator that you want to find the value of *P*.



Then use SOLV (F6) to solve the equation for *P*. If the Lft and Rgt (standing for left and right) are identical then the calculator has found an accurate solution.

In some cases the calculator will fail to find a solution, and the Lft and Rgt values will differ. You will be instructed to Try again. The calculator may ask you to enter an approximation, so it is good to have at least a rough idea of the solution.

Using REPT (F1), you can return to the previous screen and change the values of the variables and solve again and again.

Interaction G

1. How much money would Bob need to have invested for five years in an account that pays 12% pa simple interest if he wanted to earn $300 in interest?

2. At what interest rate would Bob have to invest $20 000 for six years in an account that pays simple interest if he wanted to earn $1500 in interest?
3. Complete the following table if the principal invested is $2000.

number of years (n)	0	1	2	3	4
interest for r = 0.02 ($\$I$)					
interest for r = 0.04 ($\$I$)					
interest for r = 0.06 ($\$I$)					

Enter each row of the above table into a list of your calculator. Draw a scatterplot of I against n for each r value on the one set of axes. (Set up three `StatGraphs` and then press `EXIT` and use `SEL (F4)` to turn all three `StatGraphs On`.)

If a straight line was drawn through the points for each r value, one could determine the gradient of the line. Describe the relationship between the gradient, the principal and the rate of interest.

Describing a multiplicative pattern (exponential)

Another simple type of number pattern is formed by* multiplying *the previous value in a pattern by a constant amount. If these patterns were to start with zero the result would be a little boring.

Consider the following:

$$2,\ \ 2\times5,\ \ 2\times5\times5,\ \ 2\times5\times5\times5,\ \ 2\times5\times5\times5\times5, ...$$

or

$$2,\ 10,\ 50,\ 250,\ 1250,\ \ldots$$

Such patterns are called exponential patterns.

To help describe this pattern we again assign a number to each term in the pattern so we know that number's *position* in the pattern. For ease of description we have chosen to start with position zero.

This is best displayed in a table. Let the position number be p and the value of the number in the pattern be v.

position (p)	0	1	2	3	4
value (v)	2	10	50	250	1250

As well as the *multiplying by 5* pattern that can be seen in a horizontal manner, we should be able to find a pattern in a vertical sense as we did in the additive pattern. It is, however, not as easy to see. The following table will help.

position (p)	0	1	2	3	4
value (v)	2 $=2\times5$	10 $=2\times5$ $=2\times5^1$	50 $=2\times5\times5$ $=2\times5^2$	250 $=2\times5\times5\times5$ $=2\times5^3$	1250 $=2\times5\times5\times5\times5$ $=2\times5^4$

In this table we have written the values in terms of the number of multiplications that have occurred using exponents. Notice that the exponent value matches the position number in each case.

Hence, the value of the position five number will be 2×5^5 and the value for position six will be 2×5^6 and so on. Hence for position p the value will be 2×5^p.

So we can now write down the rule:

$$v = 2 \times 5^p.$$

Note that 2 was the *starting value* of the pattern and 5 is the *constant multiplier* hence if the starting value was s and the constant multiplier was m we could develop a general rule for a multiplicative pattern:

$$v = s \times m^p.$$

Consider the pattern in the following table.

position (p)	0	1	2	3	4
value (v)	4	12	36	108	324

The first value is 4 and the constant multiplier is 3 so the rule should be

$$v = 4 \times 3^p$$

Let's check this rule for $p = 4$

$$\begin{aligned} v &= 4 \times 3^p \\ \Rightarrow \quad v &= 4 \times 3^4 \\ \Rightarrow \quad v &= 4 \times 81 \\ \Rightarrow \quad v &= 324 \end{aligned}$$

Check a few more to convince yourself it works.

As a second example, consider the pattern in the following table.

position (p)	0	1	2	3	4
value (v)	60	30	15	7.5	3.75

The first value is 60 and the constant multiplier is $\frac{1}{2}$ so the rule should be

$$v = 60 \times \left(\frac{1}{2}\right)^p$$

Let's check this rule for $p = 3$

$$\begin{aligned} & v = 60 \times \left(\tfrac{1}{2}\right)^p \\ \Rightarrow \quad & v = 60 \times \left(\tfrac{1}{2}\right)^3 \\ \Rightarrow \quad & v = 60 \times \tfrac{1}{8} \\ \Rightarrow \quad & v = 7.5 \end{aligned}$$

Check a few more to convince yourself it works.

Interaction H

1. Consider the following number pattern

position (p)	0	1	2	3	4
value (v)	1	4	16	32	64

a) what is the constant multiplier?

b) Write down a rule that describes this pattern. Check it works for at least two values.

2. Consider the following number pattern

position (p)	0	1	2	3	4
value (v)	400	100	25	6.25	1.563

a) what is the constant multiplier?

b) Write down a rule that describes this pattern. Check it works for at least two values.

3. Consider the following number pattern

position (p)	0	1	2	3	4
value (v)	5600	6003.2	6435.4304	6898.781389	7395.493649

a) what is the constant multiplier?

b) Write down a rule that describes this pattern. Check it works for at least two values.

Investigation:

If you were to have $20 000 invested and it was to grow either by a constant addition of $1000 per year or by a constant multiplication of 1.04, which option would you take if the investment period was 10 years? Would your answer change if the duration of the investment was 15 years?

What do your answers suggest about the speed at which multiplicative patterns grow compared to additive patterns?

Understanding compound interest

Compound interest is an alternative method to pay interest on an investment. It differs from simple interest in that the first interest instalment remains in the account and is then considered part of the principal on which the next interest instalment is calculated. Hence the second interest instalment is larger than the first. This process continues and so the interest instalments continue to increase in size.

It is common to calculate the future value of the investment (abbreviated to *FV*) at some number of *periods* (n) from when the investment began. *The money must be in the account for a full period before it gains any interest*. The word *period* is used instead of year as the interest is often calculated and added to the principal at intervals other than a year depending on the terms of the account. For example, it is quite common for the interest to be compounded monthly (ie. interest is calculated and added to the previous principal at the end of 1 month).

If \$5600 was invested in an account that paid 7.2% pa compounded annually, we could determine the future value at the end of year one by *increasing* \$5600 by 7.2%. This is most simply done by multiplying by 1.072 $\left(\text{or } \frac{107.2}{100}\right)$.

So, at the end of year 1:

$$FV = 5600 \times 1.072$$
$$\Rightarrow \quad FV = \$6003.20$$

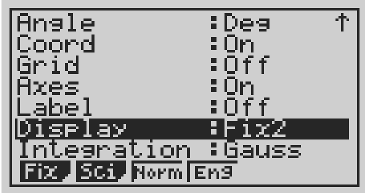
at the end of year 2:

$$FV = 5600 \times 1.072 \times 1.072$$
$$\Rightarrow \quad FV = 6003.20 \times 1.072$$
$$\Rightarrow \quad FV = \$6435.43$$

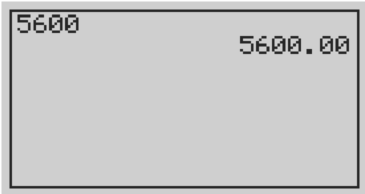
at the end of year 3:

$$FV = 5600 \times 1.072 \times 1.072 \times 1.072$$
$$\Rightarrow \quad FV = 6435.43 \times 1.072$$
$$\Rightarrow \quad FV = \$6898.78$$

Repeated operations like multiplication can be done quickly on the calculator.

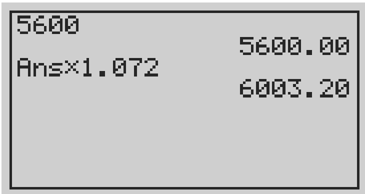


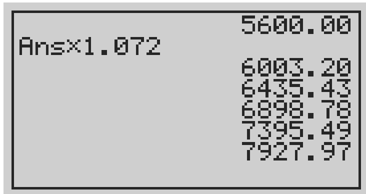
Enter RUN mode then enter SET UP (SHIFT then MENU) to set all calculations to be displayed correct to 2 decimal places as is appropriate when dealing with money. Arrow down to Display and use Fix (F1) and then 2 (F3).



Press EXIT and then enter the initial principal and press EXE. The calculator now thinks of 5600 as an answer.

Now press × and then enter 1.072 and press EXE. The *FV* at the end of the first year ($6003.20) is given. Now simply press EXE again and the next *FV* is given. Continued pressing of EXE repeats the multiplication of 1.072.





Interaction I

1. Check that the investment of $5600 at 7.2% p.a. compounded annually is worth more than$10 000 after nine years. For how long must the $5600 be invested for to have a value of over $20 000?
2. Find the value of an investment of $6400 after 10 years if the interest paid is 5.3% pa compounded annually.
3. Jillian invested an inheritance of $4500 at 6.8% pa compounded annually. How long will it take for this investment to be worth at least $8000?

To calculate an investment's value in the future (*FV*), one could simply multiply the present value (*PV*) by 1 plus the interest rate (r), expressed as a decimal, the appropriate number of times (n).

So an appropriate formula could be,

$$FV = PV\,(1 + r)^n.$$

Let us develop this another way. First, look at the future values of the investment over time.

number of periods (n)	0	1	2	3	n
future value (FV)	5600	6003.20	6435.43	6898.78	?

This table is virtually identical to that from question 3 in Interaction H. You should have realised by now that the pattern formed by the future values is multiplicative. Hence, using the theory of multiplicative patterns, we could say that:

$$FV = 5600\,(1.072)^n.$$

This rule allows us to work out FV after any number of periods in the same way we use the simple interest formula.

What if the interest rate had been 12% pa (0.12) compounded annually?

$$FV = 5600\,(1.12)^n = 5600\,(1 + 0.12)^n$$

What if the interest rate had been 6.8% pa (0.068) compounded annually?

$$FV = 5600\,(1.068)^n = 5600\,(1 + 0.068)^n$$

What if the interest rate had been r% pa (written as a decimal) compounded annually?

$$FV = 5600\,(1 + r)^n$$

Finally if the principal was an unknown value, say PV (which stands for present value) then we could say that:

$$FV = PV\,(1 + r)^n$$

which is the compound interest formula.

Usually, compound interest is not added every year, but is added several times per year, such as monthly or quarterly (every 3 months). In such cases, you will get a bit more interest, since you will earn interest on the interest earlier than for annual compounding.

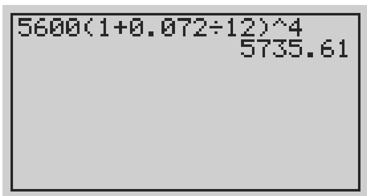
To see an example of this, suppose the investment of \$5600 at 7.2% per annum uses compound interest compounded monthly.

Then the interest rate every month is 0.072 ÷ 12 = 0.06. The compound interest formula becomes:

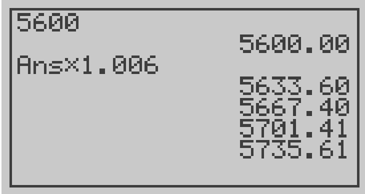
$$\begin{aligned} FV &= PV\,(1+r)^n \\ \Rightarrow \quad FV &= 5600\left(1 + \frac{0.072}{12}\right)^n \end{aligned}$$

So after 4 periods (months in this case)

$$\begin{aligned} FV &= 5600\left(1 + \frac{0.072}{12}\right)^4 \\ \Rightarrow \quad FV &= \$5735.61 \end{aligned}$$



To perform the last calculation in run mode you must take care with the use of brackets. Only one set is necessary. We have left the i value $\frac{0.072}{12}$ as in some cases the division will result in a non terminating fraction. It is a more accurate way to deal with the situation.



An alternative to using the formula is to use the repeated multiplication shown earlier. On the calculator, the Principal is entered as before.

The amount after every *month* is given by multiplying by 1.006 each time, as shown. The screen shows that, after four months, the investment is worth $5735.61.

After 12 months of interest have been added in this way, the investment is worth $6016.78. This is a little more ($13.58) than was the case for annual compounding.

Interaction J

1. Check that the FV of an investment of $5600 at 7.2% pa, compounded monthly, is $8018.02 after five years.
2. Compare the FV of $5600 after 12 months if it earns 7.2% compounded:
   - i) annually
   - ii) quarterly
   - iii) monthly
   - iv) fortnightly
3. Kellie invested $5600 at 7.2% pa compounded quarterly. What was the value of her investment after four years?
4. Simon invested $5600 at 7.2% pa compounded semi-annually (i.e. twice every year). Predict whether his investment would accumulate, in four years, to more or to less than that of Kellie in the previous question. Use your calculator to check your prediction.

Comparing simple and compound interest

Recall the investigation that preceded Understanding compound interest. You were asked the question:

If you were to have \$20 000 invested and it was to grow by a constant addition of \$1000 per year or by a constant multiplication of 1.04 per year, which option would you take if the investment period was 10 years? Would your answer change if the duration of the investment was 15 years?

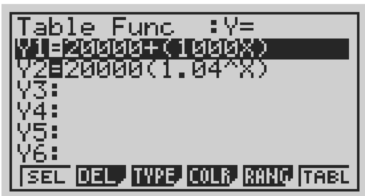
If we let the FV be the future value of both investment and n be the number of periods (years in this case) then for the simple interest case:

$$FV = 20\,000 + (1000n)$$

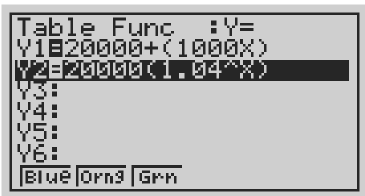
And for the compound interest case:

$$FV = 20000\,(1.04)^n$$

We can use the calculator to efficiently calculate and display the future value of these investments for a series of years.

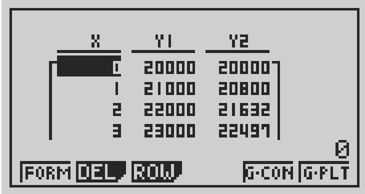


Enter the `TABLE` mode on your calculator and define `Y1` as 20 000 + (1000x). Press `EXE` and then define `Y2` as 20000 (1.04)^x. Note that the exponent is entered using the upside down `V` symbol and that multiplication symbols are not required.

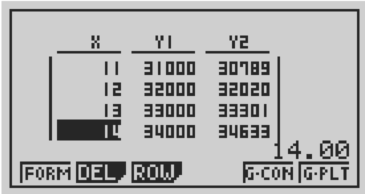


Use the up arrow key to highlight `Y2` and then use `COLR` (`F4`) and then `Orng` (`F2`) to change to orange. When we draw a graph, the graph of `Y2` will be in orange while `Y1` will be in blue. Press `EXIT`.

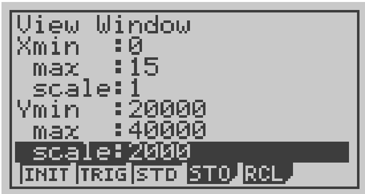
Use `RANG` (`F5`) to set the range of years for which we want the calculator to display future values. Display the values for year 0 (the beginning of the investment period) to year 15. Pitch refers to the gap between future values. We want one year in this case.



Press EXIT and then use TABL (F6) to produce the table of values. Use the up and down arrow keys to explore the table.

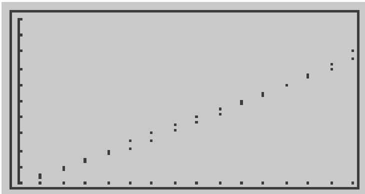


It appears that around year 12 the second scenario becomes the better option!

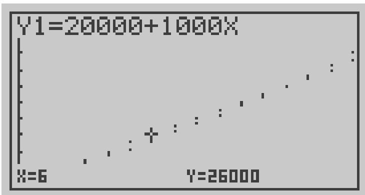


We can produce a graphical display (value against year) for each investment scenario. First we must set up the scales for the axes.

Use V-Window (SHIFT then F3) to set the parameters as shown. You should have explored the table sufficiently to have been able to work suitable values out.



Press EXIT and then use TABL (F6) to produce the table again and then use G·PLT to draw the graph. The points show the value of the investments at the end of each period.



Now use Trace (SHIFT then F1) and the left and right arrow keys to trace along the plots. The up and down arrow keys change between plots. The coordinates of the points are shown at the base of the screen.

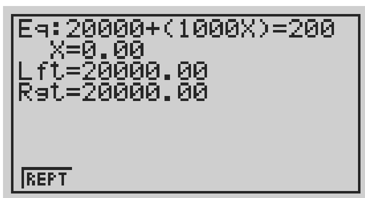
We could also determine when the value of one investment overtakes another by solving an equation. The investments will have equal value when

$$20\,000 + (1000n) = 20000\,(1.04)^n$$

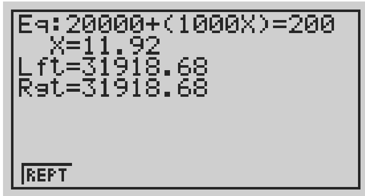
This a rather complex equation that is rather difficult to solve with an algorithm. We can however use the Solver in EQUA mode to help.

Enter EQUA mode and then the Solver option. Refer to the instructions earlier in this book if you have forgotten how to delete and enter equations.

The first screen below shows the equation before we have asked the calculator to solve it. Notice that it does not all fit on the screen, shown below. The value for X at present is zero and is used by the calculator as its first estimation of the solution. After pressing SOLV (F6) to solve the equation the second screen is produced. Of course X = 0 is a solution to the equation but not the one we required.



Press REPT and enter a better estimation to the wanted solution by changing the X value to a higher value, say 8 and then solve the equation again. The result is seen below.



We must think about the solution carefully. The decimal part of 11.92 really means nothing in the context of this problem. Interest is paid at the end of the investment periods, not continually throughout the the period. We can, however, use this solution to tell us that at the end of the twelfth period the second investment will be worth more than the first.

Interaction K

1. Determine from what year onwards an investment of $10 000 invested in an account that earns 4% pa simple interest is worth less than an investment of $8000 in an account that earns 3.2% pa compounded annually. [If a negative answer is returned by the calculator, it is a solution, but not the one we desire. Use REPT (F1) to repeat the calculation, but change the initial estimate to about 20.]
2. Determine from what time onwards an investment of $10 000 invested in an account that earns 4% pa compound interest compounded monthly is worth less than an investment of $8000 in an account that earns 4.5% pa compounded monthly.
3. Determine from what time onwards an investment of $10 000 invested in an account that earns 4% pa compound interest compounded semi-annually is worth less than an investment of $8000 in an account that earns 8% pa compounded semi-annually.
4. Determine from what time onwards an investment of $10 000 invested in an account that earns 4% pa simple interest is worth less than an investment of $8000 in an account that earns 3.2% pa compounded monthly.
   [Hint: The simple interest function will be FV = 10 000 + (33.33n) where n = the number of months.]

Answers

Some of the questions that have been asked do not have a single correct answer. In such cases, MPA (which stands for many possible answers) will be the answer supplied. In many cases, some supporting comment is supplied.

Interaction A

1. \$274.34
2. \$191.66
3. \$316.64, \$220.16

Interaction B

2. \$201.67
3. MPA, e.g., not unless it is edited to change \$8.41 to \$9.81.

Interaction C

1. a) 3 b) 3 c) $v = 3p$
2. a) 120 b) 120 c) $v = 120p$
3. a) −10 b) −10 c) $v = -10p$
4. a) a b) a c) $v = ap$
5. Yes. In these number patterns the position number corresponds to the number of additions of the constant adder from zero. Hence, as two additions of five is equivalent to 2 lots of five or 2 × 5, the observation is no coincidence.

Interaction D

3. MPA, e.g., the value for b corresponds to that for v when $p = 0$, which is 5 in this case.

Interaction E

1. \$8019.20 is less than \$8680. The first must be invested for 36 years while the second must be invested for 47 years.
2. \$979.20
3. 12 years

Interaction F

1. \$84
2. \$672
3. \$303.33

Interaction G

1. \$500
2. 1.25%
3.

number of years (n)	0	1	2	3	4
interest for r = 0.02 (\$$I$)	0	40	80	120	160
interest for r = 0.04 (\$$I$)	0	80	160	240	320
interest for r = 0.06 (\$$I$)	0	120	240	360	480

The gradient of each line will equal the product of the principal p and the rate r.

Interaction H

1. a) 4 b) $v = 4^p$
2. a) $\frac{1}{4}$ b) $v = 400\left(\frac{1}{4}\right)^p$
3. a) 1.072 b) $v = 5600\,(1.072)^p$

Interaction I

1. \$10 469.87 is greater than \$10 000, 19 years.
2. \$10 726.64
3. 9 years.

Interaction J

2. i) \$6003.20 ii) \$6014.22
   iii) \$6016.78 iv) \$6017.47
3. \$7449.93
4. A bit less than Kellie's (\$7431.32)

Interaction K

1. 35 years.
2. 538 months.
3. 12 periods (semi-annual).
4. 405 months.

INSTALMENT SCHEDULE

	Weekly Instalment					Weekly Instalment					Weekly Instalment			
Weekly earnings	With tax free threshold with leave loading	With tax free threshold no leave loading	No tax free threshold	No tax file number	Weekly earnings	With tax free threshold with leave loading	With tax free threshold no leave loading	No tax free threshold	No tax file number	Weekly earnings	With tax free threshold with leave loading	With tax free threshold no leave loading	No tax free threshold	No tax file number
181	16.75	16.20	49.30	87.80	256	33.70	32.10	75.95	124.15	331	52.30	51.20	102.55	160.55
182	17.00	16.40	49.65	88.25	257	34.10	32.50	76.30	124.65	332	52.50	51.40	102.90	161.00
183	17.20	16.60	50.00	88.75	258	34.50	32.90	76.65	125.15	333	52.70	51.60	103.25	161.50
184	17.40	16.80	50.40	89.25	259	34.95	33.30	77.00	125.60	334	52.95	51.85	103.65	162.00
185	17.60	17.00	50.75	89.70	260	35.35	33.70	77.35	126.10	335	53.15	52.05	104.00	162.45
186	17.80	17.20	51.10	90.20	261	35.75	34.10	77.70	126.60	336	53.40	52.25	104.35	162.95
187	18.00	17.40	51.45	90.70	262	36.15	34.50	78.05	127.05	337	53.60	52.45	104.70	163.45
188	18.20	17.60	51.80	91.20	263	36.55	34.90	78.40	127.55	338	53.80	52.70	105.05	163.95
189	18.40	17.80	52.15	91.65	264	36.95	35.30	78.80	128.05	339	54.05	52.90	105.40	164.40
190	18.60	18.00	52.50	92.15	265	37.35	35.70	79.15	128.50	340	54.25	53.10	105.75	164.90
191	18.80	18.20	52.85	92.65	266	37.75	36.10	79.50	129.00	341	54.45	53.35	106.10	165.40
192	19.00	18.40	53.20	93.10	267	38.15	36.50	79.85	129.50	342	54.70	53.55	106.45	165.85
193	19.20	18.60	53.55	93.60	268	38.55	36.90	80.20	130.00	343	54.90	53.75	106.80	166.35
194	19.40	18.80	53.95	94.10	269	38.75	37.30	80.55	130.45	344	55.10	54.00	107.20	166.85
195	19.60	19.00	54.30	94.55	270	38.95	37.70	80.90	130.95	345	55.35	54.20	107.55	167.30
196	19.80	19.20	54.65	95.05	271	39.20	38.10	81.25	131.45	346	55.55	54.40	107.90	167.80
197	20.00	19.40	55.00	95.55	272	39.40	38.50	81.60	131.90	347	55.80	54.60	108.25	168.30
198	20.20	19.60	55.35	96.05	273	39.65	38.70	81.95	132.40	348	56.00	54.85	108.60	168.80
199	20.45	19.80	55.70	96.50	274	39.85	38.95	82.35	132.90	349	56.20	55.05	108.95	169.25
200	20.65	20.00	56.05	97.00	275	40.05	39.15	82.70	133.35	350	56.45	55.25	109.30	169.75
201	20.85	20.20	56.40	97.50	276	40.30	39.35	83.05	133.85	351	56.65	55.50	109.65	170.25
202	21.05	20.40	56.75	97.95	277	40.50	39.55	83.40	134.35	352	56.85	55.70	110.00	170.70
203	21.25	20.60	57.10	98.45	278	40.70	39.80	83.75	134.85	353	57.10	55.90	110.35	171.20
204	21.45	20.80	57.50	98.95	279	40.95	40.00	84.10	135.30	354	57.30	56.15	110.75	171.70
205	21.65	21.00	57.85	99.40	280	41.15	40.20	84.45	135.80	355	57.55	56.35	111.10	172.15
206	21.85	21.20	58.20	99.90	281	41.35	40.45	84.80	136.30	356	57.75	56.55	111.45	172.65
207	22.05	21.40	58.55	100.40	282	41.60	40.65	85.15	136.75	357	57.95	56.75	111.80	173.15
208	22.25	21.60	58.90	100.90	283	41.80	40.85	85.50	137.25	358	58.20	57.00	112.15	173.65
209	22.45	21.80	59.25	101.35	284	42.05	41.10	85.90	137.75	359	58.40	57.20	112.50	174.10
210	22.65	22.00	59.60	101.85	285	42.25	41.30	86.25	138.20	360	58.60	57.40	112.85	174.60
211	22.85	22.20	59.95	102.35	286	42.45	41.50	86.60	138.70	361	58.85	57.65	113.20	175.10
212	23.05	22.40	60.30	102.80	287	42.70	41.70	86.95	139.20	362	59.05	57.85	113.55	175.55
213	23.25	22.60	60.65	103.30	288	42.90	41.95	87.30	139.70	363	59.25	58.05	113.90	176.05
214	23.45	22.80	61.05	103.80	289	43.10	42.15	87.65	140.15	364	59.50	58.30	114.30	176.55
215	23.70	23.00	61.40	104.25	290	43.35	42.35	88.00	140.65	365	59.70	58.50	114.65	177.00
216	23.90	23.20	61.75	104.75	291	43.55	42.60	88.35	141.15	366	59.95	58.70	115.00	177.50
217	24.10	23.40	62.10	105.25	292	43.75	42.80	88.70	141.60	367	60.15	58.90	115.35	178.00
218	24.30	23.60	62.45	105.75	293	44.00	43.00	89.05	142.10	368	60.35	59.15	115.70	178.50
219	24.50	23.80	62.80	106.20	294	44.20	43.25	89.45	142.60	369	60.60	59.35	116.05	178.95
220	24.70	24.00	63.15	106.70	295	44.45	43.45	89.80	143.05	370	60.80	59.55	116.40	179.45
221	24.90	24.20	63.50	107.20	296	44.65	43.65	90.15	143.55	371	61.00	59.80	116.75	179.95
222	25.10	24.40	63.85	107.65	297	44.85	43.85	90.50	144.05	372	61.25	60.00	117.10	180.40
223	25.30	24.60	64.20	108.15	298	45.10	44.10	90.85	144.55	373	61.45	60.20	117.45	180.90
224	25.50	24.80	64.60	108.65	299	45.30	44.30	91.20	145.00	374	61.65	60.45	117.85	181.40
225	25.70	25.00	64.95	109.10	300	45.50	44.50	91.55	145.50	375	61.90	60.65	118.20	181.85
226	25.90	25.20	65.30	109.60	301	45.75	44.75	91.90	146.00	376	62.10	60.85	118.55	182.35
227	26.10	25.40	65.65	110.10	302	45.95	44.95	92.25	146.45	377	62.35	61.05	118.90	182.85
228	26.30	25.60	66.00	110.60	303	46.15	45.15	92.60	146.95	378	62.55	61.30	119.25	183.35
229	26.50	25.80	66.35	111.05	304	46.40	45.40	93.00	147.45	379	62.75	61.50	119.60	183.80
230	26.70	26.00	66.70	111.55	305	46.60	45.60	93.35	147.90	380	63.00	61.70	119.95	184.30
231	26.95	26.20	67.05	112.05	306	46.85	45.80	93.70	148.40	381	63.20	61.95	120.30	184.80
232	27.15	26.40	67.40	112.50	307	47.05	46.00	94.05	148.90	382	63.40	62.15	120.65	185.25
233	27.35	26.60	67.75	113.00	308	47.25	46.25	94.40	149.40	383	63.65	62.35	121.00	185.75
234	27.55	26.80	68.15	113.50	309	47.50	46.45	94.75	149.85	384	63.85	62.60	121.40	186.25
235	27.75	27.00	68.50	113.95	310	47.70	46.65	95.10	150.35	385	64.10	62.80	121.75	186.70
236	27.95	27.20	68.85	114.45	311	47.90	46.90	95.45	150.85	386	64.30	63.00	122.10	187.20
237	28.15	27.40	69.20	114.95	312	48.15	47.10	95.80	151.30	387	64.50	63.20	122.45	187.70
238	28.35	27.60	69.55	115.45	313	48.35	47.30	96.15	151.80	388	64.75	63.45	122.80	188.20
239	28.55	27.80	69.90	115.90	314	48.60	47.55	96.55	152.30	389	64.95	63.65	123.15	188.65
240	28.75	28.00	70.25	116.40	315	48.80	47.75	96.90	152.75	390	65.15	63.85	123.50	189.15
241	28.95	28.20	70.60	116.90	316	49.00	47.95	97.25	153.25	391	65.40	64.10	123.85	189.65
242	29.15	28.40	70.95	117.35	317	49.25	48.15	97.60	153.75	392	65.75	64.30	124.20	190.10
243	29.35	28.60	71.30	117.85	318	49.45	48.40	97.95	154.25	393	66.10	64.50	124.55	190.60
244	29.55	28.80	71.70	118.35	319	49.65	48.60	98.30	154.70	394	66.45	64.75	124.95	191.10
245	29.75	29.00	72.05	118.80	320	49.90	48.80	98.65	155.20	395	66.80	64.95	125.30	191.55
246	29.95	29.20	72.40	119.30	321	50.10	49.05	99.00	155.70	396	67.15	65.15	125.65	192.05
247	30.20	29.40	72.75	119.80	322	50.30	49.25	99.35	156.15	397	67.50	65.35	126.00	192.55
248	30.45	29.60	73.10	120.30	323	50.55	49.45	99.70	156.65	398	67.85	65.70	126.35	193.05
249	30.85	29.80	73.45	120.75	324	50.75	49.70	100.10	157.15	399	68.20	66.05	126.70	193.50
250	31.25	30.00	73.80	121.25	325	51.00	49.90	100.45	157.60	400	68.60	66.45	127.05	194.00
251	31.70	30.20	74.15	121.75	326	51.20	50.10	100.80	158.10	401	68.95	66.80	127.40	194.50
252	32.10	30.50	74.50	122.20	327	51.40	50.30	101.15	158.60	402	69.30	67.15	127.75	194.95
253	32.50	30.90	74.85	122.70	328	51.65	50.55	101.50	159.10	403	69.65	67.50	128.10	195.45
254	32.90	31.30	75.25	123.20	329	51.85	50.75	101.85	159.55	404	70.00	67.85	128.50	195.95
255	33.30	31.70	75.60	123.65	330	52.05	50.95	102.20	160.05	405	70.35	68.20	128.85	196.40

Example

An employee's weekly earnings are $363.60. To work out the correct amount of tax to deduct go to column 1 and find $363. If the employee:

- is claiming the tax-free threshold and is entitled to a leave loading, scan across to column 2 to find the correct amount of tax to deduct — $59.25
- is claiming the tax-free threshold and is not entitled to a leave loading, scan across to column 3 to find the correct amount of tax to deduct — $58.05
- is not claiming the tax-free threshold, whether or not entitled to a leave loading, scan across to column 4 to find the correct amount of tax to deduct — $113.90

• *Do not allow any FTA, rebates or Medicare levy variation.*

- has not supplied a tax file number, scan across to column 5 to find the correct amount of tax to be deducted — $176.05

• *Do not allow any FTA, rebates or Medicare levy variation.*