

Pup Tent Investigation

A note to teachers:

This is a 'real-world' algebraic modelling investigation. In order to successfully attempt this activity you or your students will need to be competent with generating tables of values and drawing graphs with the 9860. These skills can be learned via the 'Self Guided TABLE-GRAPH' document available on this site. Also available on this site are other, simpler algebraic modelling investigations.

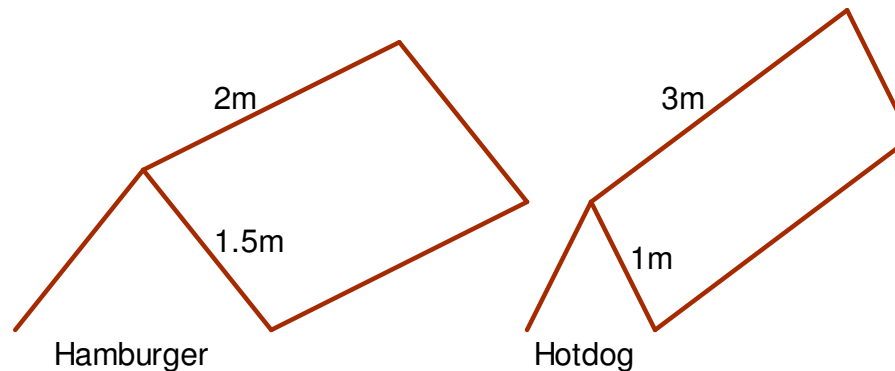
NOTE: When running this investigation it may be advantageous to hand out Investigation Two after posing the opening question of Investigation Two. This is because the answer to the question appears in the opening paragraph.

The purposes of this activity include:

- To engage students mathematically
 - To give students experience of applying algebra to a problem
 - To represent the problem through multiple mathematical displays
 - To efficiently generate a table of values:
 - To enable students to 'see the problem' represented in the numbers
 - To expose students to finding a solution within a table
 - To efficiently generate graph/s for the problem:
 - To enable students to 'see the problem' represented in the graph/s
 - To find a graphical solution
- (and arguably the most important of all)
- To afford the teacher multiple opportunities to ask leading, higher-order-thinking questions

Which pup tent is best? An algebraic modelling exercise

You are going camping with a small group of friends. Rather than hire or buy tents you have decided to carry one lightweight tarp which you will use to make a pup tent (see below). The tarp measures 2m by 3m. There are two ways you can set up the pup tent. One method (hamburger) is to fold the tarp in half along the 2m length and place it over a tight rope. The second method (hotdog) is to fold the tarp in half along the 3m length and place it over the tight rope. The 'hotdog' is obviously a longer, thinner pup tent than the 'hamburger'. The slant heights of the 'hamburger' and 'hotdog' are 1.5m and 1m respectively.



Types of Pup Tent

INVESTIGATION ONE:

NOTE: A piece of A4 paper has dimensions approximately in the same ratio as the tarp (2:3)

1) (Suggested time: 10 - 15 min) Working with a colleague use two pieces of A4 paper to help you answer the following question:

Which pup tent do you think will give the greatest volume?

You need to justify your answer clearly, with diagrams and clear explanations. Refer to the 'Things to think about' listed below. You do not have to use any algebra for this activity.

Things to think about:

- How many pup tents with different dimensions can be made from this tarp?
- Are all pup tents which can be made from this tarp worthy of being considered? If yes, explain. If not, why not?
- What aspects of the tent affect its volume?

2) Share your findings with others / or the whole class.

3) Now we will investigate this problem more precisely with the use of a graphic calculator. NOTE: This section may be done as a whole class activity, individually, in pairs or a combination of the above, depending upon student competence with algebra and with the graphic calculator.

Suggested steps to be used for this investigation:

- a) Draw a clear, good-sized diagram for the hamburger and hotdog pup tents.
- b) Develop a formula for volume for each. NOTE: Your formula must have only two variables, one for volume and one other variable.
- c) Convert the formulae so that the only variables are Y (for volume) and X
- d) Enter the formulae into TABLE mode, generate a table of values, inspect the values to gain a close idea of the dimensions of the hamburger which give the maximum volume. Repeat for the hotdog.
- e) Set up the axes in preparation for graphs. Turn Axes and Coordinates on in Set Up.
- f) Generate a graph for the hamburger and the hotdog. Explain the shape of the graph.
- g) Make your conclusion: Which pup tent type gives the maximum volume? (was your prediction correct?) Are there any other aspects which make one tent better than the other in your opinion. Explain.

INVESTIGATION TWO:

Is the maximum volume the most important factor in determining which type of pup tent to use for your camping trip? Is there a different measurement that might prove more relevant to your choice of tent in which you will sleep? It could be argued that floor-area, rather than volume is a more useful measurement, given that you will be sleeping on the ground (a 2-dimensional space)

1) (Suggested time: 10 - 15 min) Working with a colleague, and with the help of 2 pieces of A4 paper answer the following question:

Which pup tent do you think will give the greatest floor area?

You need to justify your answer clearly, with diagrams and clear explanations. You do not have to use any algebra for this activity. NOTE: There will be some important issues/considerations which will arise when investigating this problem.

2) Share your findings with others / or the whole class.

3) Now we will investigate this problem more precisely with the use of a graphic calculator. NOTE: As for Investigation One, this section may be done as a whole class activity, individually, in pairs or a combination of the above.

Suggested steps to be used for this investigation:

- a) Draw a clear, good-sized diagram for the hamburger and hotdog pup tents.
- b) Underneath each diagram draw another diagram of the floor area.
- c) Develop a formula for the floor area for each pup tent type. NOTE: Your formula must only have two variables, one for area and one for the width of the floor.
- d) Convert the formulae so that the only variables are Y (for area) and X
- e) Enter the formulae into TABLE mode and generate a table of values. Inspect the values to gain a close idea of the dimensions of the hamburger which give the maximum area. Repeat for the hotdog. What is significantly different about this situation compared to the volume investigation? NOTE: You will need to decide on a pup tent height for both hamburger and hotdog. Use the same heights that you found for the maximum volume calculations in Investigation One.
- f) Set up the axes and draw the graphs.
- g) Make your conclusion: Which pup tent type gives the maximum floor area for your chosen tent height? Was your prediction correct?

Pup Tent Investigation – Solutions and Instructions

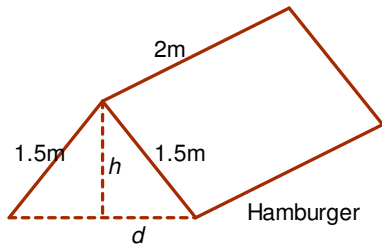
Instructions

Screenshots

Investigation One

Q3a) and b)

Applying algebra to the problem:



Area of triangle:

$$A = 2 \times \frac{1}{2} \times h \times d$$

$$= hd$$

Volume:

$$V = 2hd$$

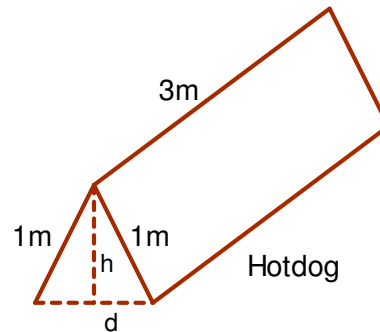
Finding d in terms of x:

(using Pythagoras):

$$d = \sqrt{(1.5^2 - h^2)}$$

Substituting d into V:

$$V = 2h \times \sqrt{(1.5^2 - h^2)}$$



Area of triangle:

$$A = 2 \times \frac{1}{2} \times h \times d$$

$$= hd$$

Volume:

$$V = 3hd$$

Finding d in terms of x:

(using Pythagoras):

$$d = \sqrt{(1^2 - h^2)}$$

Substituting d into V:

$$V = 3h \times \sqrt{(1^2 - h^2)}$$

3c) CONVERTING THE FORMULAE:

Hamburger:

$$Y_1 = 2X \times \sqrt{(1.5^2 - X^2)}$$

Hotdog:

$$Y_2 = 3X \times \sqrt{(1^2 - X^2)}$$

3d) ENTERING INTO TABLE MODE:

Enter the formulae into Table Mode NOTE: Use α, θ, T for X(Fig1)

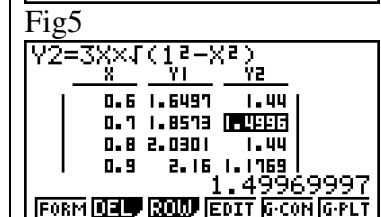
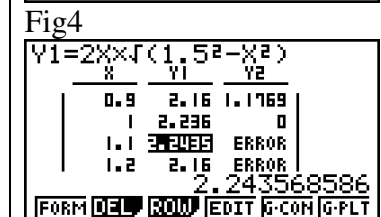
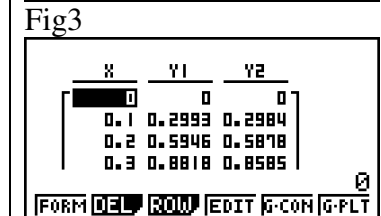
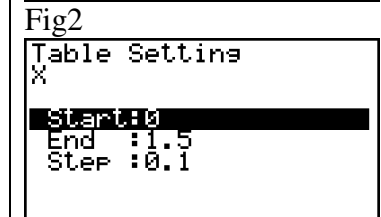
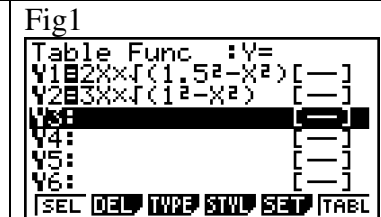
Press SET (F5) and enter the Table Settings as per Fig 2, pressing EXE each time.

Press EXIT and then TABL (F6) to generate the table (Fig3)

What do the numbers mean? What do the numbers tell us?

We can see in Fig4 the maximum volume for the hamburger appears to be around 2.24 cubic metres when the height of the Hamburger Pup Tent is 1.1m

In Fig5 an approximation for the Hotdog maximum volume is shown to be around 1.5 cubic metres when the height of the tent is 0.7m



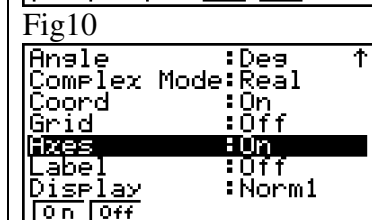
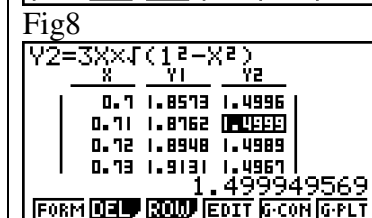
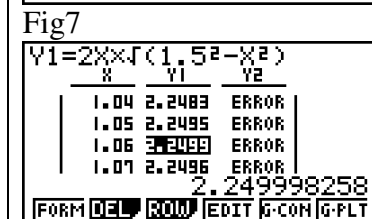
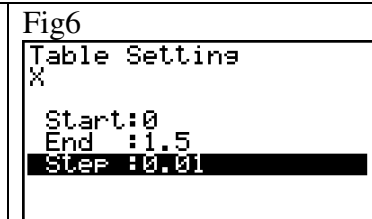
If we re set the Step values for x to 0.01 (Fig6) we will gain more accurate values for the maximum volume.
 (To do this press EXIT SET (F5) enter 0.01 and press EXE EXIT TABL (F6))

Figs 7 and 8 show the more accurate values of maximum volumes for Hamburger and Hotdog respectively.

3e) SETTING UP THE AXES:

Press EXIT, then go to V-Window (SHIFT F3) to set the max and min axes values as per Fig9 NOTE: There is no need to enter scale and dot values. Note also that the reason for entering negative values for the x and y minimums is so that the axes are displayed on the screen.

Press EXIT. Go to SET UP by pressing SHIFT MENU. Scroll up and ensure Coord and Axes are turned On using F1 (Fig10)



3f) GENERATING A GRAPH:

Press EXIT MENU (Fig11)

Enter GRAPH mode (Fig12) Note that neither of the functions are selected (Fig12)

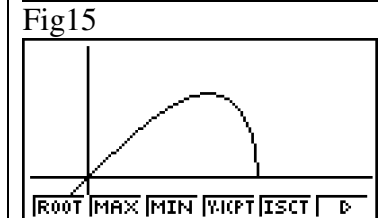
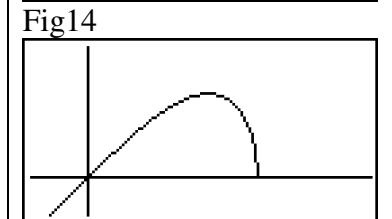
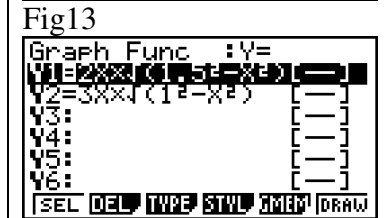
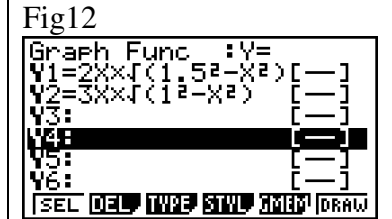
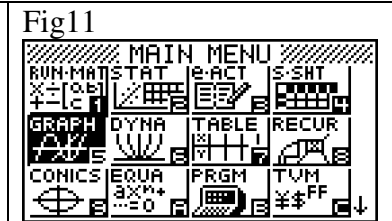
Select Y1 by scrolling to Y1 and pressing SEL (F1) (Fig13)

Press DRAW (F6) (Fig14)

We need to find the maximum value for the Hamburger Volume (Y1)

Press Trace (SHIFT F1) and trace the graph using left and right arrow to find an approximation for the maximum.

To find the 'exact' maximum press SHIFT G-Solv (F5) (Fig15)



Press MAX (F2) (fig16)

We can see that the maximum hamburger volume is 2.25 m^3 when the height is 1.1 m

We now will find the maximum volume for hotdog.

Press EXIT. Scroll to Y2 and press SEL (F1)

Press DRAW (F6) (Fig17)

Press G-Solv (SHIFT F5) (Fig 18)

Press MAX (F2) The calculator, via the Fig19 screen, is 'asking' if we want to find the max of Y1.

No, we don't.

Press the down arrow  (Fig20)

Fig16

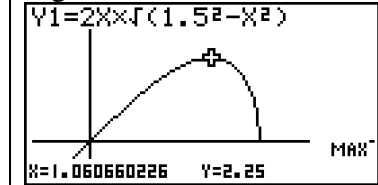


Fig17

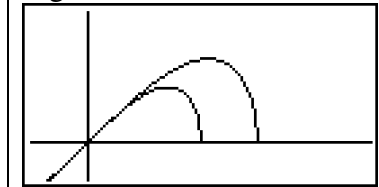


Fig18

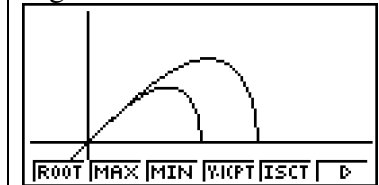


Fig19

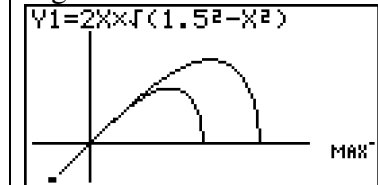
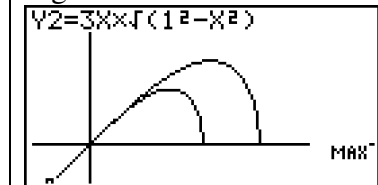


Fig20



Y2 is the graph we want to find the maximum value of. Therefore press EXE (Fig21)

The max volume for hotdog is 1.5 m^3 when the height is 0.7 m

THE GRAPH SHAPE:

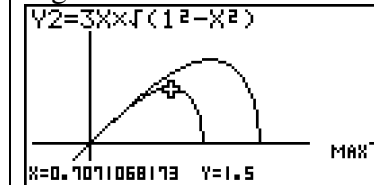
The graph shape (which is similar for the volumes of both tents) can be explained as follows: when the height of the tent is zero the volume is zero. As the height increases the volume increases at a fairly consistent rate, tapering off as it reaches the height required for maximum volume. (We observed this with our paper model). As the height increases past the height for maximum volume the sides of the tent move very quickly together and this reduces the volume rapidly, hence the near vertical down curves of the graphs.

3g) CONCLUSION:

The Hamburger gives the greater volume (2.25 m^3 compared to 1.5 m^3 for the hotdog, a difference of 0.75 m^3) Interestingly the Hamburger will be 0.36 m taller than the Hotdog (ie 36 cm) and therefore would be slightly easier to move around inside as a result. However, the Hotdog has the advantage of being less open to the wind (smaller triangular openings at each end which are at a greater distance from each other.)

Based upon volume, the Hamburger would be the most suitable tent.

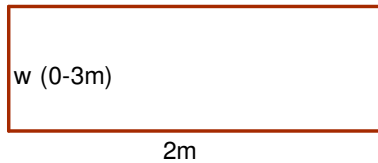
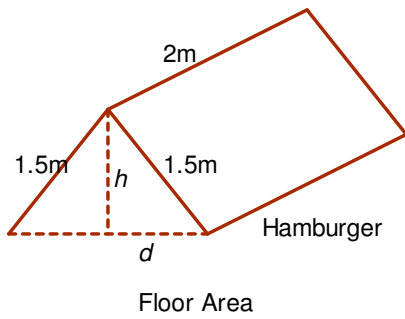
Fig21



Investigation Two

Q3a, b and c

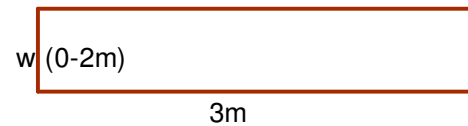
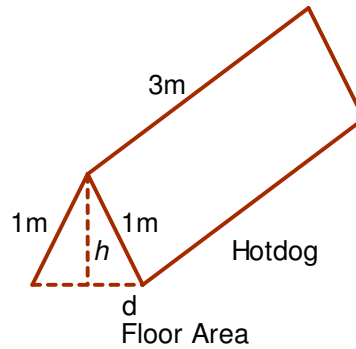
Applying algebra to the problem:



Floor Area:

$$A = 2w$$

$$Y_3 = 2X \text{ (X is the width)}$$



Floor Area:

$$A = 3w$$

$$Y_4 = 3X \text{ (X is the width)}$$

3e) ENTERING INTO TABLE MODE:

Let's consider the numbers for floor area, firstly for the Hamburger Pup Tent.

Enter TABLE mode. Deselect Y1 and Y2. Enter 2X for Y3 (using x, θ, T for X)(Fig22)

Press SET (F5) and enter the values shown in Fig23 (The End value for X is 3 because this is the widest the tent (tarp) can be)

Press EXIT and TABL (F6) (Fig24)

What do the numbers mean? What do the numbers tell us? What is the maximum floor area?

Let's add the Hotdog Pup Tent numbers.

Press EXIT and enter 3X at Y4 (using x, θ, T for X) (Fig25)

Press TABL (F6) (Fig26)

Fig22

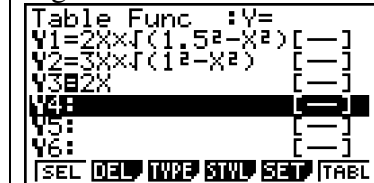


Fig23

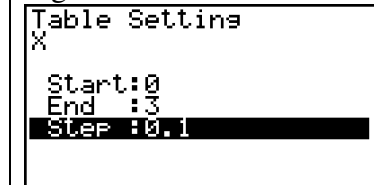


Fig24

X	Y3
0	0
0.1	0.2
0.2	0.4
0.3	0.6

Fig25

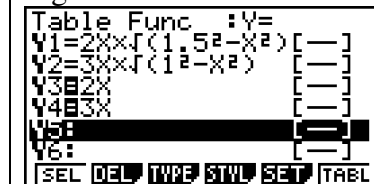


Fig26

X	Y3	Y4
0	0	0
0.1	0.2	0.3
0.2	0.4	0.6
0.3	0.6	0.9

Scroll to the bottom of the table. It looks like the Hotdog has the greatest floor area. (Fig27) Could this be correct?

The interpretation of this table is critical. X is the width of the Pup Tent. The Hamburger (Y3) has a maximum width of 3m which gives a maximum floor area of $6 m^2$. But the Hotdog (Y4) only has a maximum width of 2m! Scrolling up we see the floor area of Y4 at X=2 to be ... $6 m^2$ also! (Fig28)

Why is this?

Because it is the same tarp! And in both these cases the tarp is flat (ie the tarp is a tarp on the ground, not a tent!)

This situation is different to the volume investigation because the areas of each pup tent floor keep increasing until the tarp is extended flat on the ground. We therefore need to choose an acceptable height for the tent. We will need to find an expression for width in terms of height. Then we can find a formula for area in terms of height.

NOTE: These Solutions will now start again at 3a)

To determine the widths of the two tents we will use Pythagoras' Theorem.

Fig27

Y4=3X	Y3	Y4
2.7	5.4	8.1
2.8	5.6	8.4
2.9	5.8	8.7
3	6	9

FORM DEL ROW EDIT G-COM G-PLT

Fig28

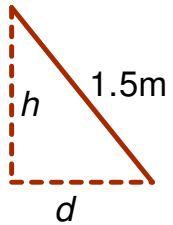
Y4=3X	Y3	Y4
1.9	3.8	5.7
2	4	6
2.1	4.2	6.3
2.2	4.4	6.6

FORM DEL ROW EDIT G-COM G-PLT

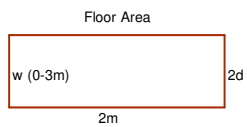
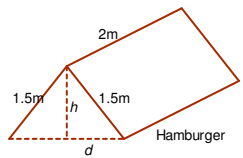
3a), b) and c)

Referring to the small triangle from the diagram on P13:

Hamburger:



$$d = \sqrt{(1.5^2 - h^2)}$$



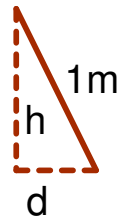
The width = $2d$

$$w = 2\sqrt{(1.5^2 - h^2)}$$

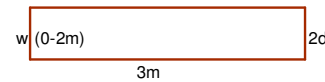
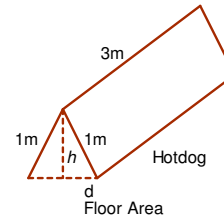
Substituting into $A=2w$:

$$A = 4\sqrt{(1.5^2 - h^2)}$$

Hotdog:



$$d = \sqrt{(1^2 - h^2)}$$



The width = $2d$

$$w = 2\sqrt{(1^2 - h^2)}$$

Substituting into $A=3w$:

$$A = 6\sqrt{(1^2 - h^2)}$$

3d) CONVERTING FOR Y AND X:

$$Y_5 = 4\sqrt{(1.5^2 - X^2)}$$

$$Y_6 = 6\sqrt{(1^2 - X^2)}$$

This is an expression for area (Y) in terms of height (X)

3e) ENTERING TABLE MODE:

Enter the two equations for area (above) into TABLE mode. Deselect all functions except for Y5 and Y6 (Fig29)

Enter the table settings as per Fig30

Press EXIT and TABL (F6) (Fig31-32)

Scroll through the table to understand the numbers For example:

- Why are the maximum areas both 6 when X=0?
- Why are there error messages at the bottom of the table for Y6?
- Do the numbers change at the same rate as you scroll up or down the table?

We will need to decide on a height for each tent in order to see which has the most suitable floor area.

Fig29

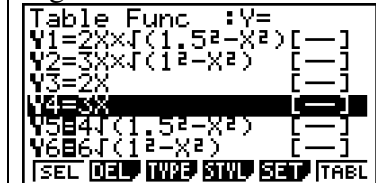


Fig30

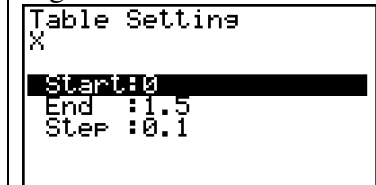


Fig31

X	Y5	Y6
0	6	6
0.1	5.9866	5.9699
0.2	5.9464	5.8787
0.3	5.8787	5.7236

Fig32

X	Y5	Y6
1.2	3.6	ERROR
1.3	2.9933	ERROR
1.4	2.154	ERROR
1.5	0	ERROR

We will first use the same heights that determined the maximum volumes. I.e:

Hamburger Pup tent height: 1.1m

Hotdog Pup Tent height: 0.7m

We can find the respective areas in the table. The hamburger area at height 1.1m has a floor area of $4.1 m^2$ (Fig33)

The Hotdog area at a height of 0.7m has a floor area of $4.3 m^2$ (Fig34)

Does this mean the Hotdog has the biggest floor area? Careful! Look at the Y5 (Hamburger) area one-up from 4.0792 (Fig33) The area rises sharply as the height is slightly reduced. The same can be said for Y6 (Hotdog) (Fig34)

We need to look more closely to make our decision based on floor area.

It needs to be realised these differences in floor area at these heights is negligible. However, the taller tent (Hamburger) would definitely be preferable, (unless you were a group of exceedingly short people.)

A final investigation would be premised by the question 'What if both heights were equal ie what is the floor area of the Hamburger when the height of the tent is also 0.7m? In Fig35 we can see the Hamburger tent is more than $1 m^2$ greater than the Hotdog, again making the Hamburger the pup tent of choice.

3f) SETTING AXES, DRAWING GRAPHS:

Go to V-Window (SHIFT F3) and set the max and min X and Y settings to: X – 0 to 1.5; Y – 0 to 7 (Ignore scale and dot)

Press EXIT, go to GRAPH mode and highlight only Y5 and Y6 (Fig36)

Fig33

Y5=4√(1.5²-X²)		
X	Y5	Y6
0.9	4.8	2.6153
1	4.4721	0
1.1	4.0792	ERROR
1.2	3.6	ERROR
		4.079215611

Fig34

Y6=6√(1²-X²)		
X	Y5	Y6
0.6	5.499	4.8
0.7	5.3065	4.3
0.8	5.0754	3.6
0.9	4.8	2.6153
		4.284857057

Fig35

Y5=4√(1.5²-X²)		
X	Y5	Y6
0.7	5.3065	4.2848
0.8	5.0754	3.6
0.9	4.8	2.6153
1	4.4721	0
		5.306599665

Fig36

Graph Func :Y=	
Y1=2X√(1.5²-X²)	[—]
Y2=3X√(1²-X²)	[—]
Y3=2X	[—]
Y4=X	[—]
Y5=4√(1.5²-X²)	[—]
Y6=6√(1²-X²)	[—]

Press EXIT and DRAW (F6) (Fig37)

Turn the Trace on. The maximum floor areas are shown in Fig37 when the tents have a height of zero (ie tarp-on-the-ground)

To view the floor areas at a height of 0.7m perform an X-Calculation. Simply enter 0.7 (Fig38)

Press EXE (Fig39) We can see the floor area for Hamburger is 5.3 at a height of 0.7m.

By pressing the down arrow (Fig40) we see the Hotdog floor area to be 4.3 at the same height.

3g) CONCLUSION:

Once again, this time based on floor area at a height of 0.7m, the Hamburger seems to be the pup tent of choice as it gives an extra square metre of floor space compared to the Hotdog.

