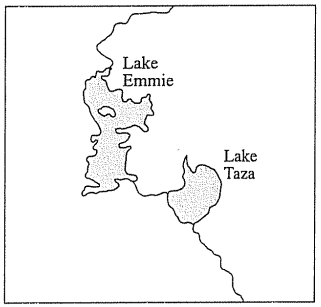
The Polluting of Lake Emmie - Algebraic Modelling Activity

In the following lake system a river flows through Lake Emmie to Lake Taza. Many people use this lake system for recreation.



A harmful pollutant was spilt in Lake Emmie. The concentration of the pollutant was measured every second day for forty days after the spill.

A table of the concentration (C) of the pollutant at a given time (t) is given below.

t (days)	0	2	4	6	8	10	12	14	16	18	20
C (mg/m^3)	10	9.10	7.95	7.36	7.01	6.16	5.54	5.00	4.32	4.25	3.64
t (days)	22	24	26	28	30	32	34	36	38	40	
C (mg/m^3)	3.71	3.40	2.77	2.22	2.08	1.95	1.97	1.56	1.72	1.69	

Use your Graphics Calculator to draw a Scatter Plot of this data.

1) Describe how the concentration of pollution in Lake Emmie changed over the 40 days.

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_____

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_____

Simone lives near Lake Emmie and regularly swims in it. Simone used the internet to research safe pollution levels for swimming and concluded that she would not continue swimming in Lake Emmie until the concentration of pollution dropped below 1 mg/m^3.

2) By tracing the scatter graph, make a prediction for the number of days Simone will need to wait before being able to swim in Lake Emmie again.

_____ days

3) Is this easy to do from the scatter graph. If not, why not.

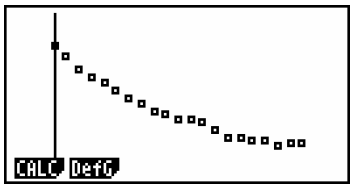
_____

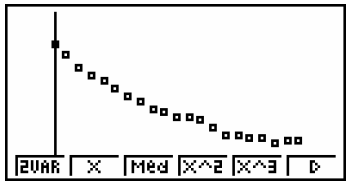
_____

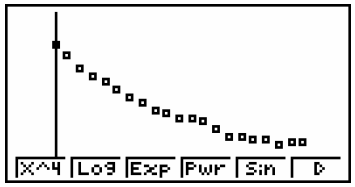
_____

A better way to predict

Redraw the scatter graph (without trace) and press CALC (F1). The F buttons (except F1) now give various graph options that the calculator can use to generate a graph-of-best-fit (F6 gives you more options).







Investigate the various graph options by drawing the graphs over the scatter plot and choosing your 'graph of best fit'. Eg to investigate the x-squared graph type, press X^2 (F4), then DRAW (F6). Use the arrows to extend beyond the scatter plot. After each graph you will need to EXIT twice and then redraw the graph.
NOTE: When you overlay an algebraic model over a scatter plot and then trace, use the up-down arrows to flip between the 2 graphs. You will need to have the algebraic model selected to investigate values beyond the scatter plot (using the right arrow).

4) Explain why the X^3 (F5) graph type is not a good model. (Hint: Use the right arrow to greatly extend the graph)

_____

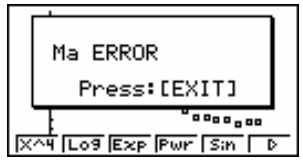
_____

_____

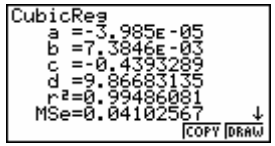
5) Choose an appropriate model (graph) and predict more accurately how many days Simone will need to wait until she can swim in Lake Emmie again.

Notes on selecting algebraic models:

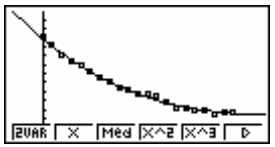
- Some graph options will not fit certain data and this will bring up a 'Ma ERROR'



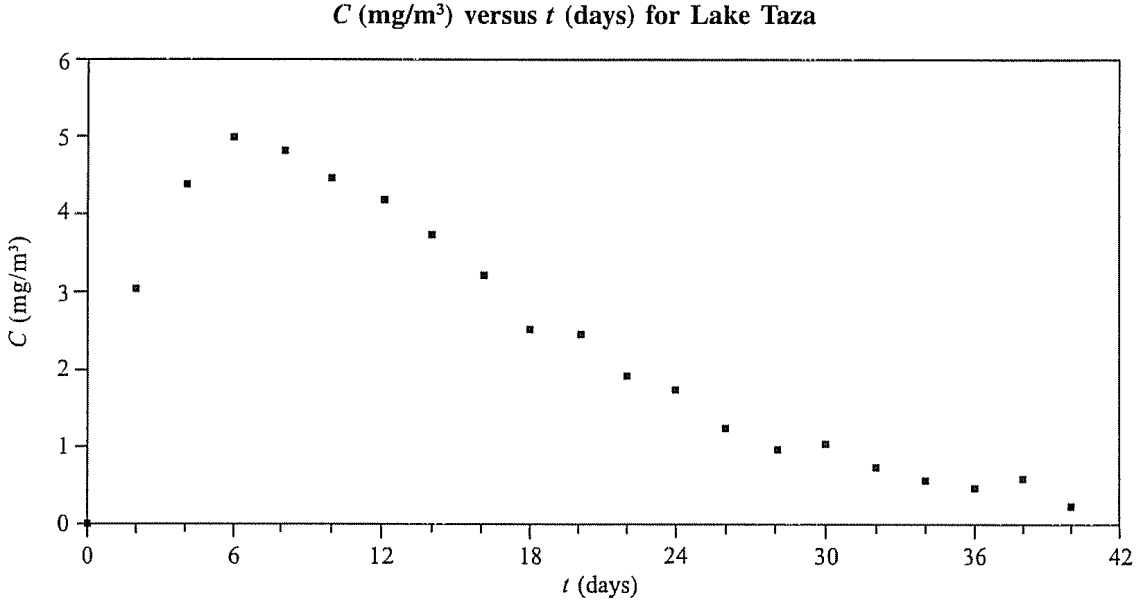
- Where a graph option is possible for a set of data, the first screen will give the equation.



- If you press DRAW (F6) the graph (algebraic model) will display with the scatter plot.



Lake Taza also became contaminated by the pollutant. The concentration of the pollutant in this lake was also measured every second day after the spill in Lake Emmie. A graph of concentration (C) of the pollutant versus time (t) for Lake Taza is given below:



6) Write a likely explanation for the shape of the graph of the Concentration of Pollution in Lake Taza.

7) Now write a un-likely explanation for the shape of this graph.

The Polluting of Lake Emmie – Solutions and Instructions

Instructions for Casio 9860 Graphic Calculator	Screenshots

To draw a scatter plot of the data:

Turn the calculator on, press MENU then 2.
If pre-existing data is present you can either delete the data or use the right arrow to find some empty lists.
NOTE: For basic instructions in STAT mode the document Self-Guided_9860_STAT exists at www.casioed.net.au

Enter the data as per Fig1 (If necessary refer to Self-Guided_9860_STAT re typing headings)

Press GRPH (F1) (Fig2) then press SET (F6) (Fig3)

StatGraph1 is the default graph and this will suffice.

Arrow down to Graph Type and press Scat (F1) (Fig4)

Fig1

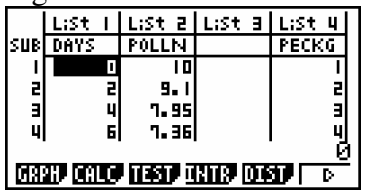


Fig2

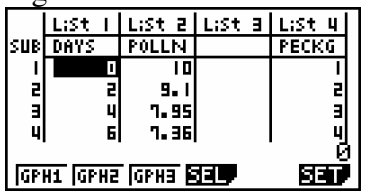


Fig3

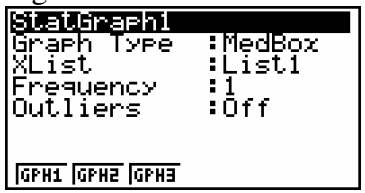
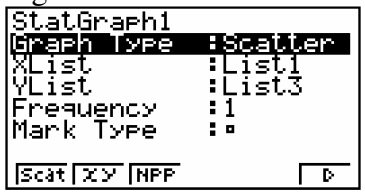


Fig4



To complete the settings as per Fig5:
Arrow to XList. Press LIST (F1) 1 EXE
Arrow to YList. Press LIST (F1) 2 EXE
Arrow to Frequency. Press 1 (F1)
Choose your Mark Type

Press EXIT (Fig6)

Turn the axes On in SET UP (**press SHIFT MENU then arrow up. Use F1 to turn the Axes On.** (Fig7) Also ensure **Coord is On**.

Press EXIT (back to Fig6)

Press GPH1 (F1) (Fig8)

Q1) The concentration of pollution has decreased over the 40 days. But the rate of decrease has been slowing down over that time. Ie the pollution level was falling quickly at the start and very slowly by day 40.

Q2) Predicting how long it will take for the pollution to drop to 1 mg/m^3 is very difficult to determine by estimation. It appears the pollution could take 50 to 60 days to drop to the desired level.

Q3) No, because the scatter graph stops well short of the 1 mg/m^3 level.

Fig5

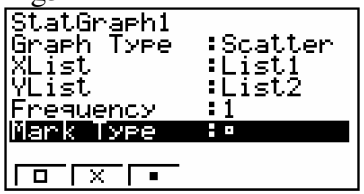


Fig6

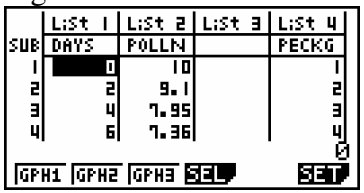


Fig7

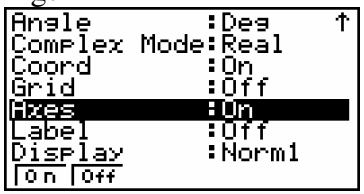
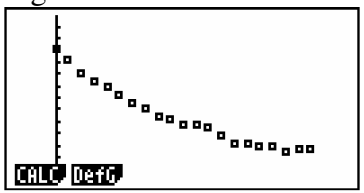
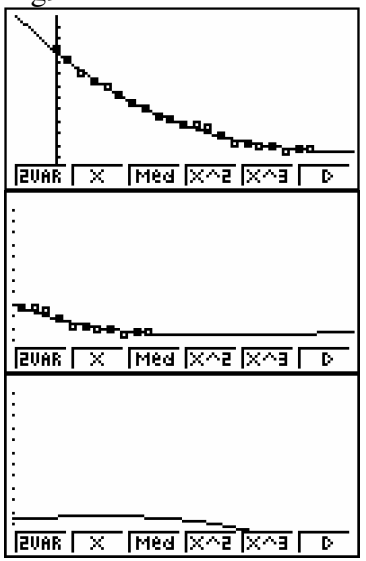


Fig8



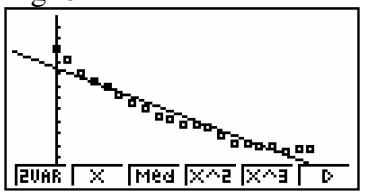
Q4) By scrolling to the far right we see that the graph starts to drop rapidly, and then cuts the x axis. This implies that the pollution levels, after remaining very low for a while, would start to drop more suddenly and than become negative, which is impossible.

Fig9



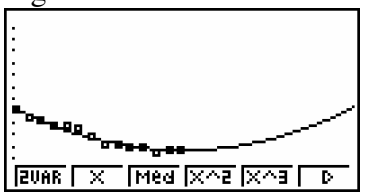
Q5) The appropriate model is not a straight line (Fig10) because the decrease in pollution is clearly a curve.

Fig10



The appropriate model is not a quadratic (X^2) (Fig11) because the pollution will not suddenly increase as the curve indicates.

Fig11



The appropriate model appears to be exponential (Fig12).

To find the number of days when the concentration of pollution falls below 1 mg/m^3 we need to trace the model.

However, if we Trace the curve by arrow-ing to the right the cursor stops before we reach our solution (Fig13).

Press EXIT and redraw the graph using GPH1. Redraw the exponential **model (CALC (F1) ▷ F6 Exp (F3) then DRAW** (F6)) (Fig12)

Before turning Trace on again we need to 'push' the graph forward. **Press the right arrow several times** until you think the solution (at Y=1) is somewhere on the screen (Fig14)

Now Trace **(SHIFT F1) and scroll forward** until Y first drops to below 1 (Fig15)

We can see that the model suggests the pollution will first fall to the safe level on the 48th day.

Q6) Likely explanation is that it took several days for the pollution to flow from Lake Emmie into Lake Taza and therefore the pollution levels in Lake Taza took a few days to rise to their maximum. From there they fell in a similar fashion as they did in Lake Emmie.

Q7) Many possible scenarios. Here is one: The pollution was deliberately put into Lake Emmie. The polluters wanted to contaminate Lake Taza as well. They measured the levels in Lake Taza a few days after contaminating Lake Emmie and much to their surprise pollution levels in Lake Taza were negligible. For reasons that puzzled the authorities the polluters emptied small amounts of the pollution into the middle of Lake Taza every few hours over a six day period. Then they stopped.

Fig12

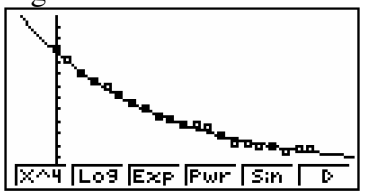


Fig13

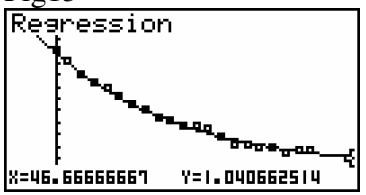


Fig14

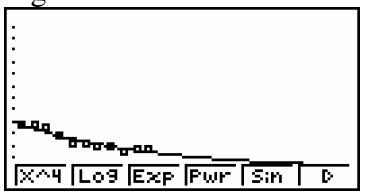


Fig15