### Investigating Algebraic Systems in TABLE and GRAPH Modes

One of the real advantages of the Graphic Calculator is the speed at which it can generate tables. Generally speaking tables are rarely used to investigate systems of algebra in traditional lessons because 1) they are traditionally too tedious to generate and 2) activities based upon tables provided on paper tend not to engage students. The speed and interactivity of a Graphic Calculator makes the act of 'looking at the numbers in a table' much more interesting and an obvious precursor to 'investigating the graph'.

### Vital Buttons for GRAPH Mode:

<table>
<thead>
<tr>
<th>Button</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trace:</td>
<td>(SHIFT F1) Tracing usually is the first action performed after drawing a graph. Tracing the graph gives X-Y values at any point. IMPORTANT: In order for Trace to work the 'Coord' setting in SETUP needs to be ON.</td>
</tr>
<tr>
<td>View Window:</td>
<td>(SHIFT F3) This is where the axes are set up.</td>
</tr>
<tr>
<td>G-Solve:</td>
<td>(SHIFT F5) For finding maxima, minima, intersection points, intercepts, X-Calculations, Y-Calculations, etc after generating a graph.</td>
</tr>
</tbody>
</table>
GRAPH Mode:

Setting the graph TYPE:
(Not knowing this can be extremely frustrating for beginners)
Enter the Graph Function screen by pressing MENU. Then scroll to GRAPH. Press EXE.
Delete any pre-existing equations using DEL (F2 then F1)
The default setting in both the Graph Function and Table Function screens is 'Y='. However there are other types of graphs that can be entered.

The common point of confusion: (This WILL happen to you or one of your students at some stage!)
If you want to enter, for example, Y=3X+1 but your screen is set to something different, for example, 'r=' (FigA), then you will need to change the Graph Function screen to 'Y=' (FigC)

Experiment with pressing TYPE (F3) then 'r=' (F2) OR TYPE (F3) then 'Y=' (F1) (Figs A and C)

Entering Functions:
To enter Y=3X+1:
A function can be entered on any of the lines offered. (Use the up-down arrows to move the cursor). NOTE: Do not enter either 'Y' or '=' because these already exist..
With the cursor on 'Y1:' enter 3X+1 ie press 3 + 1 EXE (FigD)

(NOTE: X must be used for X in GRAPH and TABLE modes) … (not Alpha then +)

Let's say we want to see where Y=3X+1 intersects with X=5. We will need to change the graph TYPE to enter X=5.
Press TYPE (F3) (FigE)

Press X=c (F4) (FigF)
NOTE: It appears that our first function has disappeared, but one press of the up arrow will find it!

Enter 5 then EXE (FigG)
Setting the Calculator to Draw Graphs:

We will need to action two types of settings. Firstly, **go to the SET UP (SHIFT MENU)** and arrow up to Coord (FigH) **Set Coord, Grid, Axes and Label as in FigH.**

Press EXIT

Now we need to set up the axes to match our graphs. We do this through V-Window.

**Press V-Window (SHIFT F3) and set Xmin and max and Ymin and max as in FigI**, ignoring scale, dot and the other settings.

**Press EXIT.** You should be back at the Graph Function screen (Fig G)

Now **press DRAW (F6)** (FigJ)

Of course in this demonstration example the intersection point can be easily determined algebraically. However, by pressing **Trace (SHIFT F1)** and scrolling we can get close to the intersection point (FigJ-2). Note that the up-down arrows move the cursor between graphs.

To find the exact intersection point we need press **G-Solv (SHIFT F5)** (FigJ-3)

With most sets of graphs ISCT (F5) will display the intersection point. However, in the case of the X=c graph ISCT will not work. (Try it! To redraw the graph press EXIT and DRAW (F6))

Because we know the intersection point has an X value of 5, we can perform a Y-Calc to find the intersection point.

**Press G-Solv (SHIFT F5) then **$b$** (F6)** (FigJ-4) **NOTE: It soes not matter if Trace is On or Off.**
Now press Y-CAL (F1) and enter 5 for X (FigJ-5)
Press EXE

The intersection point is (5, 16) (FigJ-6)

**Inequalities:**
Return to the Graph Function screen (EXIT) and use SEL (F1) to deselect functions Y1 and Y2, and place the cursor in line 3 (FigK)

Entering Y>X:

Press TYPE (F3) then \( \geq \) (F6) (FigL)

Press Y> (F1) and press X using \( x, \theta, t \) then EXE (FigM)

To draw the inequality:
Press V-Window (SHIFT F3) and press INIT (F1) (This gives us a 'square grid') Press EXIT
You should be back at the Graph Function screen (FigM)

Press DRAW (F6) (FigN)
Finding Intersections of Inequalities:
Let's find the intersection between \( Y > X \) and \( Y < 2 \)
Using **EXIT** **TYPE** (**F3**) **B** **F2** enter **2** **EXE** into line 4 (FigO) NOTE: The FigO screen has been 'arrowed up'

Press **Draw** (**F6**) (FigP)

There's hours of playing to be done here!! Try intersections of inequalities using quadratics, cubics, etc.

Finding Max, Min, Intersection Points, Roots, Intercepts:

Clear the Function Screen by using **F2** and **F1**
IMPORTANT: Use the **TYPE** settings to return the screen to **Y=** (Press **TYPE** (**F3**) then 'Y=' (**F1**)
Then enter the functions shown in FigQ

Set up the axes in V-Window (**SHIFT** **F3**) as in FigR

Press **EXIT** and **DRAW** (**F6**) (FigS)

Seeing which graph is Y1, Y2, Y3:

Trace the graph using **SHIFT** **F1**, Then use left-right arrows (to get away from the Y Axis for better viewing). Then use the up-down arrows. The top of the screen will state the function name of each graph. (eg FigT)
We will now find the MAX of Y2 and the ROOTS of Y1.

**Press G-Solv (SHIFT F5) then MAX (F2) (FigU)**

Note in FigU the calculator is 'pointing' to Y1. We want Y2.

**Press the down (or up) arrow to get to Y2** (FigV)

**Press EXE** (FigW)
Predictably we see the Y2 maximum is at (0, 15) Finding the ROOTS of Y1:
Press G-Solv (SHIFT F5) then ROOT (F1), arrow to Y1 then EXE (FigW-2)
The left root is displayed (X= -3.2) To display the other root, press the right arrow (X=3.2) (FigW-3)

**Finding Intersection Points:**
Note there are numerous intersection points with three pairs of line combinations.
We will first find the 2 intersection points between $Y_2 = -X^2 + 15$ and $Y_3 = 2X+1$

**Press G-Solv (SHIFT F5) then ISCT (F5) (FigX)**
Note the calculator is displaying Y1.

**Press the down arrow**, Y2 will show.
**Press EXE**, This has selected Y2. Now Y3 will show. **Press EXE again.**
We have now selected both Y2 and Y3. One of the intersection points of Y2 and Y3 is shown (FigY)

To determine the other intersection point press the right arrow (FigZ)
To return to the first intersection point, press the left arrow.

Practice these procedures by finding the other intersection points.

**USING TABLE AND GRAPH MODES TOGETHER:**

The examples below are simple and are used as an attempt to highlight the process of moving from TABLE mode to GRAPH mode and to explain the associated steps. This process is a good protocol to follow for investigating algebraic systems. Having worked through these pages we recommend the interesting algebraic modelling investigations available at www.casioed.net.au

**Investigating the effect of Radius on the Volume of a Sphere and Cone:**

Firstly we will generate a table of values.

**Enter TABLE mode and delete any pre-existing formulae** using F2 and F1

Using the \( x, \theta, t \) button for X **enter the formula** for the volume of a sphere as shown in Fig1

**Press SET (F5) and enter the settings** shown in Fig2

**Press EXIT and TABL** (Fig3)

Scroll through the values. Are the values increasing at an even rate? Are they increasing at an increasing rate? What would your students say? How could we investigate the rate of increase from the table? We could take some numbers and operate on them in RUN mode but let's draw the graph.

In order to generate a graph we need to set up the axes. TABLE mode is a good place from which to set up axes because the numbers we need to use are right there on the screen. We need to set X between 0 – 16 and Y between 0 – 20 000

To set the axes go to **V-Window** (SHIFT F3)

**Enter the settings** according to Fig4, ignoring the values for scale and dot.

Note that -5000, rather than zero, has been used for the Ymin. This is to allow the lower section of the graph to continue to display when the graph is being traced. **Press EXIT**
To turn the axes on go to SETUP (SHIFT MENU) and arrow up 3 places. Use F1 to turn axes on if necessary. Set Coord, Grid, and Label according to Fig5. Press EXIT (NOTE: Coord=ON allows the graphs to be traced. Label simply puts an X and Y on the screen)

Press MENU, scroll to GRAPH and press EXE (Fig6)

Note that the function we entered in TABLE appears in GRAPH mode. Note also that the graph is not selected. Select the graph using SEL (F1) (Fig7)

Now press DRAW
Turn the trace on by pressing SHIFT F1 (Fig9)

An overview of what we will do next:
We are going to investigate the ratios of Y values for X values in which the X values are in the ratio 2:1 ie Y (when X=2) ÷ Y (when X=1); Y (when X=4) ÷ Y (when X=2); Y (when X=6) ÷ Y (when X=3); etc.
We will find Y values for whole-number X values and transfer these to RUN mode to perform the divisions. In each case we will use the memory function.

Performing a Y Calculation:
With the trace on as in Fig9 press 2 (Fig10)
NOTE: By entering a number with the trace on the calculator readies itself to perform a Y calculation, reading the entry as the chosen X value.

Press EXE (Fig11)
We can see that Y=33.5
We now need to recall this in RUN mode and store it.
NOTE: The RUN screenshots display the Linear (not Math) Input Mode. To set RUN to Linear: Enter RUN (MENU 1), go to SET UP (SHIFT MENU), press Linear (F2).

Go to RUN (press MENU then 1)
Type Y (ALPHA Y) then EXE
To commit this to a memory press ← then choose a letter (eg ALPHA A) EXE (Fig12)

Now we need to return to GRAPH and perform a Y Calc on X=1
Press MENU then 5
Press DRAW then Trace (Fig13)

Press 1 and EXE
We can see that Y=4.2 when X=1 (Fig14) NOTE: X=1 is obscured by the cursor
We need to return to RUN, recall our first number and then divide it by this one ie perform the division Y(x=2) ÷ Y(x=1)

Return to RUN (MENU then 1)
Press ALPHA A then EXE
Press ÷ ALPHA Y EXE (Fig15)
We can see the quotient is 8. (Perhaps not surprising for maths teachers but probably surprising for students!) (Fig15)

Let's perform Y (at X=4) ÷ Y (at X=2)
Go to GRAPH and press DRAW
Turn on Trace and press 4 and EXE (Fig16)

Go to RUN
Press ALPHA Y EXE
Press ← then (choose a letter) ALPHA B
Press EXE (Fig17)
Remember that $Y$ (at $X=2$) is on memory A.

So press $\div$ APLPA A and EXE (Fig18)
Once again we see the quotient is 8 (Again, possibly quite surprising to students!)

Now repeat for $Y$ (at $X=6$) $\div$ $Y$ (at $X=3$); $Y$ (at $X=8$) $\div$ $Y$ (at $X=4$); $Y$ (at $X=16$) $\div$ $Y$ (at $X=8$)

You should see that each quotient is 8.

The obvious question to ask students is 'Why is the quotient consistently 8?'

If we were to continue this investigation it would be wise to investigate another formula with an $X^3$ term.

Let's consider the volume for a cone:

Go to TABLE first and deselect Y1 using F1.

Enter the formula as in Fig19. NOTE: In Fig19 at Y2 we have used the fraction button this time instead of $(1 \div 3)$

We will use the same table setting as last time ($X$ from 0 – 16 with a step of 1)

Press TABL and scroll through to look at the values. (Fig20)

Are the values increasing at an even rate? Are they increasing at an increasing rate? What would your students say?

What is the highest value for Y? (about 4300) Go to V-Window and set as per Fig21

Now we will go to GRAPH mode and then perform the 'number crunching' in RUN as we did for the sphere.

Go to GRAPH mode. Using SEL (F1) de-select Y1 and select Y2 (Fig22)

Press DRAW and Trace (Fig23)
Now to perform the ratio divisions:

Press 2 and EXE (Fig24)

Go to RUN mode.
Recall the Y value with ALPHA Y and EXE
Commit this to the memory using \( \frac{Y}{X^3} \) then (choose a letter) ALPHA A then EXE (Fig25)

Return to GRAPH, press DRAW then Trace
Press 1 and EXE (Fig26)

Go to RUN
Press ALPHA A EXE ÷ ALPHA Y EXE (Fig27)

Repeat for Y (at X=4) ÷ Y (at X=2), etc

If investigating this with students and they still don't see why the quotient is always 8 for 'X-cubed expressions, investigating an area formula would be a good next step.
We will investigate the area of a circle.

Enter TABLE mode, de-select Y2 and enter the circle area formula (Fig28)
The last table setting will suffice so press TABL, and once again have a look at the Y values. Note the maximum Y=800+

Go to V-Window and change the settings according to Fig29
Now enter GRAPH mode.
Deselect Y2 and select Y3.
Press DRAW and Trace. (Fig30)

Now the same procedure to perform Y(x=2) ÷ Y(x=1):
Enter 2 EXE enter RUN mode press ALPHA Y EXE
Press ← then ALPHA A then EXE (Fig31)
Enter GRAPH, press DRAW and Trace
Enter 1 then EXE
Enter RUN press ALPHA A then EXE ÷ ALPHA Y EXE

Note the result is 4 (Fig32)

Repeat for Y (at X=4) ÷ Y (at X=2), etc
All quotients will be 4
(Then obviously draw out from the students the relationship between the quotient and the X^2 expressions and the quotient and the X^3 expressions)

The intent of these TABLE-GRAPH instructions is to give you a 'feel' for what the calculator can do and hopefully for you to realize there are endless situations in which these skills can be utilized in order to give greater meaning to the relationships between a problem, the associated algebra and the related tables and graphs.
