

Assessing chance

A unique learning experience – Student Booklet



A product of the Noel Baker Centre for School Mathematics

WIP (Work in progress)

***LUMAT-NSW (2003) is the initiative of the
Noel Baker Centre for School Mathematics and CASIO AUSTRALIA.***



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About using these materials – the method in the madness

A constructivist approach – the holy grail.

Higher-level thinking is encouraged

Student thinking drives lessons

Students are engaged in dialogue with the teacher and each other

Student autonomy and initiative are accepted and encouraged

Students reflect on experiences and actions and then predict future outcomes

Students are involved in real-world possibilities using primary sources, raw data, physical and interactive materials

Student inquiry is encouraged through asking open-ended questions and allowing *wait time for responses*

Students are engaged in experiences that encourage conjectures, discussion and a search for the truth

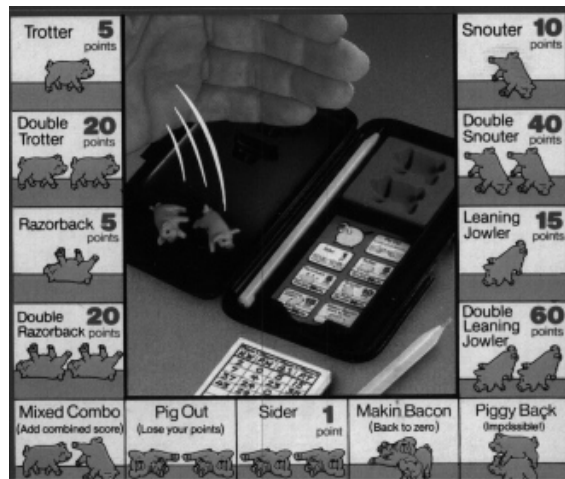
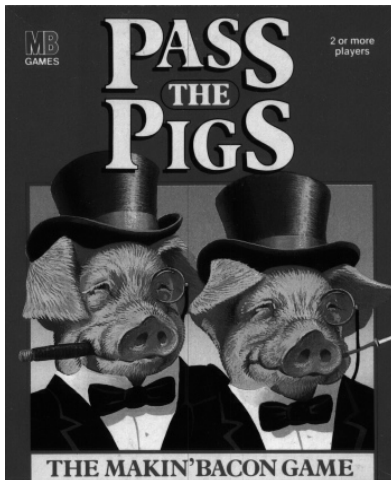
Harradine's learning framework

Stenduser – LJ etc

Game time

Time to kick back, relax and play some cool games. If time permits have a go at all of the games on offer. When you are finished, be sure to answer the 'reflection questions' at the end of each game and at the end of the section.

Pass the Pigs (for real)



Read the rules of the game and then get in it having some fun. Play just one game with two or three people. Be sure to record your results in a table like that given overleaf.

If you do not have the pigs?

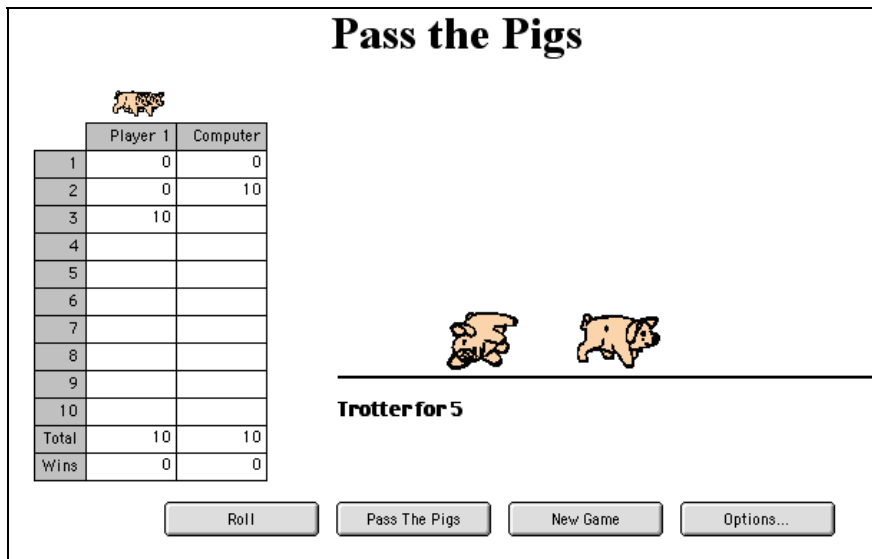
The pigs are commercially available. Most games shops should have them. Alternatively, contact Partington Agencies, 16 Reese Ave. Richmond, SA, ph (08) 8234 7811.

Alternatively you can make a cheap imitation – see Appendix 1.

Reflection Questions (answer after the game)

1. Why do you think the points offered are different in value for each outcome?
2. Name the positions that seem to come up most often? Are they allocated the least, most or and in between point score?
3. Name the positions that seem to come up the least? Are they allocated the least, most or and in between point score?
4. Note that a Double Trotter will get you 20 points but a Double Snouter will get you 40 points? What does this suggest about the relative difficulty of these two outcomes?
5. Does the point scheme seem fair to you? Justify your response by referring to what you saw happened when you played the game.
6. Do you think you have played the game enough to make an accurate comment on question 5? Explain

Pass the Pigs (virtually – a simulation of the real thing)



The game Pass the Pigs is also available as a *simulation* of the real thing. Go to:

<http://www.censusonline.net/games/pigs/oink.html>

Have a few games against the computer, try the different 'computer levels' – under the options window – enjoy.

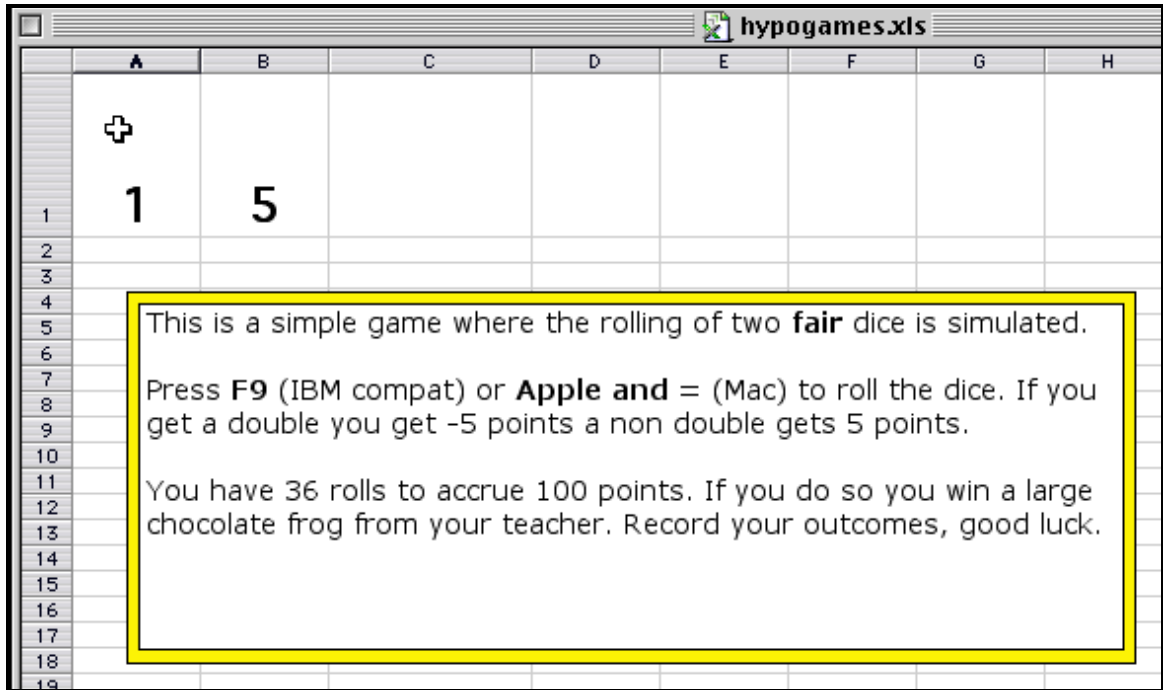
Reflection Questions (answer after the game)

1. Do the virtual pigs seem to behave the same way as the real pigs?
2. How do the different computer levels change the pig behaviour?
3. How is the behaviour of the virtual pigs determined?
4. Do you prefer the real game or the virtual game? Why?

Hypogames anyone?

Go to the computer room and access the spreadsheet called *hypogames.xls*.

Play each game – I hope you win a frog or two.

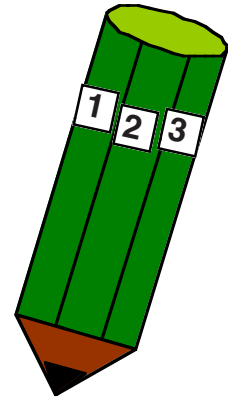


Reflection Questions (answer after the game)

1. How did you feel about the first game?
2. Did you feel differently about the second game than the first?
3. Do you think the games are fair?

Cricket anyone? Well a simulation of the real thing.

Grandpa says school was coll in his day too. He says that when he went to school, during mathematics classes he used to play cricket with just his pencil. The idea was to mark the faces of the hexagonal cross-section pencil " 1", "2", "3", "4", "6" for the runs scored per ball – and a "5" for a wicket (i.e. when a batsman is out). The team accumulates runs until the tenth batsman is out. Grandpa claims that he (or his team) often made scores of 400 or 500 – provided the teacher didn't catch them playing the game!



Use a die (more convenient than a pencil!) to play the game once.
Record ***all*** results.

Reflection Questions (answer after the game)

1. Compare your results with those of other students. What seems to be a ***typical*** score?
How could we improve our estimate of the average score?
2. What do you think a team could ***expect*** to get.
3. Do you think that Grandpa's '400 or 500 claim' is a bit like 'the fish that got away'? Explain your reasoning.
4. I think you would agree that the game as it is at present is a poor ***simulation*** of the real thing. Suggest some ways in which you would change it to make it more realistic. Make sure to indicate the reasons for each change. Maybe a simple dice does not suit the purpose.
5. If possible play your improved version of the game once and record the result.
Do you think that the average team score for the improved game will be greater or less than the average team score of the original game? Explain your reasoning.

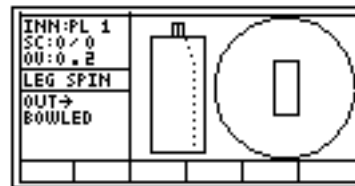
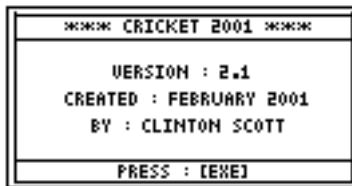
Cricket anyone? Well an electronic simulation of the real thing.

Ask your teacher to load the 'cricket' program (all of its parts) into your graphic calculator. This program was written by Clinton Scott – February 2001.

Go to the PRGM menu and put the cursor on the program called CRICKET and use EXE (F1) to run the program.



Follow the self explanatory instructions and enjoy.



Reflection Questions (answer after the game)

1. Did you enjoy this game? Why or why not?
2. Can you start to imagine how the calculator is doing what it does?

Coin on the board

Some of you may have played a game where you throw a coin of some denomination onto a board that has squares drawn on it. If the coin lands entirely in a square then you win and if it lands crossing a boundary then you lose.

A win is normally rewarded with a pay out of more money than you threw.

Take the 'grid' attached as **Appendix 2** and place it on the floor in between you and your partner's desk.

The rules of the game are as follows:

- *A five-cent coin is to be dropped onto the grid from standing height.*
- *If the coin lands such that no part of it covers a line on the grid then you are a winner and you receive your **coin back plus another two five cent pieces.***
- *If the coin lands such that it covers, in anyway, a line then you lose your coin.*
- *Your teacher will pay up whatever you win.*

Play the game 80 times – you might make enough to buy lunch! Keep an accurate record of the outcome of each throw. Use a table like that given overleaf for this purpose. Use the concept of *directed numbers* to keep a tally of how much money you are winning or losing.

Throw No.	Win or Lose	Bank balance	Throw No.	Win or Lose	Bank balance
1			41		
2			42		
3			43		
4			44		
5			45		
6			46		
7			47		
8			48		
9			49		
10			50		
11			51		
12			52		
13			53		
14			54		
15			55		
16			56		
17			57		
18			58		
19			59		
20			60		
21			61		
22			62		
23			63		
24			64		
25			65		
26			66		
27			67		
28			68		
29			69		
30			70		
31			71		
32			72		
33			73		
34			74		
35			75		
36			76		
37			77		
38			78		
39			79		
40			80		

Reflection Questions (answer after the game)

1. How many wins and how many loses did you have – how many did you expect.
2. How much money did you have after 80 throws.
3. What would you expect to happen if you played the game 1600 times?
4. Now clearly this would be a trying task. So pool our results with the rest of the class, document them in tables like that given below. We will pool then for different totals to see. How close was your expectation to the result?

Number of wins from 80	Number of loses form 80	% of wins	Bank balance at end

Number of wins from 400	Number of loses form 400	% of wins	Bank balance at end

Number of wins from 800	Number of loses form 800	% of wins	Bank balance at end

Number of wins from 1600	Number of loses form 1600	% of wins	Bank balance at end

5. Comment on how the results from 80 throws compare to the results of 1600 throws.

After match debrief

Reflection Questions – answer after all games played.

1. What makes the games fun?
2. Why do we play them?
3. How do the virtual games actually work?

Stenduser – Sorry teacher, I am late again – but it is not always my fault.

Are you often late to school? Why? Does your group teacher get upset with you? Is it ever your fault?

The situation

Ken is often late to school. His group teacher is unhappy with the situation and asks for an explanation. Ken says:

Picture of the rear end of a bus disappearing into the distance.

I attempt to catch the 7.00am bus every morning, my father drops me off on his way to work. Some of the time the bus is early, some of the time it is late, some of the time my father is early and some of the time he is late. Unfortunately there are times when the variation in our arrival times result in me missing the bus. It is just randomness Miss – I can't help it. OK?

The group teacher asks him to explain the bus arrival and his arrival pattern in detail.

Ken tells her that the bus comes up to 3 minutes early and as much as 3 minutes late and that his father arrives as much as 1 minute early and as much as 3 minutes late. Ken says the buses behaviour seems random – he knows his father's is!

The teacher wants to know how often Ken should be late if his claims are true and from this she will make her decision as to whether she is happy with the situation. You need to help her out.

Task One

The teacher could simply let Ken continue and see how often he is late for the year. What would be the problem with this approach?

Task Two

Assume that the bus can only arrive at the stop 'on the minute'. How many possible arrival times exist for the bus?

Task Three

Assume that the Ken's father can only arrive at the stop 'on the minute'. How many possible arrival times exist for him, and hence Ken?

Task Four

What is the chance that:

- the bus is early?
- the bus late?
- Ken is early?
- Ken is late?

Task Five

Assuming both the bus and Ken can only arrive on the minute, describe all possible situations, indicating which of them result in Ken missing the bus and which result in him catching the bus.

Can you now determine the proportion of the time that Ken will catch the bus?

Task Six

Use electronic technology simulate Ken's situation for *one term* of the school year and determine the proportion of time Ken is late. Assume both the bus and Ken can only arrive on the minute.

Compare your result with those of your classmates.

Task Seven

The assumption that both the bus and Ken can only arrive on the minute is a little unrealistic, but helped us to get a start.

Refine your simulation to make is more closely resemble the actual problem. Use the refined simulation to determine the proportion of time Ken will be late in one term.

Task Seven

Build a theoretical model that closely models the real situation from which you can determine the probability that Ken will be late for school.

Unpredictable events

There are many events that we know *can and do* happen, but for which the actual moment of occurrence is completely unpredictable. Some of these events occur frequently, others are quite rare. An unpredictable event can be a freak event but *not* necessarily.

Cartoon of a bird pooing in mouth while yawning.

Taking letters out of a letter box – saying “only two today.”

Many freak events are often referred to as acts of nature and definitely fall into the category of unpredictable. Many humans spend their lives affected by acts of nature – whether they be pre or post birth. The (in)famous J.T Barnum made a living from exhibiting humans who were misfortunate enough to be born looking different to most. The website

<http://www.missinglink.free-online.co.uk> will give you an insight of a past era.

Some examples of unpredictable events are:

- The hiccups starting/stopping
- A burning piece of wood pops and hit you right in the eye
- A fly flies in your mouth while yawning
- Getting an A in a maths test (oops – this is not unpredictable, is it?)
- Throwing snake eyes
- Finding a four leafed clover
- Contracting a cold
- You are involved in a car crash
- Your house or car are broken into
- You win the lottery



Interaction A

1. Given we all have very different lives and life experiences each of us will have very different examples of unpredictable events to offer. Make a list of unpredictable events that you have seen and share them with your class.
2. Make a cartoon of the funniest event shared and display it in your classroom.

How likely is the occurrence of an unpredictable event?

So if these things are unpredictable is there any value in trying to analyse them?

Can we determine' how likely' a given outcome is even if we can't predict whether or not it will, or even when it will happen.

You will have heard statements like:

- How likely is that?
- What's the chance that?
- It is a 50-50 chance
- You have a 10% chance
- Your chance is about 1 in 100
- Very unlikely
- It is a certainty

While it is impossible to predict when an unpredictable event will occur, mathematicians have developed a science called *probability* that is the **study of building theoretical models of real situations** that determine the chance that an unpredictable event will occur.

Why bother determining the *chance*?

Fortunes have been won and lost on attempting to predict unpredictable events. People's lives are affected every day by unpredictable events. Humans seem to have an innate desire to study the unpredictable. And the results can lead to lots of fun!



Interaction B

Consider and discuss the following facets of life and how they relate to 'the chance that':

- gambling
- insurance
- slip, slop, slap
- installing an alarm to your car or home
- using deodorant
- having fun

Some real situations where probability was important.

Read and discuss each of the following real situations that involve the concept of 'determining the chance that'.

Each is taken from a booklet called "Statistics: a job for professionals" released in June 2003 on the website of SSAI. The booklet can be downloaded at:

<http://www.statsoc.org.au>

Statistical miscarriage

British mother Sally Clarke was convicted of murdering her two baby sons, because the court believed it was highly improbable she could lose two babies to cot death.

Medical experts, unqualified in statistics, had testified that the chance of her losing two babies to SIDS was 1 in 73,000,000 and the jury believed them. The Royal Statistical Society later rejected their estimate, stating it was without statistical basis, likely to contain a very large error and was 'probably false'.

Deadly lottery

Fifty thousand Americans died in the Vietnam War, many of them conscripts. Later, statisticians concluded that the 1970 birth date lottery by which conscripts were drafted was flawed and unfair. The lottery planned to select birth dates in random order. Birth date capsules were placed in a bowl a month at a time, starting with January and finishing with December. Because the capsules weren't mixed thoroughly enough, men born later in the year had a higher chance of selection (capsules nearer the top). Undoubtedly some of these men were among the war's casualties, while others, who were not called up, escaped.

Catching drug cheats at the Sydney Olympics

An Australian statistician helped protect the good name of the Sydney Olympics and sport in general by devising a test to detect athletes abusing the drug erythropoietin. The drug is hard to detect because it occurs naturally in the human body. However when the synthetic drug is used it causes red blood cell numbers to climb sharply – then drop again sharply when usage stops or declines.. A statistician devised a testing regime that gave a one-in-four or one-in- three chance of catching drug cheats. Sports officials believe this was enough to deter would-be abusers and keep the Olympics as clean as possible.

Holes in the missile shield

Simple statistics were sufficient to blow holes in the US Government's plans for a national missile defence shield. Nobel physics laureate Burton Richter pointed out that if there is an 80 per cent chance of an interceptor killing an incoming warhead, then if five warheads are launched, the probability of hitting all five is only 33 per cent, meaning that some are bound to get through. To reduce the chance of a warhead striking the US to 10 per cent, the interceptors have to be at least 98 per cent reliable, he pointed out. Warheads can be deployed with decoys that reduce the intercept rate, so the chance of getting all the active warheads without one getting through becomes highly problematic. A case where a sensible look at the statistics might save a lot of political argument and expense.

Viewing and studying randomness

The word **random** is used to describe any process, the outcome of which is **not pre-determined** and hence, is unpredictable – such a process is called a **random process**.

The concept of randomness is the key to the science of determining ‘the chance’.

An example of a random process is the rolling of a 6-faced die. Such a process will have an **outcome**, there are 6 possible outcomes (1,2,3,4,5, and 6) and the outcome of the rolling is, of course not pre-determined before rolling.

Is rolling a pig (from Pass the Pigs) a random process? What are the possible outcomes of rolling a pig?

The word outcome is often replaced with **event**, so rolling a 5 is an example of a **random event**. Not that **rolling a die** would be called a **random process**.

Most things in the world have some random aspect in them, which is why it is so hard to predict the future accurately.

Truly random processes – you have experienced lots already.

The essential element to a truly random process is that (as far as humans can determine) the outcome is not predetermined. Example of this are:

- The time you arrive at a set of traffic lights in your car in peak hour traffic
- The amount of coke in a 375 ml can
- Brownian motion – ask your science teacher about this
- Radio active decay rates
- Electronic noise currents (see <http://moebius.psy.ed.ac.uk/REG H.html>)



Interaction D

1. Make a list of truly random processes that you have experienced and list the all the possible outcomes for each
2. Toss the die twenty times, **recording** the number on the uppermost face each time in a list. Which number(s) occurred most often? Check with other students.
3. Toss the die another twenty times and **record** your results. Can you see any patterns in the numbers that result?
4. How many times has each number resulted? Compare your answers with other people.
5. Each person in your class tossed a die 20 times. How many fives resulted altogether. Check how many times the other numbers resulted. Describe anything you notice.

When you roll a die, the number that is uppermost will *vary* in an *unpredictable manner*.

When I did Interaction D, my results were:

1,3,2,4,4,5,1,2,1,3,6,4,3,2,1,4,4,3,3,2

This is a string of number that has been produced by a random process and so the number that is uppermost is called a *random variable*.

Random variables are abbreviated with capital letters. So, let the number that is uppermost be X . This is a different concept to the variables from your study of algebra – they describe *predictable variation*.

The possible values for the random variable X are 1,2,3,4,5,6, **Pool your results** with the rest of your class from Interaction D **so you have a total of 120 rolls**. Record your results in a table like that below.

X	frequency (f)
1	
2	
3	
4	
5	
6	
	$\Sigma f = 120$



Interaction E

1. Describe the feature(s) of a dice that makes rolling it a random process.
2. List other devices that give rise to a random process.

Pseudo random processes (PsRP) – lets experience one, lots of times

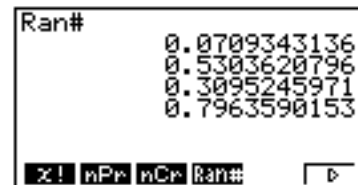
Pseudo random processes *share many of the same properties of truly random processes*, but are totally predictable – if you know some insider information. Pseudo random processes are of great use (as you will see later).

You may have heard of ‘random numbers’, either in the form of random number tables or the **RAND#** feature on computers/calculators. The production of a so-called random number is an example of a PsRP.

You can use your calculator to generate numbers using a pseudo random process* (sort of equivalent to rolling a dice). Enter RUN mode (by pressing MENU and 1), press OPTN and then access the PROB menu (F6 followed by F3). Ran# (F4) is a command that produces a number larger than zero and less than 1 by a pseudo random process*.

The process is just as likely to produce a number in the 0.3's as it is in the 0.9's.

Press EXE continually after Ran# is entered.



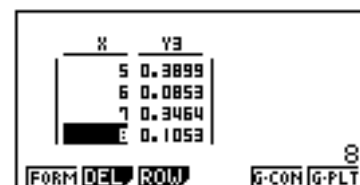
(* The process employed in the calculator to produce the number when the EXE key is pressed is an example of a *pseudo random process*. The numbers are actually the result of an algorithm based on a ‘seed’ value. The output is totally predictable, but the result mirrors truly random events well (certainly suitable more our needs). Many whole books and much research have been devoted to the study of processes that attempt to mirror genuine random processes and the quest to invent better ones. To find out a little more about the ‘pseudo random process’ used in calculators and the like’ visit: <http://www.brianhetrick.com/casio/gmrandom.html> or <http://www.ticalc.org/archive/files/fileinfo/293/29358.html> or www.howstuffworks.com/question697.htm

To see a lot of numbers that have been produced ‘randomly’, it is easier to make a table than to press EXE many times.

Enter TABLE mode (by pressing MENU and 7). Enter the Ran# command in the function list and use RANG (F5) to specify a table of 100 values, as shown below.



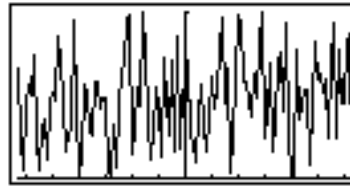
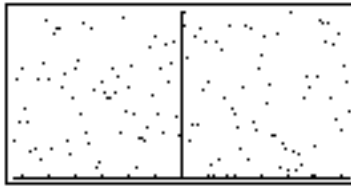
Press EXIT and then activate TABL (F6) to generate the table of 100 numbers, which you can explore with the up and down arrow keys.



You can get a picture of the 'random' process by drawing a graph of the table output. Set up the viewing window (Shift then F3) so that the y values are between 0 and 1, as shown opposite.

```
View Window
Xmin :-6.3
max :6.3
scale:1
Ymin :0
max :1
scale:1
INIT TRIG STD STO RCL
```

Press EXIT and then produce the table of numbers again (F6). The use G.PLT or G.CON (F6 or F5). Your graph will look a bit different from ours. Each time you make a new table the set of numbers will be different.



Notice that the graph on the right above is drawn with the points connected, which does not make much sense here; the graph on the left is drawn with the points plotted (not connected). Even though the connection does not make sense, it does allow a good view of the situation.



Interaction F

1. In RUN mode, use Ran# to generate six random numbers. How many are larger than 0.5? Repeat this experiment a few times. Compare your results with other students.
2. Make some tables of the Ran# function. Do this a few times. Write a paragraph to describe the numbers generated.
3. Draw some graphs of the Ran# function. Notice that each one is different. (Check this by looking carefully at one part of the graph each time - like the top right corner.) How would you describe the sets of numbers produced?
4. **When many numbers are produced, the Ran# function should produce numbers evenly spread between 0 and 1.** For example, about one tenth of the time the result should be ≥ 0 but ≤ 0.1 , one tenth of the time the result should be ≥ 0.1 but ≤ 0.2 and so on.
Does this seem to be the case for your tables and graphs? How did you work this out? **(The SORT or LMEM command on your calculator may be useful.)**

The ‘long run’ – studying the behaviour of random processes over time

While the outcome of a random process is unpredictable interesting things can be discovered if the process is studied for a long period of time, or in the *long run*.

Cartoon of a person getting older and older and older while rolling a die.

The only trouble with such studies is that they can take a very long time (and can be tedious)

The simplest random processor to study is a die. Earlier you and your class rolled a die quite a number of times. With a different class, **one student’s results** from a class were as follows:

<i>X</i>	frequency (<i>f</i>)	relative frequency (<i>rf</i>)
1	4	0.20
2	2	0.10
3	4	0.20
4	6	0.30
5	2	0.1
6	2	0.10
	$\Sigma f = 20$	$\Sigma rf = 1$

another one student’s results were:

<i>X</i>	frequency (<i>f</i>)	relative frequency (<i>rf</i>)
1	1	0.05
2	2	0.10
3	5	0.25
4	2	0.10
5	3	0.15
6	7	0.35
	$\Sigma f = 20$	$\Sigma rf = 1$

when the class results were pooled they found :

<i>X</i>	frequency (<i>f</i>)	relative frequency (<i>rf</i>)
1	17	0.142
2	20	0.167
3	25	0.208
4	18	0.15
5	24	0.20
6	16	0.133
	$\Sigma f = 120$	$\Sigma rf = 1$



Interaction G

1. Study the results of the three tables above carefully and describe any points of interest you can see in the results.
2. What do you think the results might look like if one was to keep rolling and keep rolling and keep rolling and?

A group of students did decide to keep on rolling and collectively rolled a dice 1200 times (lucky for you hey!) Their results were as follows:

X	frequency (f)	relative frequency (rf)
1	190	0.158
2	205	0.171
3	218	0.182
4	194	0.162
5	187	0.156
6	206	0.172
	$\Sigma f = 1200$	$\Sigma rf = 1$

As you can see the relative frequencies seem to be *less variable* with the more rolls that are done. It seems like each relative frequency is settling down to be somewhere between 0.15 and 0.19

So we could say that *over time (or in the long run) it is likely that each face will result roughly the same number of times.*

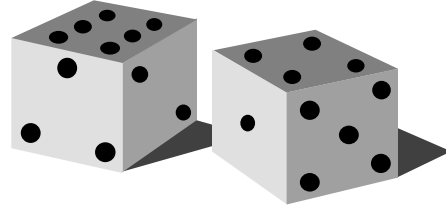
We could even say: *in the long run it seem likely we will roll a 1 about as frequently as any other number – about $\frac{1}{6}$ or $\frac{100}{6}\%$ the time – note that $\frac{100}{6}\% = 16\frac{2}{3}\% = 0.1\bar{6}$*

The ‘long run’ results of a die explained – building a theoretical model.

What makes the rolling of a die a random process?

Essentially it is the rolling act – it bounces all over the place and ‘where it stops nobody knows’! If we were to slide the die – a random process it would not be.

What characteristic(s) of the die make the outcomes close to equally likely, as we have seen in the previous tables?



A dice is a cube and as such, if the material it is made of is equally distributed throughout, it will have multiple planes of symmetry passing through its centre of gravity. Hence, each face is equally like to be ‘fallen on’.

So, ***in theory***, we can say that there are **6 *equally likely*** outcomes that are possible – such a die is called a ***fair die***.

The full set of outcomes or the ***sample space*** (S) and can be written as a set:

$$S = \{1,2,3,4,5,6\}$$

Since we have a *random process* with *equally likely* outcomes the fraction of the time *we would expect* to roll a 1 *over many rolls (in the long run)* is $\frac{1}{6}$ or $0.1\bar{6}$, which is remarkably close to the relative frequencies. The ***theoretical relative frequency*** is called the ***probability***.

By definition, each member of a sample space is as equally likely to result as any other.

What is in the box above is called a ***theoretical model***. We know it does not ***perfectly*** describe the random process of rolling a die many times, **but is a good model of the real thing.**

Some points for reflection:

- If we can build a satisfactory theoretical model of a random process we are analysing, it saves a great deal of time. The alternative is to do the real thing over and over again. In the adult world time is money!
- Sometimes we have to do the *real thing for a while* to get the feel of the problem before we can build a model. This way we also get an idea of what probability value the theoretical model should produce. This could mean a lot of repetition.
- Sometimes we might build a theoretical model first and then *do the real thing* to verify our theory is correct.
- Sometimes we use the theoretical model to make games to amuse us – because games of chance seem to be attractive to humans.



Interaction H – Sample spaces of random processes

1. What does the word ‘probability’ mean?
2. Write down the set that represents the sample space of outcomes for each of the following random processes – consider the coins and dice to be fair.
 - the toss of a coin
 - the roll of a four sided die
 - the spinning of a spinner with 12 equal divisions number 1- 12
 - the spinning of a spinner with 11 divisions numbered 1 – 11, with divisions 1-10 equal in size with the 11th being twice the size as all others.
 - a fair coin is tossed and a fair dice is rolled simultaneously
 - a coin is thrown onto a chess board
3. What is the probability that when a die is rolled and a coin is tossed simultaneously that a head and a 6 result?

Some formality

The probability of an event E , denoted $P(E)$, can be calculated by the following law:

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{number of outcomes for the event}}{\text{number of outcomes possible}}$$

note that the each of the possible outcomes must be equally likely



Interaction I

1. What is the smallest probability possible – what could you call an event with the minimum probability? Give four examples of such events
2. What is the largest probability possible – what could you call an event with the maximum probability? Give four examples of such events.
3. Determine the probability of each possible event when two coins are tossed simultaneously and sum the four probabilities. Was your result expected? Explain your answer.
4. Repeat the process seen in question 3 for two other situations.

Time to run some laps

Exercise 6B Page 179 of your text-book

Doing the real thing over and over as opposed to the simulating the real thing over and over electronically.

We can make use of pseudo random processors (like RAND# on your calculator) to simulate the real thing when we:

- Wish to make games for our enjoyment (maybe even non-electronically)
- Wish to determine chances involved and we can't build a theoretical model
- To test/validation theoretical models

Note that actually doing the real thing or simulating non-electronically may:

- take to long or
- be too hard or
- be too expensive or
- simply make no sense.

Experiment with the following commands on your graphic calculator in RUN mode:

1. **6Ran#**
2. **6Ran# + 1**
3. **Int(6Ran# + 1)** To obtain the **Int** command press OPTN and then use the DESmore arrow (F6) and then use the NUM (F4) menu to reveal the INT (F2) command.
4. **Int(6Ran#) + 1**
5. **Int(Ran# + 0.5)**
6. **Int(Ran# + 0.9)**



Interaction J

1. For each command above, state the numbers generated and whether or not the outcomes are equally likely.
2. Use the appropriate command to simulate rolling a dice. Does it seem to be a good replacement for the real thing?
3. Fred has determined from past records that his chance of being put 'on hold' when he calls a certain company is 80%. Simulate him ringing the company 20 times using your calculators Rand# function. Are the results what you would expect?

See Appendix 3 to help you get into 'cricket gaming' on the Casio 9850 GB Plus.

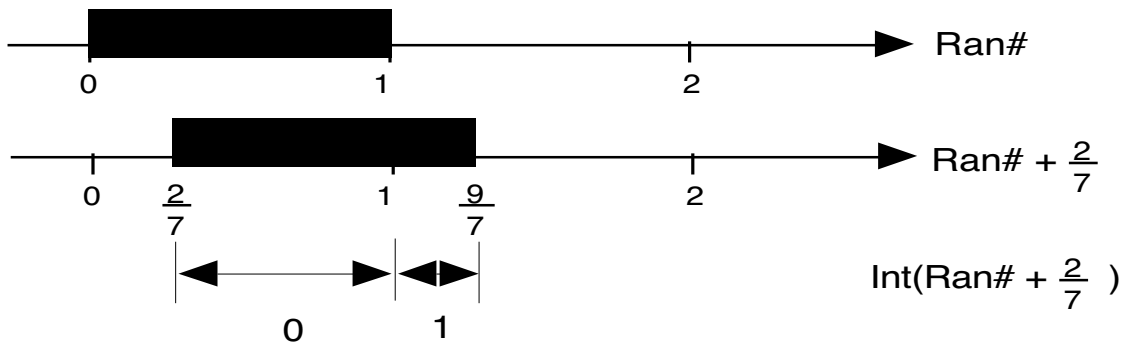
Coding Sensibly

Random processes are a useful in playing games of chance. They are also useful when public credibility is important – such as in drawing "lotto" numbers, making the draw for a tournament, determining who is chosen for military conscription (!) or deciding who kicks to which end in a footy game or the like.

We could use some physical random processor in situations like those above (like a coin at the footy) or we could numerically code the possible outcomes so that a random number generator (like our calculator) could be used instead.

Example: If a day of the week is chosen in a random manner, what is the chance it is a weekday? Obvious answer here (5/7), but how could we simulate many 'choosings'

- We reason that since 5 days in each 7 day week is a weekday we could draw one card from a well shuffled pack of 7 cards of which 5 are marked WEEKDAY and 2 are marked WEEKEND.
- Since Ran# gives a randomly chosen number in the interval (0, 1) then $\text{Ran\#} + 2/7$ gives a randomly chosen number in the interval $(2/7, 9/7)$. Hence $\text{Int}(\text{Ran\#} + 2/7)$ returns either 0 or 1, *but not equally often*. The diagram below shows that 0 is returned, in the long run, 5/7 of the time.



Lets check it out.

Enter TABLE mode and define Y1 as shown opposite.

```
Table Func :Y=
Y1|Int (Ran#+2,7)
Y2:
Y3:
Y4:
Y5:
Y6:
[SEL DEL TYPE COLR RANG TABL
```

Set the range of the table RANG (F4) as shown

```
Table Ranse
X
Start:1
End :210
Pitch:1
```

Press EXIT and use TABL (F6) to produce the table of numbers.

X	Y1
2	1
3	1
4	0

FORM DEL ROW G-COM G-PLT 1

To record the numbers for analysis, place the cursor in List 2, and press OPTN, then LIST (F1) and then LMEM (F2) and choose List1 (F1). The data will then be stored in List 1 of Stat mode.

Y1=Int. (Ran#+2.7)

X	Y1
1	0
2	1
3	1
4	0

List1 List2 List3 List4 List5 List6 0

Enter STAT mode and you will see your data

	List 1	List 2	List 3	List 4
1	0			
2	1			
3	1			
4	0			
5	1			

GRPH CALC TEST INTR DIST D

To determine the number of weekends picked we could now just add up List 1

Put the cursor into the first cell of List 2 and then press OPTN, then LIST (F1) then DES (F6) twice and enter the command see below (you will need to use DES again). Pressing EXE gives the sum.

	List 1	List 2	List 3	List 4
1	0			
2	1			
3	1			
4	0			
5	1			

Sum List 1
List L→M Dim Fill Seq D

	List 1	List 2	List 3	List 4
1	0	60		
2	1			
3	1			
4	0			
5	1			

List L→M Dim Fill Seq D

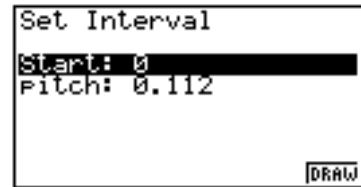
I got 60 from 210, which is amazingly $\frac{2}{7}$. How did you go? What did the rest of the class get?

The other way is to draw a graph of your data and trace it. This is useful for more complex cases.

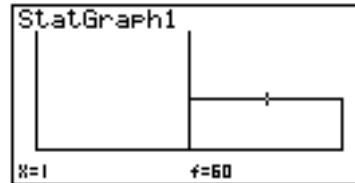
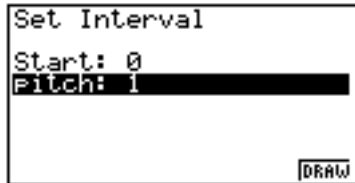
Press EXIT twice and then GRPH (F1) and SET (F6). Set up StatGraph1 as shown below.

StatGraph1	
Graph Type	:Hist
XList	:List1
Frequency	:1
Graph Color	:Blue
Hist Box Box N-Dis Brkn D	

Pressing EXIT and then GPH 1(F1) gives



Set the pitch to be 1 and press EXE then use DRAW (F6) to produce the graph. Using Trace (SHIFT then F1) you can see how many of each you have.



Interaction K

1. One in 12 people who book a plane fare are "no-shows" - i.e. they don't turn up for the trip. Many passengers are likely to be on a fully booked flight of 200 people? **Answer this question using both simulation and theoretical methods.** How do airlines cope with this?

2. In a 'blind' darts game you always hit the "four square" target shown (or else you get a re-throw) despite the results being quite random! For each group of three darts what might be your expected score?

2	4
1	3

3. Two dice are rolled – what is the most like sum? Can you answer this using a theoretical approach?

4. What does the following statement mean? “There is a 5% chance that a condom will fail.” Use your calculator to simulate the use of a condom – how did you fair?

5. Ask your teacher to put the program called ‘**LONGRUN**’ into your calculator. The code for the program is in Appendix 4. Run the program and experience the ‘long run’ graphically. Look at the code and be sure you can understand what it is doing. How does the program illustrate ‘winning streaks’. Then try out the spreadsheet called **longrun-schmude.xls** for a more in depth look.

6. Do you think you could have a go at some of the Stenduser now? If so have a go!

Making theoretical models

At the heart of the study of probability is the law:

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{number of outcomes for the event}}{\text{number of outcomes possible}}$$

where each of the possible outcomes are equally likely

Hence, when determining a theoretical model it must:

- identify all possible equally likely outcomes (and obviously illustrate how many)
- identify the number of outcomes that are associated with the event in which we are interested.

So, a theoretical model is really just a tool to help us count. So far, we have looked at cases where we could count in our head – there are many more difficult situations – like the stenduser.

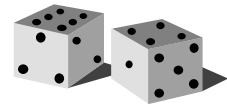
Compound Events - Introduction

Consider the following random processes:

- rolling two dice simultaneously
- rolling one die twice
- tossing a head and then a die

How do these random processes differ to those you have encountered so far?

Let's now focus on the ***rolling of two dice*** and the ***sum of the upper most faces***.



Interaction K

Form a group of four.

Two of you are to simulate the rolling of two dice 240 times on you calculator and determine the proportion of times each possible sum resulted. Document your findings with a **suitable table and graph** that displays the ***relative frequency*** of each possible outcome.

The other two should attempt to develop a theoretical model that clearly **displays all the possible outcomes** and using this model determine the ***probability*** of rolling each of the possible outcomes.

Once complete, compare the relative frequency values to the probability values.

Let's now focus on the *rolling of three dice* and the *sum of the upper most faces*.



Interaction K

1. Form a group of four (swap jobs from last time).

Two of you are to simulate the rolling of two dice 216 times on your calculator and determine the proportion of times each possible sum resulted. Document your findings with a **suitable table and graph** that displays the *relative frequency* of each possible outcome.

The other two should attempt to develop a theoretical model that clearly **displays all the possible outcomes** and using this model determine the *probability* of rolling each of the possible outcomes.

2. Once complete, compare the relative frequency values to the probability values.
3. How many *equally likely outcomes* are there, 16 or 216. Explain your answer.

Compound Events - Grids and Tree Diagrams as theoretical models

Grids (or Dot diagrams)

Some random events that consist of two *unconnected actions* can be theoretically modelled with a *grid*.

By *unconnected* we mean that the outcome of one action is in no way affected by the other action.

See Example 1 and 2 on page 188 of your text book.

Tree Diagrams

If a random event consists of more than 2 unconnected actions, a dot diagram does not have enough dimensions to cope. In such cases a *tree diagram* may be of assistance. This allows us to model as many unconnected actions as we like.

See Example 1 and 2 on page 189 of your text book.

Time to run some laps

Exercise 6D Page 190 of your text-book.

Back to that Stenduser

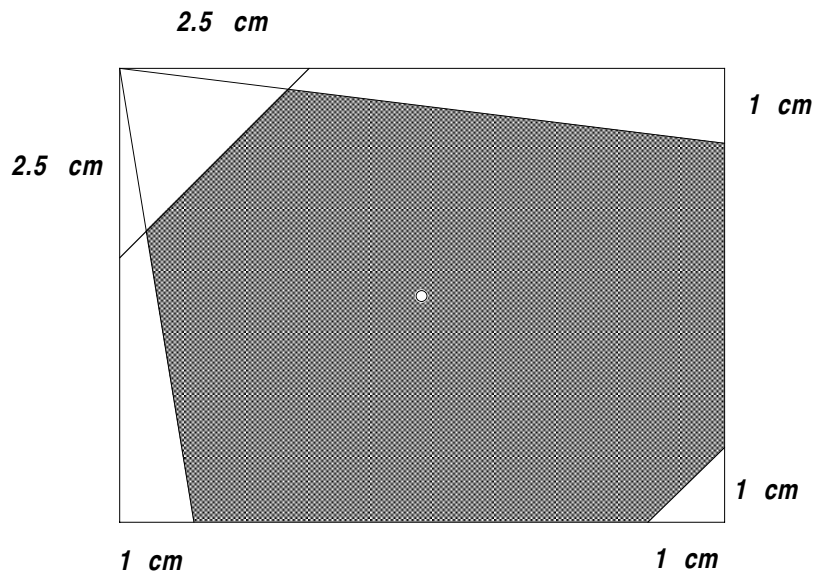
Well, if you have not already done so, go back and solve that stenduser.

You should be equipped to do so using a simulation approach. Your simulation may not be an exact replica of the actual situation, but that is OK, as in the real world of simulations most are not exact replicas.

You should also be able to provide a reasonable theoretical model of the situation as well. Reconsider the solution to the 'Coin on the Board' problem – it just may help.

Appendix One – A cheap imitation of a pig.

From a piece of stiff card **8 cm by 7 cm** make a shape as directed below. Glue a match or toothpick into the centre hole and you have a pig **spinner** – nowhere near as much fun, but it has all the correct characteristics to achieve the same learning outcomes – except extreme enjoyment. Name each edge as you like, and develop a game from the spinner similar to Pass the Pigs. *Diagram not to scale*, but the dimensions have been tested and work quite well.



Appendix Two – The coin board

See next page.

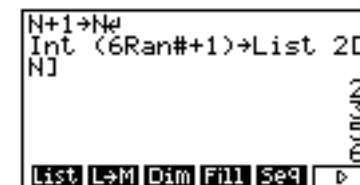
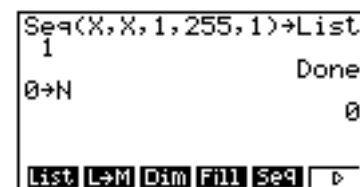
Appendix Three – Getting into ‘cricket gaming’.

You could play cricket in run mode by entering $\text{Int}(6\text{Rand}\#+1)$ and continually pressing the EXE key, but keeping track of the total runs and the number of wickets (players "out") is still fairly tedious.

One way in which our graphics calculator can help is to save all the random numbers as a list and then to add up the list after the tenth wicket falls.

Method:

- Start in RUN mode
- Enter **255** into **Dim List 1** by pressing $\boxed{2} \boxed{5} \boxed{5} \boxed{\rightarrow}$ and then $\boxed{\text{OPTN}} \boxed{\text{F1}}$ to get the List menu, then select Dim with $\boxed{\text{F3}}$ and List 1 with $\boxed{\text{F1}} \boxed{1}$ and the press $\boxed{\text{EXE}}$.
[This tells the calculator that List 1 may contain up to 255 numbers, the maximum that the calculator can accept.]
- Repeat for List 2
- we will use List 1 to record the "ball number" so fill it with the numbers 1, 2, 3 ... by pressing $\boxed{\text{OPTN}} \boxed{\text{F1}}$ to get the List menu, then select Seq with $\boxed{\text{F5}}$, enter $\text{Seq}(\mathbf{X}, \mathbf{X}, \mathbf{1}, \mathbf{255}, \mathbf{1})$ and press $\boxed{\text{EXE}}$
- Enter $\mathbf{0} \rightarrow \mathbf{N}$ to store 0 as the value of the list place counter N.
- Enter $\mathbf{N+1} \rightarrow \mathbf{N} \downarrow$ [Use $\boxed{\text{SHIFT}} \boxed{\text{EXE}}$ for \downarrow]
 $\text{Int}(6\text{Rand}\# + 1) \rightarrow \text{List 2}[\mathbf{N}]$ (You must use "square" brackets here.)
Then press $\boxed{\text{EXE}}$
[This updates the list place counter and calculates the first random number which is placed in position 1 of List 2.]
- Keep pressing $\boxed{\text{EXE}}$ until 5 has appeared for the tenth time
- Now press $\boxed{\text{MENU}} \boxed{2}$ to switch to STAT mode and observe that List 1 contains the ball number sequence and List 2 contains your random numbers.



- Locate the cursor on the "List 3" header
- Press **OPTN** **F1** to get the List menu and then **F6** **F6** **F3** to get the Cuml menu. Then enter List 2 by pressing **F6** **F1** **2** and **EXE** What do the entries in List 3 show?

	List 1	List 2	List 3	List 4
1				
2	1	2		
3	2	3		
4	3	4		
5	4	5		
	5	6		
		2		

Cuml List 2

List L→M Dim Fill Seq | ▷

	List 1	List 2	List 3	List 4
1				
2	1	2	2	
3	2	3	5	
4	3	4	10	
5	4	5	16	
	5	6	18	
		2		

List L→M Dim Fill Seq | ▷

- Find the last non-zero entry in List 2. (It should be a 5 – if it is not you have batted the 11th man like I did below and got a 6!) What is represented by the corresponding entry in List 3? What is the team total?

	List 1	List 2	List 3	List 4
41				
42	41	6	140	
43	42	5	145	
44	43	6	151	
45	44	0	151	
	45	0	151	

List L→M Dim Fill Seq | ▷

Better still - a program

You might think that the previous approach is easier on the memory but harder on your typing fingers! This is especially true if we play the game several times and have to enter the "instructions" each time we start a new game (or make a mistake in typing!). Your computer or graphics calculator can of course remember instructions if they are written in the form of a **program**. You were entering lines of a program in the last activity. The code for this program is as follows:

DCRICKET

```
0→A:0→W:0→T↵
ClrText↵
" "↵
" "↵
"xxxxxxxxxxxxxxxxxxxxxxxx"↵
"SCORE:"↵
" "↵
"    WICKET FOR"↵
Lbl 1↵
Int (6Ran#+1)→A↵
If A=5↵
Then Locate 6,1,"OUT.!!"↵
W+1→W↵
If W>1↵
Then Locate 12,6,"S FOR"↵
IfEnd↵
Goto 3↵
IfEnd↵
T+A→T↵
Locate 6,1,A↵
If A=1↵
Then Locate 7,1,".RUN."↵
Goto 3↵
IfEnd↵
Locate 7,1,".RUNS"↵
Lbl 3↵
Locate 3,6,W↵
Locate 18,6,T↵
If W=10↵
Then Goto 2↵
IfEnd↵
Goto 1↵
Lbl 2↵
Locate 8,7,"GAME OVER"↵
```

- Import the program "Dice Cricket" into your calculator (seek the program from your teacher) and use it play a one innings match against your neighbour. Record your win/loss record here.

Wins:	Losses
-------	--------

 Can you improve the game?
- Copy and modify the program to play your improved version of Dice Cricket. Play a one innings match against your neighbour.

Record your win/loss record here.

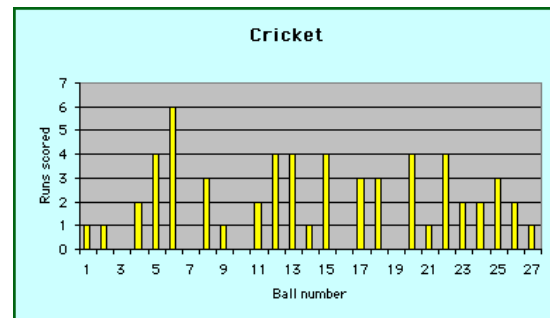
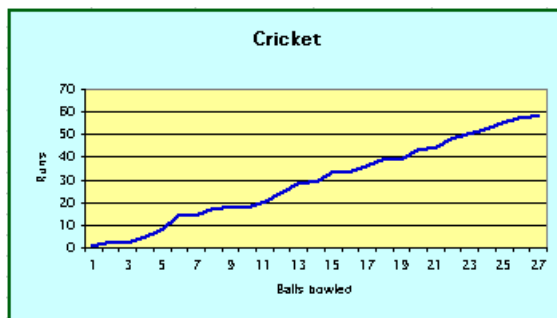
Wins:	Losses
-------	--------

A computer spreadsheet can also do the job.

- Construct a spreadsheet similar to the one shown on the next page and use it to play one innings of the basic Dice Cricket game.
- Modify your spreadsheet to play one-innings of your improved form of the game.
- Use your spreadsheet to draw the kind of statistical report graphs that you see on the cricket coverage on television:
 - (1) a line graph showing the cumulative score
 - (2) a histogram showing ball by ball results
- Can you make your spreadsheet play Team Australia against Team Visitors and declare a winner? Is this a good model of international cricket?

Example spreadsheet

	A	B	C	D	E	F
1	CRICKET					
2						
3	Rules: 5=out, {1,2,3,4,6}=runs, 10 out = end of innings					
4						
5	Rand#	Runs	Wickets	Total Wickets	Total Runs	
6	1	1	0	0	1	Play continues
7	1	1	0	0	2	Play continues
8	5	0	1	1	2	Play continues
9	2	2	0	1	4	Play continues
10	4	4	0	1	8	Play continues
11	6	6	0	1	14	Play continues
12	5	0	1	2	14	Play continues
13	3	3	0	2	17	Play continues
14	1	1	0	2	18	Play continues
15	5	0	1	3	18	Play continues
16	2	2	0	3	20	Play continues
17	4	4	0	3	24	Play continues
18	4	4	0	3	28	Play continues
19	1	1	0	3	29	Play continues
20	4	4	0	3	33	Play continues
21	5	0	1	4	33	Play continues
22	3	3	0	4	36	Play continues
23	3	3	0	4	39	Play continues
24	5	0	1	5	39	Play continues
25	4	4	0	5	43	Play continues
26	1	1	0	5	44	Play continues
27	4	4	0	5	48	Play continues
28	2	2	0	5	50	Play continues



Appendix Four – LONGRUN program.

This program was written by Barry Kissane – Murdoch University.

Visit Barry Kissanes website for more neat programs:

<http://wwwstaff.murdoch.edu.au/~kissane>

```
GridOff↵
-0.1→Ymin↵
1.1→Ymax↵
0.1→Yscl↵
0→Xmin↵
126→Xmax↵
10→Xscl↵
"PROBABILITY"?→P↵
Horizontal P↵
0→S↵
0→K↵
0→X↵
While X<126↵
Int (Ran#+P)→T↵
If T≠1↵
Then Goto 1↵
Else S+1→S↵
IfEnd↵
Lbl 1↵
K+1→K↵
PlotOn X,S÷K↵
X+1→X↵
WhileEnd↵
"RELATIVE FREQUENCY"↵
S÷K↵
```