

Financial Mathematics Investigation - Reducing your mortgage with a lump sum

A note to teachers:

This investigation contains some interesting conclusions for engaged students. The investigation is designed for Yr 12 General Mathematics students in NSW. It would be an advantage for students to have completed the investigation 'Everyone Wants A Mortgage' (or similar) prior to tackling this investigation. This is available at <http://www.casioed.net.au/teachers/fx9860/ready.php>

Section B requires students to discuss some options which are then investigated in Section C. It therefore would be advantageous to distribute Section C after Section B is completed.

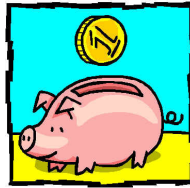
This investigation assumes students are competent with TVM and with recalling TVM values from RUN mode.

If you are not yet familiar with TVM or RUN modes you may like to first work through 'Self-Guided_9860_TVM' and/or Self-Guided_9860_RUN. Both are at <http://www.casioed.net.au/teachers/fx9860/start.php>

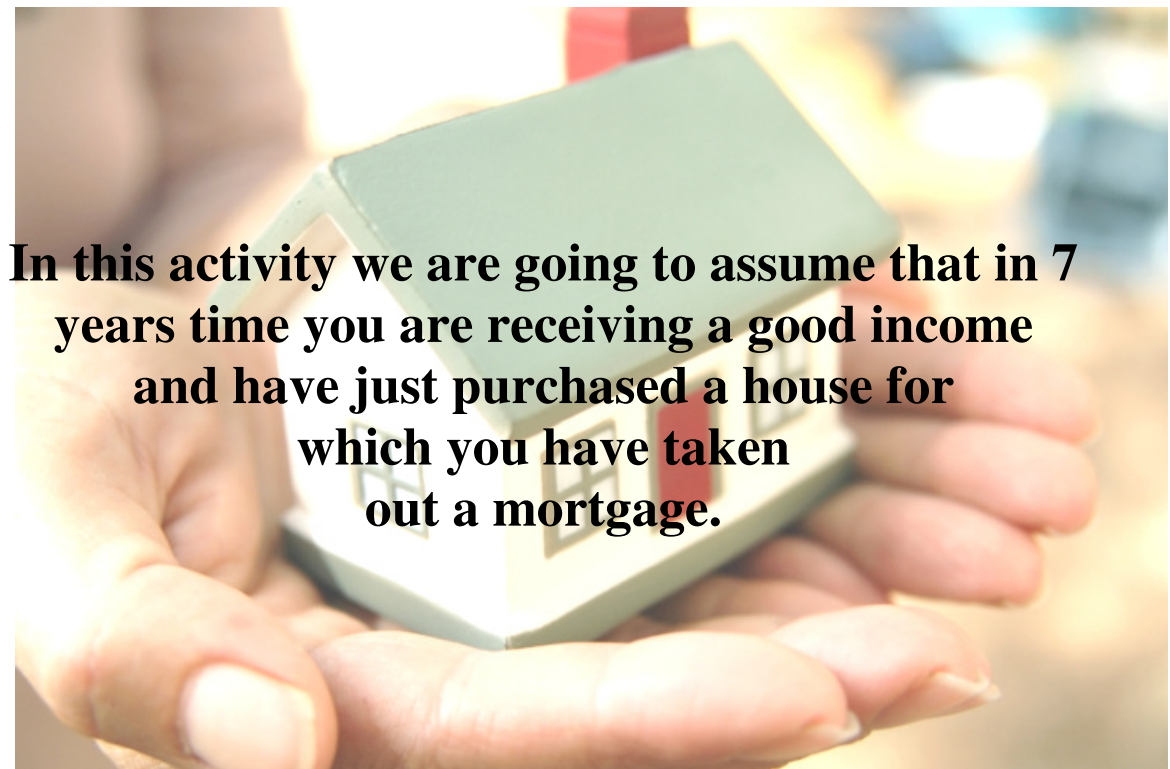
NOTE: If you desire to modify this activity and therefore desire the original word document you may request it by emailing casio.edusupport@shriro.com.au

Financial Mathematics Investigation

Reducing your mortgage with a lump sum



What is the best way of utilizing a \$110 000 inheritance if you have a mortgage?



In this activity we are going to assume that in 7 years time you are receiving a good income and have just purchased a house for which you have taken out a mortgage.

Section A)

We will firstly calculate the payments on this mortgage.
The conditions of the mortgage are:

- \$385 000 loan
- 7.1% pa interest rate, compounding monthly
- 40 years duration

a) Using TVM calculate your monthly repayments. (NOTE: Copy your TVM screen below showing all values)

And now for the main part of the investigation:

What if, 5 years into your mortgage payments, you receive a significant sum of money. What are your options for investing this money? Which option will be the most financially rewarding?

b) How many years will remain on your mortgage from the time this money is received?

c) How much money will you still owe on your mortgage at the time this money is received?

Strategy Two: Pay the \$110 000 into the mortgage as a lump sum. (ie reduce the amount you owe by \$110 000). Keep making the same repayments. This will result in you paying off your mortgage much sooner than the originally planned 40 years. Once your mortgage is fully paid continue making the same payments into an annuity at 6% compounding monthly until the 40 year period expires.

2a) How much will be remaining to pay on the mortgage after you have paid the \$110 000 into it?

b) How many months will it take to finish paying the mortgage from this point? (Answer to nearest whole number)

c) Show that the number of mortgage repayments (months) has been reduced by 243 (nearest whole) payments due to the injection of the \$110 000 into the mortgage.

d) By how many years has your mortgage been reduced? _____

e) As soon as the mortgage is paid off you continue making the same payments into an annuity (6% pa compounding monthly) until a full 35 years has passed from that time. Calculate the final value of that annuity.

f) How much of this money (answer to e) is money you paid into the annuity and how much of it is interest?

g) Why is Strategy Two (reduce the mortgage by \$110 000, invest the payments until the original mortgage period expires) financially superior to Strategy One (investing the \$110 000 over 35 years)?

Strategy Three: Pay the \$110 000 into the mortgage as a lump sum. Recalculate the payments. Invest the monthly difference into an annuity (at 6% pa compounding monthly) for 35 years

3a) Calculate the new monthly repayment amount for this scenario.

b) Calculate the difference between the new payment and the original.

c) You are to invest this difference in a 35 year annuity at 6% pa compounding monthly. Calculate the size of the annuity when your mortgage is paid off.

d) How much of this money (answer to c) is money you paid into the annuity and how much of it is interest?

e) Why is Strategy Three (reduce the mortgage by \$110 000, reduce the payments, invest the monthly difference in payments over 35 years) financially superior to Strategy One (investing the \$110 000 over 35 years)?

f) List the 3 Strategies in order of financial return, and give a numeric comparison for each.

Reducing your mortgage by a lump sum SOLUTIONS

Section A

a) \$2 420.53 per month

b) 35yrs

c) \$374 763.72

Section C Strategy One

Q1)a) \$893 590.66

b) This amount is relative to the real value of money in 35 year's time. Inflation will cause this amount to be worth much less than the same amount today.

Strategy Two

Q2) a) (ANS from Section A part c Minus \$110 000) = \$264 763.73

b) 177 months

c) 243 (NOTE: to recall 'n' (or any TVM value) press VARS, F6 then TVM (F4), then choose n, PMT, FV, etc.)

d) $243 \div 12 = 20$ years (nearest whole)

e) \$1 142 539.69 (!)

f) Amount paid into annuity =
\$588 187.81

```
Compound Interest:End
n = 480
I% = 7.1
PV = 385000
PMT = -2420.52598
FV = 0
P/Y = 12
↓
n I% PV PMT FV AMT
```

```
Compound Interest:End
n = 60
I% = 7.1
PV = 385000
PMT = -2420.52598
FV = -374763.7157
P/Y = 12
↓
n I% PV PMT FV AMT
```

```
Compound Interest:End
n = 420
I% = 6
PV = -110000
PMT = 0
FV = 893590.6643
P/Y = 12
↓
n I% PV PMT FV AMT
```

```
374763.72-110000
264763.72
↓
PMT
```

```
Compound Interest:End
n = 176.5993401
I% = 7.1
PV = 264763.73
PMT = -2420.52598
FV = 0
P/Y = 12
↓
n I% PV PMT FV AMT
```

```
420-n
243.4006599
↓
n I% PV PMT FV b
```

```
420-n
Ans÷12
20.28338832
↓
n I% PV PMT FV b
```

```
Compound Interest:End
n = 243
I% = 6
PV = 0
PMT = -2420.52598
FV = 1142539.685
P/Y = 12
↓
n I% PV PMT FV AMT
```

```
nX-PMT
588187.8131
↓
n I% PV PMT FV b
```

Section C (cont)

Q2f) cont

Interest = \$554 351.87

g) In Strategy One \$110 000 has been invested over 35 years compound monthly. In Strategy Two although the investing period is shorter (20 years) more than \$588 000 is invested in total thereby making Strategy Two far more effective.

Strategy Three

Q3a) \$1710.06 per month

b) \$710.47 (Note: the new payment has been added to the old payment because the new payment is a negative)

c) \$1 012 213.93 (!)

d) Amount paid into annuity =
\$298 397.40

Interest = \$713816.53

e) Both Strategies One and Three are of 35 years in duration. However, in Strategy Three you make 420 monthly investments of \$710 for 35 years. It only takes 115 months to reach \$110 000 (the Strategy One investment). So for 305 months there is more money invested in Strategy Three (and growing) than in Strategy One.

f) In order of financial return:
Strategy Two: return = \$1 142 539.69
(130 326 better than Strategy Three and \$248 949 better than Strategy One)
Strategy Three: return = \$1 012 213.93
Strategy One: return = \$893 590.6

```
nX-PMT
588187.8131
FV-Ans
554351.8721
↓
n I% PV PMT FV b
```

```
Compound Interest:End
n = 420
I% = 7.1
PV = 264763.73
PMT = -1710.067458
FV = 0
P/Y = 12
↓
n I% PV PMT FV AMT
```

```
2420.53+PMT
710.4725421
↓
n I% PV PMT FV b
```

```
Compound Interest:End
n = 420
I% = 6
PV = 0
PMT = -710.47
FV = 1012213.926
P/Y = 12
↓
n I% PV PMT FV AMT
```

```
nX-PMT
298397.4
↓
n I% PV PMT FV b
```

```
nX-PMT
298397.4
FV-Ans
713816.526
↓
n I% PV PMT FV b
```