

# Simple and Compound Interest Investigation

## A note to teachers:

This is a very simple algebraic modelling investigation, excellent for junior secondary students to explore as part of their financial mathematics studies.

This investigation uses TABLE, GRAPH and RUN modes. Extra support using these modes can be gained via the Self-Guided Instruction Sheets on this site at <http://www.casioed.net.au/teachers/fx9860/start.php>

Further algebraic modelling investigations are available at <http://www.casioed.net.au/teachers/fx9860/ready.php> and [http://www.casioed.net.au/services/tuition/fx9860/fx9860\\_tuition.php](http://www.casioed.net.au/services/tuition/fx9860/fx9860_tuition.php)

## The purposes of this activity include:

- To engage students mathematically
- To give students experience of applying algebra to a problem
- To represent the problem through multiple mathematical displays
  - A table of values:
    - To enable students to 'see the problem' represented in the numbers and expose students to finding a solution within a table
  - Graphical representations:
    - To enable students to 'see the problem' represented graphically and find a graphical solution

(and arguably the most important of all)
- To afford the teacher multiple opportunities to ask leading, higher-order-thinking questions which facilitate conceptual understanding in students

NOTE: If you desire to modify this activity and therefore desire the original word document you may request it by emailing [casio.edusupport@shriro.com.au](mailto:casio.edusupport@shriro.com.au)

## Simple and Compound Interest Investigation

### Exercise A: Investigating Simple Interest

Consider the Simple Interest formula  $I = P \times R \times N$  where:

- $I$  = the amount of interest earned
- $P$  = the amount of money invested
- $R$  = the rate of simple interest per year
- $N$  = the number of years

Now consider the formula  $A = P + (P \times R \times N)$

1) What do you think this formula will give us? Explain your answer.

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2) Consider investing \$10 000 ( $P$ ) at 6% pa simple interest.

Using the formula  $A = P + (P \times R \times N)$  we will use TABLE mode to investigate the effect of time ( $N$ ) on the final value ( $A$ ) of the investment for up to 50 years.


Converting the formula for the graphic calculator and entering the above values will give  $Y = 10000 + (10000 \times 0.06 \times X)$  where  $Y$  is the final value and  $X$  is time.

Enter TABLE mode and in the Y1 line enter  $10000 + (10000 \times 0.06 \times$  

NOTE: The  button must be used to enter  $X$ .



Table Settings
X
Start:0
End:50
Step:1

Press SET (F5) and enter the following values , pressing EXE after each entry. Press EXIT and TABL (F6)

a) Use your table to approximate how long will it take for the investment to grow to \$16 500 \_\_\_\_\_

b) How much does the investment increase by in the first ten years ie between  $X = 0$  and  $X = 10$  ? \_\_\_\_\_

c) How much does the investment increase by in the last ten years ie between  $X = 40$  and  $X = 50$  ? \_\_\_\_\_

d) If we were to sketch a graph of how simple interest affects an investment what do you think it would look like? Using the diagram below and without placing any numbers on the axes make a 'sketch-guess' showing the shape of the graph. Let 'the number of years' (X) be the horizontal axis and 'the value of the investment' (Y) be the vertical axis.



3) Now let's see the graph for real on the Graphic Calculator.

First we need to set up the axes.

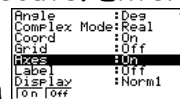
Go to V-Window and enter the following values (ignore the values for



scale and dot)

Press MENU and go to GRAPH mode and execute. Enter SETUP (SHIFT

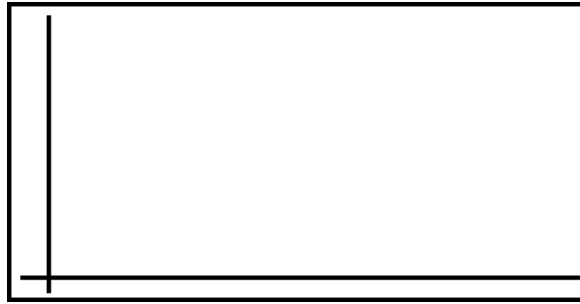
MENU), scroll up and turn Coord & Axes On Press EXIT.



Note that our formula is there, but we need to select it. Press SEL (F1) to select. Now press DRAW (F6). Trace the graph (SHIFT F1)

a) Now lets find  $Y = 16500$  by tracing. What is the closest whole number of years that will cause the investment to be 16500? \_\_\_\_\_

b) Actually, using a straight line graph for simple interest isn't quite accurate. Sketch a more accurate graph and write a sentence explaining why. (HINT: Is the value of the investment in June of the tenth year really bigger than it was during the month before?)



## Exercise B: Investigating Compound Interest

We will now investigate investing \$10 000 at 6% pa but this time the interest will compound annually.

The compound interest formula is  $A = P(1 + r)^n$   
However we will need to convert it to  $Y = 10000(1+0.06)^X$   
(Remember to use  $X, \theta, T$  for X)

4) Again we will generate a table first.  
Go to TABLE mode.

Turn the simple interest formula OFF using SEL (F1)  
(don't delete it, we will use it again later).

Into the Y2 line enter the formula  $10000(1+0.06)^X$ . There will be no need to change the SET (from Q2) because we are investigating the same range of years. Press TABL (F6)

a) After how many years will the investment first exceed \$16500? (It took 11 years with the simple interest model) \_\_\_\_\_

b) How much does the investment increase by in the first ten years ie between  $X = 0$  and  $X = 10$ ? (Remember it increased \$6000 with simple interest) \_\_\_\_\_

c)i) How much does the investment increase by in the last ten years ie between  $X = 40$  and  $X = 50$  ? (Again, it increased \$6000 with simple interest) \_\_\_\_\_

c)ii) Compare this to your answer in 2c) Why does compound interest generate so much more money than simple interest between the 40<sup>th</sup> and 50<sup>th</sup> years?

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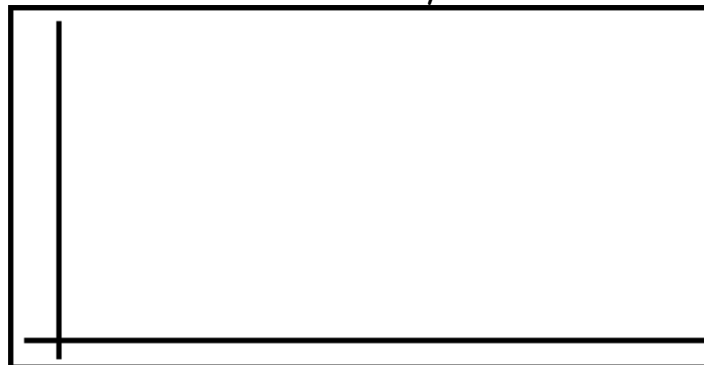


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d) Now predict what shape of graph your graphic calculator will draw for the compound interest model. Using the diagram below make a sketch-guess showing the shape of the graph with the number of years (X) along the horizontal axis and the value of the investment (Y) on the vertical axis. (Ignore the fact that interest is only added at the end of each year)



5) Go to GRAPH and press EXE. Set up the axes in V-Window and enter

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View Window
Xmin : 0
max : 50
scale: 1
dot : 0.39682539
Ymin : -100000
max : 250000
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the values shown ignoring scale and dot. (NOTE: Setting the Y min at -5000 prevents the X axis from disappearing when the trace is turned on)

Using SEL (F1), de-select the Simple Interest graph (Y1) and select the Compound Interest graph (Y2). Now press DRAW (F6)

Trace the graph using SHIFT F1

a) Using the right arrow, after how many years will the investment first pass \$100 000? \_\_\_\_\_

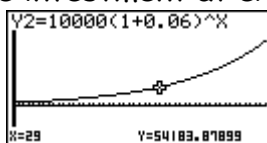
b) After how many years will the investment first pass \$250 000? \_\_\_\_\_

### Performing a 'Y Calculation' (Y-CALC)

Tracing a graph rarely gives us the exact values we are looking for.

However, there is another way.

Let's find the value of the investment at exactly 29 years.



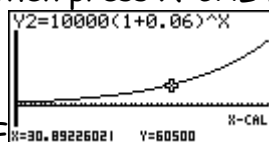
Simply press 29 and EXE.

**NOTE:** if an error message appears, it means  $X = 29$  is off the screen. Use your arrows to trace the graph. Keep 'pushing' the graph until  $X = 29$  is on the screen.

Repeat this process to find  $Y$  when  $X$  is exactly 33.

### Performing an X-CALC

To find the  $X$  value when  $Y = 60500$  ... go to  $G$ -Solv (SHIFT F5) then press  $\left[ \text{F6} \right]$  (F6) then press X-CAL (F2)



Enter 60500 EXE (make sure  $Y=60500$  is on the screen)

6) Using X-CAL and Y-CAL give answers to the following questions to 1 decimal place: (Remember the values must be showing on the screen first)

a) When  $X=9$        $Y=$  \_\_\_\_\_      d) When  $Y=31\ 220$        $X=$  \_\_\_\_\_

b) When  $X=34$        $Y=$  \_\_\_\_\_      e) When  $Y=60\ 000$        $X=$  \_\_\_\_\_

c) When  $X=48$        $Y=$  \_\_\_\_\_      f) When  $Y=105\ 000$        $X=$  \_\_\_\_\_

## Exercise C: Comparing Simple and Compound Interest

Now its time to compare Compound Interest with Simple Interest graphically.

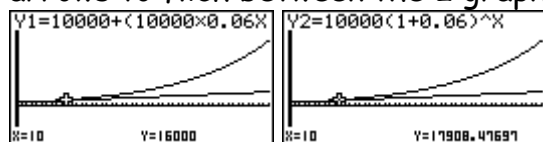
Press EXIT to go back to the Graph Function screen and turn on both



graphs Press DRAW (F6)

Now we will use Y-CAL to compare simple interest investment with the compound interest investment at  $X = 10$  years.

With the graphs drawn and the trace on press 10 and use the up-down arrows to flick between the 2 graphs (see below)



ie when  $X=10$  the SI and CI investments are worth \$16000 and \$17908 respectively. (NOTE: It looks like the cursor does not move when using the up-down arrows but this is because the graphs are so close together at  $X=10$ )

7)a) Use Y-CAL to complete the following table (whole dollars only)

No of Years (X)	Value of SI Investment	Value of CI Investment
20	\$	\$
30	\$	\$
40	\$	\$
50	\$	\$

b) What is surprising about these figures?

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## Exercise D: Investigating the effect of different compound interest rates

Just how much difference would a 15% compound interest rate make compared to, say, a 5% compound interest rate on the same investment over 40 years?

To investigate this we will need to use the version of the formula shown

here:  $A = P\left(1 + \frac{R}{100}\right)^n$  because we will need  $r$  to represent whole-number percentages.

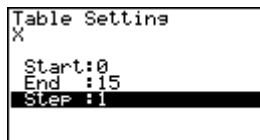
We will again consider an investment of \$10000 but this time over 40 years. The version of the formula we will enter in the Y3 line  
 $10000(1+(X\div 100))^40$

Again we will generate a table first. Enter TABLE mode.

Into the Y3 line enter  $10000(1+(\text{X},\theta,T)\div 100))^40$



Make sure all other functions are de-selected.



Press SET (F5) and enter the following values

Press EXIT and TABL (F6)

8)a) Scroll through the numbers. Why does  $Y=10000$  when  $X=0$ ?

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b) Write one or more sentences which demonstrate your understanding of the numbers in the table.

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Now we will view the graph.



Go to V-Window and enter the following values

Note that the Y values are -1 000 000 and 3 500 000. These Y values will allow you to read the graph more easily.

Now EXIT and enter GRAPH mode.

Use SEL to ensure Y3 is the only graph selected.

Press DRAW (F6)

9)a) Determine the value of the investment after 40 years for interest rates of 5% and 10%

Investment value after 40 years @ 5% =

Investment value after 40 years @ 10% =

b) Calculate how many times the investment increases when the interest rate doubles from 5% to 10% Show your working.

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c) By how many times is the investment increased if the interest rate over the entire 40 years is tripled from 5% to 15%? Show working.

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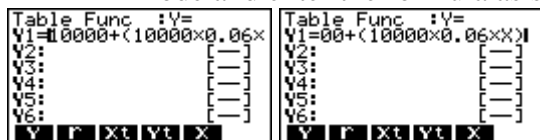
d) What interest rate (to 2 decimal places) is required to double the investment over the 40 year period? \_\_\_\_\_ (Use X-Cal)

e) What interest rate (to 2 decimal places) is required to grow the investment to one million dollars over the 40 year period? \_\_\_\_\_

# Simple and Compound Interest SOLUTIONS

1)  $A = P + (P \times R \times N)$  will give the final value of a simple interest investment.

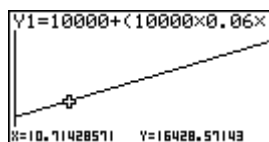
2) Enter TABLE mode and enter the formula as shown below.



X	Y1
8	14800
9	15400
10	16000
11	16600

a) It will take approx. 11 years for the investment to reach \$16500

b) \$6000    c) \$6000    d) Straight line graph drawn, positive gradient.



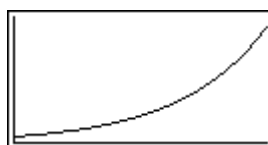
3) a) X = 11 years    b) Interest is only added at the end of each year. Therefore, the investment is constant throughout any year and 'jumps' at the end of each year when interest is added. The graph is therefore a step graph.

X	Y2
6	14185
7	15036
8	15936
9	16894

4a) 9 years (ie not much less than for the simple interest model!)

184201-102857    81344

b) \$7908    c)i) \$81344!!    c)ii) The simple interest model generates \$600 each year whereas with the compound interest model the interest is added to the principle each year. Therefore each year the amount of interest being added increases. For example in the 41<sup>st</sup> year over \$6000 interest is added.



d) Something like ...

5) a) 39.7 years    b) 55.6 years

6) a) Y=15895.8    b) Y=72510.3    c) Y=153938.7    d) X=19.5    e) X=30.7    f) X=40.4

7)

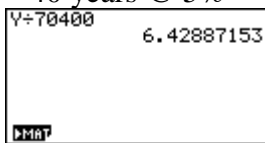
No of Years (X)	Value of SI Investment	Value of CI Investment
20	\$22000	\$32071
30	\$28000	\$57435
40	\$34000	\$102857
50	\$40000	\$184202


b) At 20 years the CI investment is approx \$10 000 more than the SI investment but at 50 years it is over \$144 000 more than the SI investment!

8a) Because  $X = \text{zero}$  means there is no interest. Therefore the investment will remain at \$10 000.

b) As the interest rate increases so does the value of the investment over 40 years. When the interest rate is 9% ( $X=9$ ) the return over 40 years ( $Y$ ) is \$314094. A 15% interest rate over 40 years returns  $\$2.7 \times 10^6 = \$2\,700\,000$  (2sf) This occurs because as the interest increases the amount added to the principle each year increases.

9)a) 40 years @ 5% = \$70400; 40 years @ 10% = \$452593



b)  6.4 times (2sf)

