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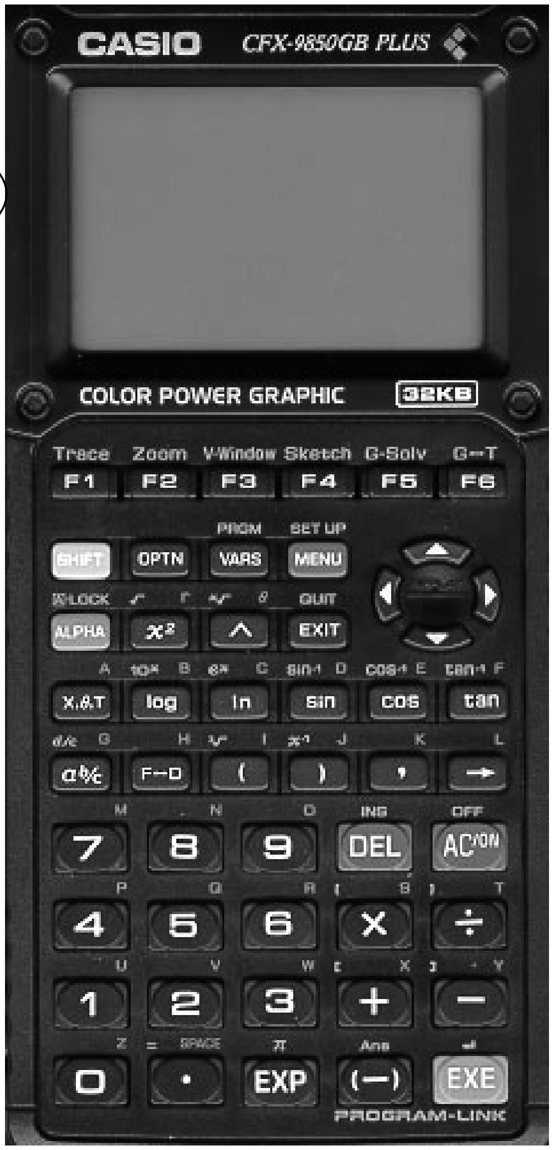
FAC FORTIA ET PATERE

Prince Alfred College
Mathematics Faculty

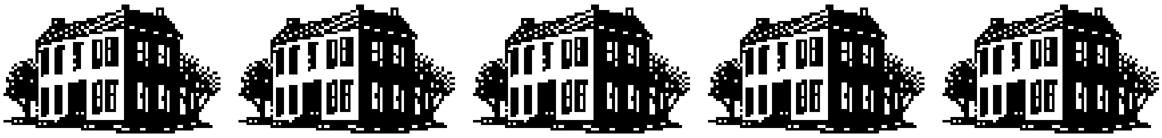
GENERALISING YOUR PATTERNS INTO AN ALGEBRAIC STATEMENT AND SOLVING PROBLEMS.

WITH HELP FROM THE CASIO 9850 GB PLUS



AN INTRODUCTION TO YEAR 8 ALGEBRA (LINK RULES)

A PROBLEM TO SOLVE



Number Neighbours

Consider the group of numbers we call the positive integers, that is

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, etc

Now consider the concept of a '*numbers first neighbour*'.

The number 5 has two **first neighbours**, 4 and 6. 4 is called the *lesser first neighbour* and 6 is called the *upper first neighbour*.

The number 24 also has two **first neighbours**, 23 and 25.

Now consider the concept of a '*numbers second neighbour*'.

The number 5 has two **second neighbours**, 3 and 7.
The number 24 also has two **second neighbours**, 22 and 26.

Third neighbours, fourth neighbours, fifth neigbours and so on exist.

TASK ONE

Complete the following table for the number 28:

neighbour number	1st	2nd	3rd	4th	5th	kth
lesser neighbour						
upper neighbour						

A Housing Development

Imagine that it is the year 3000. The moon is just beginning to be civilised and Mr. Jones, the first land developer to plan a Housing Development. He sets down the following rules for the development.

- Each street in the sub-division must have **more than one house**.
- No street is to intersect with another.
- Houses are to be on only one side of the street.
- If there are 't' houses on a street, then the numbers 1, 2, 3, 4,...,t must be used as the house numbers on that street.
- The streets are to be named, First Street, Second Street, Third Street and so on.
- On First Street, no house may be numbered so that it is next door to its **first number neighbour**.
For example, the house numbered 5 can not be next door to the house numbered 4 or 6.

  On Second Street, no house may be numbered so that it is next door to its **first or second number neighbour**.
For example, the house numbered 5 can not be next door to the house numbered 3 or 7.

  An equivalent rule applies to Third Street, Fourth Street, ... k Street.

TASK TWO

a) Determine the least number of houses which can be built on First Street.
Write down the order of the house numbers.
EXPLAIN why the number you have chosen is the least.

b) Determine the least number of houses which can be built on Second Street.
Write down the order of the house numbers.
EXPLAIN why the number you have chosen is the least.

c) Determine the least number of houses which can be built on Third Street.
Write down the order of the house numbers.
EXPLAIN why the number you have chosen is the least.

How many houses do you predict will be in the development if three streets exist and the minimum number of houses is used on each street?

d) *PREDICT* the least number of houses which can be built on Tenth Street
and the order of the house numbers.

How many houses do you predict will be in the development if ten streets exist and the minimum number of houses is used on each street?

e) *PREDICT* the least number of houses which can be built on Fiftieth Street
and the order of the house numbers.

How many houses do you predict will be in the development if fifty streets exist and the minimum number of houses is used on each street?

f) *PREDICT* the least number of houses which can be built on Kth Street and the order of the house numbers.

How many houses do you predict will be in the development if ‘k’ streets exist and the minimum number of houses is used on each street?

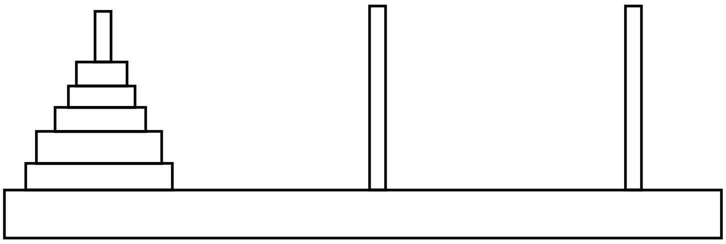
g) Write down a logical argument that *PROVES* your solution to part f) is correct.

h) What will be the name of the street containing 200 houses?

i) How many streets will there be if the development is to have a total of 2650 houses (assume minimum number of houses per street).

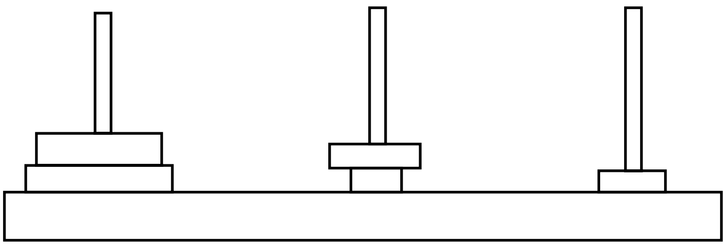
ALGEBRA - A language other than English

Investigation 1: THE TOWERS OF HANOI



The Towers of Hanoi is a very famous puzzle invented by the monks of Hanoi many many years ago. The aim is to move the tower of disks from the peg on which it is at present to another peg in the MINIMUM number of moves using the three pegs supplied, but in doing so not violating the following rules:

move only one disk at a time from one peg to another and ,
a larger disk may never be on top of a smaller disk.



This would be an illegal move.

A tower may consist of any number of individual disks of different sizes. The one pictured above has 5 disks. The towers which you will be using have at most 5 disks.

1. Make a tower of 'one' disk and determine the minimum number of moves to complete the puzzle.
2. Make a tower of 'two' disks and determine the minimum number of moves to complete the puzzle.
3. Make a tower of 'three' disks and determine the minimum number of moves to complete the puzzle.
4. Make a tower of 'four' disks and determine the minimum number of moves to complete the puzzle.
5. Make a tower of 'five' disks and determine the minimum number of moves to complete the puzzle.

6. Complete the following table. To determine the minimum number of moves for a six disk tower, see if you can discover any patterns in the table.

number of disks	1	2	3	4	5	6
minimum number of moves						

7. Describe the pattern you used to find the minimum number of moves for six disks.

P Exercise 1A
(Write your answers to these questions in your 'Problem Book')

Question 1

The monks of Hanoi are said to have a tower with 100 disks. If possible, use the pattern you have discovered to complete the following table.

If your pattern is not able to help you or results in a very labourious task, describe the feature of your pattern that has caused this. Your calculator may be able to help with the labourios task. If this is the case, look for another pattern that will make your task more simple.

The rule should LINK the 'minimum number of moves' and the 'number of disks'.

number of disks	12	20	***100***	2.12×10^{2}
minimum number of moves				

Question 2

Estimate the number of years it will take the 100 disk puzzle of the monks of Hanoi to be completed, assuming that they do so in the minimum number of moves.

TEXT-BOOK QUESTIONS

Text book questions must be done in your 'problem book', including diagrams and working out.

Haese et al, **Mathematics for Year 8,** Chapter 5

- Read page 107
- Complete Investigation 6 on page 121

MATHEMATICAL REFLECTIONS

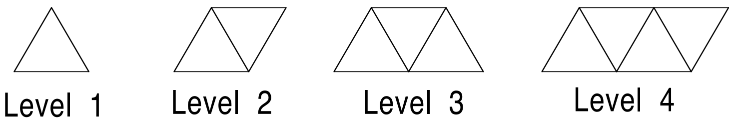
Mathematical reflections must be answered in sentences, in your exercise book.

Between your, your teacher and your classmates you should have come up with at least two different types of patterns that describe how the towers of Hanoi puzzle works. CHOOSE two of the patterns and discuss the similarities and difference between these patterns. Form a table like the following to present your ideas orderly:

SIMILARITIES	DIFFERENCES

Investigation 2: CHAINS OF POLYGONS

Consider the following sequence of triangle chains. Each chain is considered to be a different **LEVEL** of the chain.



If you were asked to build these structures with toothpicks, it would take 3 for level 1, 5 for level 2 and so on.

We could summarise this information in a table that looks as follows:

level number	1	2	3	4	5	6
no. of picks required	3	5				

1. Copy this table into your **note book** and complete it. Draw the level 5 and 6 structures if necessary.
2. Look at the tables contents and identify any patterns that are present. There is more than just one pattern. Explain why your pattern works.
3. If you can, use your pattern to complete the following table that relates to the same sequence of structures.

level number	7	8	9	100	2000	2×10^8
no. of picks required						

4. Have you proven deductively that you pattern continues on for ever or have you 'induced its truth? Explain.

Teacher Input

How did you go?

Listen carefully to the class discussion to follow and take the necessary notes in your note book that will summarise the theory behind this investigation.

RECURSIVE PATTERNS

If you had difficulty in finding the no. of picks for level 100 and greater then you have most likely identified the '**RECURSIVE**' pattern. This pattern works horizontally along the number of sticks row. This is an easy pattern to see -

each successive level requires two more sticks

We say that a **'CONSTANT ADDER' of 2** exists. Recognising this makes it easy to calculate successive values, but what about for level 100? You would need to do lots of addings!!

LINK RULES BETWEEN LEVEL NUMBER AND THE NO. OF STICKS

A more powerful pattern is one that links the level number to the number of sticks. Lets look at the table again but in a differnet way.

level number	1	2	3	4	5	6
no. of picks required	3 =(2+1) =(1×2+1)	5 =(2+2+1) =(2×2+1)	7 =(2+2+2+1) =(3×2+1)	9 =(2+2+2+2+1) (4×2+1)	11 =(2+....+1) (5×2+1)	13 =(2+....+1) (6×2+1)

Take level three for instance. 7 picks are required or 2+2+2+1. The 2+2+2+1 comes from the fact that in building the third level you have added 2 picks 3 **'from level zero'**. Some may say level zero does not exit. Fair enough but if it did, and you follow the pattern backwards then it is simply ONE stick. The 2+2+2+1 for level three then makes sense.

From here you should be able to see that :

Number of picks = level number × 2 +1

This is another pattern. The pattern has been summarised by a LINK RULE. Namely:

Number of picks = level number × 2 +1

Now mathematicians created there own language some years ago. They called it ALGEBRA. In this language we replace a quantity that can have lots of values - like number of picks, with a letter - say N and the level number with - say L. Our LINK RULE now becomes:

$$N = L \times 2 + 1 \quad \text{or}$$

$$N = 2L + 1$$

2L is the way we write 2×L or L×2 in shorthand.

This is a good rule. It allows us to calculate very simply the number of picks required for any level.

eg. How many picks will be required for level 500?

To do this we use our rule and set out the solution as follows:

$\Rightarrow$ N=2L+1

$\Rightarrow$ N=2(500)+1

$\Rightarrow$ N=1000+1

$\Rightarrow$ N=1001

Therefore we will need 1001 picks.

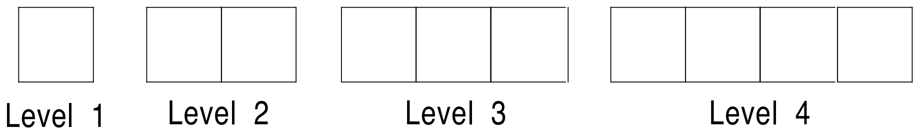
NB The $\boxed{\Rightarrow}$ sign is shorthand for the phrase ' this implies'.

P Exercise 1B

(Write your answers to these questions in your 'Problem Book')

Question 1

Consider the following sequence of square chains. Each chain is considered to be a different **LEVEL** of the chain. Imagine you need to build these chains with toothpicks.



a) Complete the following table

level number (l)	1	2	3	4	5	6
no. of picks required (N)						

b) Determine the recursive pattern for this case. Explain why it works

c) Determine the rule that links the number of picks (N) to the level number (l). Be sure to check that the rule that you have formed works for each of the levels for which you have COUNTED the number of picks required. Explain why this works.

d) Use your rule from c) to complete the following table. Set each problem out as shown earlier in the notes.

level number (l)	7	8	20	2000	2×10^8
no. of picks required (N)					

Question 2

a) Draw a sequence of pentagon chains similar to the square and triangle chains.

b) Complete the following table

level number (l)	1	2	3	4	5	6
no. of picks required (N)						

c) Determine the recursive pattern for this case. Explain why it works

d) Determine the rule that links the number of picks (N) to the level number (l). Be sure to check that the rule that you have formed works for each of the levels for which you have COUNTED the number of picks required. Explain why this works.

e) Use your rule from c) to complete the following table. Set each problem out as shown earlier in the notes.

level number (l)	7	8	20	1000	2.13×10^9
no. of picks required (N)					

Question 3

a) Draw a sequence of hexagon chains similar to the square and triangle chains.

b) Complete the following table

level number (l)	1	2	3	4	5	6
no. of picks required (N)						

c) Determine the recursive pattern for this case. Explain why it works

d) Determine the rule that links the number of picks (N) to the level number (l). Be sure to check that the rule that you have formed works for each of the levels for which you have COUNTED the number of picks required. Explain why this works.

e) Use your rule from c) to complete the following table. Set each problem out as shown earlier in the notes.

level number (l)	7	8	20	1000	2.13×10^{9}
no. of picks required (N)					

Question 4

a) Copy this table into your problem book and complete it using the knowledge gained fron question 1 -3:

Type of Chain	**Number of picks added to get to the next level.**	**Rule**
Triangle		
Square		
Pentagon		
Hexagon		

b) Look carefully at this table and induce the rule that may work for a chain of octagons. Explain why you think it will work.

c) Look carefully at this table and induce the rule that may work for a chain of decagons. Explain why you think it will work.

d) Look carefully at this table and induce the rule that may work for a chain of dodegagons. Explain why you think it will work.

Question 5

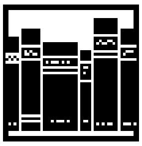
Consider the rule for the chain of squares :

$$N=4L+1$$

a) How does the 4 relate to the recursive pattern for this chain? Be sure to use the term 'constant adder' in your answer.

b) What significance does the 1 have? (Recall the level zero idea.)

c) Do you answers to a) and b) seem to be generally true for the other chains you have investigated? Give examples to support your answer.

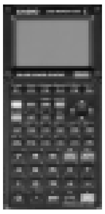


TEXT-BOOK QUESTIONS

Text book questions must be done in your 'problem book', including diagrams and working out.

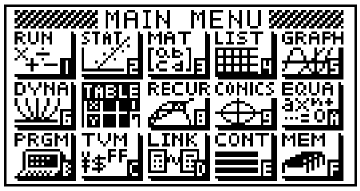
Haese et al, **Mathematics for Year 8,** Chapter 5

- Exercise 5A Question 3

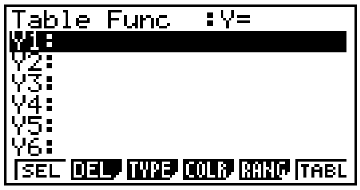


DISPLAYING PATTERNS WITH SOME TECHNOLOGICAL ASSISTANCE. (CASIO 9850GB PLUS)

With the calculator turned on and the main menu visible, use the arrow key to highlight the TABLE menu (shown opposite).

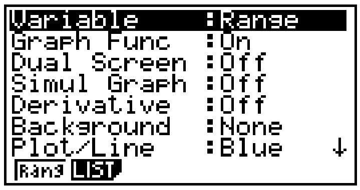


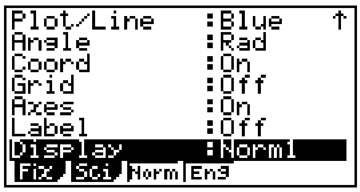
Then press the blue EXE key (alternatively, simply press 7). The following screen will result:



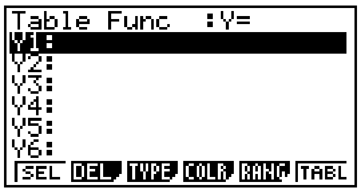
Now press SHIFT and then MENU to reveal the 'setup screen' for this module. Set each option as shown right. Use your arrow keys to scroll down to the 'hidden options'.

You can experiment with different settings later.

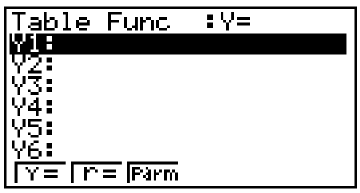




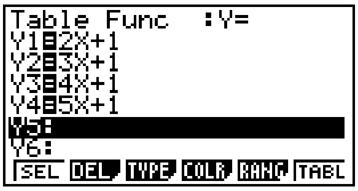
Press the black EXIT key and you will return to this sceen:



If your screen does not have Y1 and so on on the left, access TYPE (F3) and access Y= (F1)



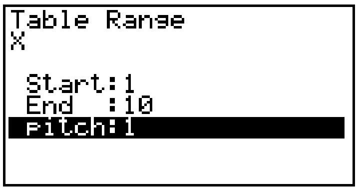
We will now enter the LINK RULES we determined for the polygon patterns.



Press 2 followed by the X,θ,T key, then + then 1, to enter y = 2x + 1, which is the same as N=2L+1.

Carry out a similar procedure to enter the link rules for the othe polygon patterns to achieve the screen opposite.

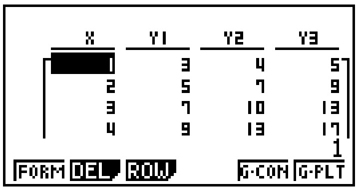
We now need to tell the calculator the x values that we wish it to use to calculate y values.



Access RANGE (F5) and set the values of start end and pitch to those shown opposite. Pitch is the incremental jumps that you wish to have in the table.

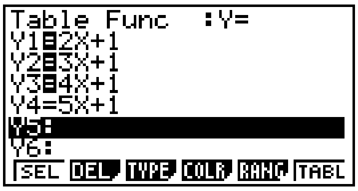
Press the EXIT key when you are done.

Access TABL (F6) to produce the table.

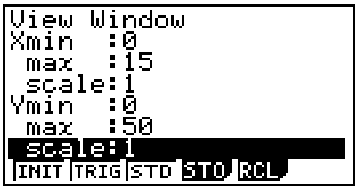


You can navigate the table using the arrow keys.

Should you not want to have all the link rules appear in the graph, press EXIT, use the arrow keys to select the rule(s) you do not want (for this exercise select Y4) and access SEL (F1). The = sign will no longer be surrounded with a dark rectangle and is said to be NOT SELECTED.



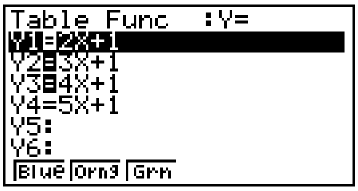
If we wsh to draw a graph that illustrates each of these link rules we must first set the scale of the axes.



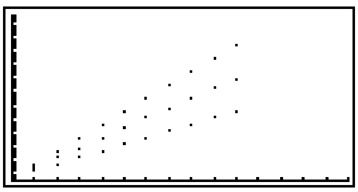
Press SHIFT and accessV-WIN and the View Window settings will appear.

Set the values as shown opposite, think about why we have set them this way.
Press the EXIT key

We can now set each of the link rules to have different coloured graphs. Sected the first rule, access COLR (F4) and then choose the colour you want by pressing the appropriate key, either F1, F2 or F3. Simply arrow down to select the other rules and choose a colour. Press the EXIT key.



Now access TABL and then G-PLT to produce the required graph.



Not seen in colour here unfortunately.

Teacher Input

THE THEORY OF LINEAR PATTERNS

Patterns that have a **'CONSTANT ADDER'** as a *recursive pattern* are said to be LINEAR PATTERNS.

From the graphs that you have just drawn, why do you think they are called linear?

All of the patterns seen in the 'CHAINS of POLYGONS' investigation were linear as they had a **'CONSTANT ADDER'** as a *recursive pattern* .

The rule that linked the number of picks to the level number for all linear patterns has something in common as well. Let us look back at the table you produced for question 4 in the previous exercise.

Type of Chain	Number of picks added to get to the next level.	Rule
Triangle	2	$N=2L+1$
Square	3	$N=3L+1$
Pentagon	4	$N=4L+1$
Hexagon	5	$N=5L+1$

Each of the rules look like the following:

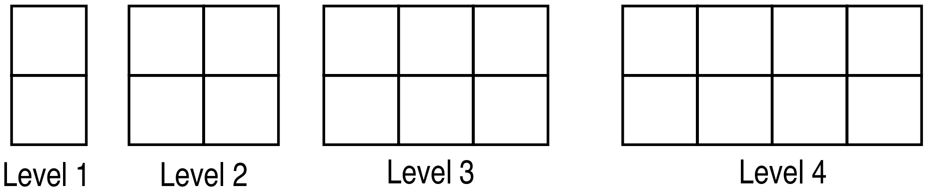
$$y = mx+c$$

where **y** is the number of things required to form the pattern
x is the level number
m is the constant adder and
c is the number of things that are required to build level 0 (even though it may not exist)

Remember that level 0's value is simply obtained by following the pattern backwards.

APPLYING THIS THEORY.

Consider the following geometric pattern:



If you were asked to build these structures with toothpicks, it would take 7 for level 1, 12 for level 2 and so on.

We could summarise this information in a table that looks as follows:

level number L	1	2	3	4	5	6
no. of picks required N	7	12	17	22	27	32

constant adder	***5***	***5***	***5***	***5***	***5***

Clearly a CONSTANT ADDER of **5** exists (ie. the recursive pattern is +5).

Therefore the rule that links the number of picks to the level number is:

$$\mathbf{N = 5L+c}$$

Now to find 'c'. We now need to go back to that 'LEVEL 0'.

level number L	0	1	2
no. of picks required N	c	7	12

constant adder	***5***	***5***	***5***

Clearly $c = 7 - 5$
$\quad\quad\;\; = 2$

Therefore the rule that links the number of picks to the level number is:

$$\mathbf{N = 5L+2}$$

Having found this rule we can then use it as we did earlier.

Consider the following table:

level number	100	2×10^8
no. of picks required		

How many pick will be required for level 100?

To do this we use our rule and set out the solution as follows:

$$N=5L+2$$
$$\Rightarrow N=5(100)+2$$
$$\Rightarrow N=500+2$$
$$\Rightarrow N=502$$

Therefore we will need 502 picks.

How many pick will be required for level 2×10^8?

To do this we use our rule and set out the solution as follows:

$$N=5L+2$$
$$\Rightarrow N=5(2 \times 10^8)+2$$
$$\Rightarrow N=1 \times 10^9+2$$
$$\Rightarrow N=1\ 000\ 000\ 002$$

Therefore we will need 1 000 000 002 picks.

P Exercise 1C

(Write your answers to these questions in your 'Problem Book')

Question 1

For the following geometric pattern,

a) Complete a table like the following:

level number L	1	2	3	4	5	6
Shaded area A						
constant adder						

b) Write down the constant adder
c) Determine the number of picks required for level 0
d) Write down the rule that links the shaded area to the level number.
e) Use your rule to complete the following table

level number L	140	300
Shaded area A		

Question 2

For the following geometric pattern,

a) Complete a table like the following:

level number L	1	2	3	4	5	6
Number of circles (N)						
constant adder						

b) Write down the constant adder
c) Determine the number of picks required for level 0
d) Write down the rule that links the number of circles to the level number.
e) Use your rule to complete the following table

level number L	140	720
Number of circles (N)		

Question 3

For the following geometric pattern,

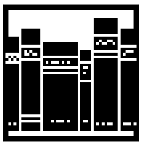
a) Complete a table like the following:

number of diamonds n	1	2	3	4	5	6
Number of intersections (I)						
constant adder						

b) Write down the constant adder
c) Determine the number of picks required for level 0
d) Write down the rule that links the number of intersections to the number of dots.

e) Use your rule to complete the following table

number of diamonds n	45	206
Number of intersections (I)		

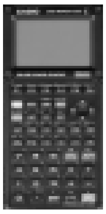


TEXT-BOOK QUESTIONS

Text book questions must be done in your 'problem book', including diagrams and working out.

Haese et al, **Mathematics for Year 8,** Chapter 5

- Exercise 5B Question 1,2 and 3.

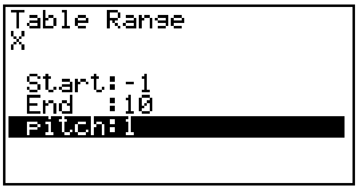


Investigation 3: The effcts of changing m and c.

To delete unwanted link rules in the Table menu of the CASIO (9850)GB PLUS, selected the rule and acess DEL (F2). You will be then be given the option to delete the rule, access YES (F1). You do not have to delete these, but be sure they are not selected.

In the table menu of the CASIO (9850)GB PLUS, enter the following rules:

$$y = 3x + 1$$
$$y = 5x + 2$$
$$y = -2x + 6$$
$$y = 4x - 5$$



Produce a table with range as shown opposite. The - on the -1 is entered with the (-) key that is next ti the EXE key.

Produce a table and G-PLOT graph of each on the same axes. Be sure to set the VIEW WINDOW before graphing.

Answer each of the following questions in your workbook.

1. What effect does the differing 'm' values have on

   i) the values in the table?
   ii) the graphs produced?

2. What effect does the differing 'c' values have on

   i) the values in the table?
   ii) the graphs produced?

Investigation 4: Sara Lee - Layer upon layer upon layer.

1. Take an A-4 (or A3 if you have one) piece of paper. Mark one face with a cross to denote this to be the uppermost face. Lay it on the table (cross upwards) and fold it in half going from left to right. Be sure to crease the fold well. Open the paper so it is A-4 sized again. It has only one crease line that has formed a valley. We will call this a valley crease. Returned the paper to the 'folded in half' position and fold it in half again. Open the paper so it is A-4 sized again. Notice this time that it has more valley creases but also some creases that form 'mountains' - we will call these mountain creases. Your job is to continue to fold in halves and keep track of the number of valley and mountain creases. Summarise your findings in your problem book using a table similar to the one below.

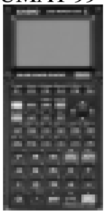
number of folds (f)	1	2	3	4	5	6
number of valley creases (V)						
number of mountain creases (M)						
total number of creases (T)						

2. Determine the recursive pattern for the number sequences for V, M and T.

3. i) Determine a link rule that links V and f
ii) Determine a link rule that links M and f
iii) Determine a link rule that links T and f

4 Use your link rules to determine how many of each type of crease will be present if the paper is folded 10 times.

5 Use your link rules to determine how many of each type of crease will be present if the paper is folded 20 times.



MATHEMATICAL REFLECTIONS (with the help of the 9850GB PLUS)

Mathematical reflections must be answered in sentences, in your exercise book.

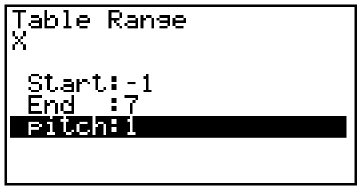
Look carefully at the way the values of the numbers in the crease patterns grow. How does this compare with the linear patterns we have looked at?

In the table menu of the CASIO (9850)GB PLUS, enter the following rules:

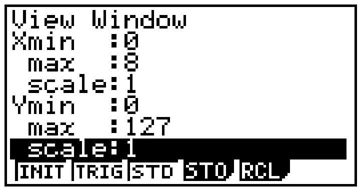
$$y = 3x + 1$$
$$y = 2^x - 1$$

The exponent of x can be entered using the ^ key that is next to the EXIT key.

Produce a table with range as shown opposite. The - on the -1 is entered with the (-) key that is next ti the EXE key.



Produce a table and G-PLOT graph of each on the same axes. Be sure to set the VIEW WINDOW as shown opposite before graphing.



Describe the difference between the growth seen in the table and the graph.

Explain why the difference exits between the values that these two rules generate.

Investigation 5: Hopping all over the world - Status Quo.

When walking through the park last weekend I observed some tiny frogs playing a game similar to leap frog on some water lily pads.

The game went as follows :

GF		**RF**

A green frog sat on one lily pad and a red frog sat on another **with an empty lily pad in between.**

The aim of the game was for the frogs to swap places by either **sliding onto an empty pad** or by **jumping over a frog onto a vacant lily pad.**

Watch carefully as your teacher demonstrates the game.

After successfully completing the game each frog called over a friend of the same colour to sit on a lily pad by their side as shown below. There was always only one lily pad in between the sets of frogs at the beginning.

GF	**GF**		**RF**	**RF**

Once again the frogs either **slide onto an empty pad** or **jumped over a frog onto a vacant lily pad** until the green frogs were where the red frogs were and vice versa.

The frogs kept on calling one more frog over to play another game once the one they were playing was finished.

GF	**GF**	**GF**		**RF**	**RF**	**RF**

1. In your problem book, draw up your own lily pads and use coins, counters or tiles to take the place of the frogs. Start with just one pair (a pair is 1 red and 1 green frog) and determine the minimum amount of moves to complete the task.

2. Play the game again but with 2 pair of frogs. Determine the minimum number of moves required to complete the game now.

3. Continue playing the game and complete the following table:

number of pairs of frogs (p)	1	2	3	4	5	6
Minimum number of moves (M)						

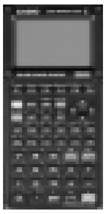
4. Determine the recursive pattern for the number sequences for M.

5. Determine a rule that links M and p.

6. Use your rule to determine the minimum number of moves for 20 pairs of frogs.

7. Use your rule to determine the minimum number of moves for 100 pairs of frogs.

8. By considering the way in which the frogs move, build a logical argument that PROVES your rule from part 5. to be correct in all cases.



MATHEMATICAL REFLECTIONS (with the help of the 9850GB PLUS)

Mathematical reflections must be answered in sentences, in your exercise book.

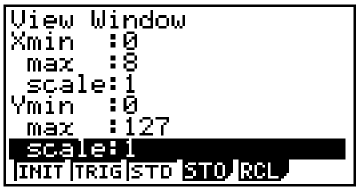
Let us now investigate how the frog link rule compares to the other types we have seen

In the table menu of the CASIO (9850)GB PLUS, enter the following rules:

$$y = 3x + 1$$
$$y = 2^x - 1$$
$$y = x(x + 2)$$

Produce a table with range as shown opposite. The - on the -1 is entered with the (-) key that is next ti the EXE key.

Produce a table and G-PLOT graph of each on the same axes. Be sure to set the VIEW WINDOW as shown opposite before graphing.



Compare and contrast the table and graph produced by these three link rules.

Teacher Input

EQUIVALENT RULES and the art of ALGEBRAIC SUBSTITUTION

Recall your rule for the total number of creases in the paper folding investigation. You may have started with

$$\begin{aligned} & T = V + M \\ \Rightarrow\quad & T = 2^{f-1} + 2^{f-1} - 1 \end{aligned}$$

Now I will suggest to you that $T = 2.2^{f-1} - 1$ is an EQUIVALENT RULE to $T = 2^{f-1} + 2^{f-1} - 1$.

By this I mean that it looks a little different but will generate the same T value for any value you choose to try for f. Remember to recall the idea of BEDMAS.

Let us try f = 1

$$\begin{aligned} & T = 2^{f-1} + 2^{f-1} - 1 \\ \Rightarrow\quad & T = 2^{1-1} + 2^{1-1} - 1 \\ \Rightarrow\quad & T = 2^{0} + 2^{0} - 1 \\ \Rightarrow\quad & T = 1 + 1 - 1 \\ \Rightarrow\quad & T = 1 \end{aligned}$$

$$\begin{aligned} & T = 2.2^{f-1} - 1 \\ \Rightarrow\quad & T = 2.2^{1-1} - 1 \\ \Rightarrow\quad & T = 2.2^{0} - 1 \\ \Rightarrow\quad & T = 2.1 - 1 \\ \Rightarrow\quad & T = 2 - 1 \\ \Rightarrow\quad & T = 1 \end{aligned}$$

and yes, they generate the same correct result.

Let us try f = 4

$$\begin{aligned} & T = 2^{f-1} + 2^{f-1} - 1 \\ \Rightarrow\quad & T = 2^{4-1} + 2^{4-1} - 1 \\ \Rightarrow\quad & T = 2^{3} + 2^{3} - 1 \\ \Rightarrow\quad & T = 8 + 8 - 1 \\ \Rightarrow\quad & T = 15 \end{aligned}$$

$$\begin{aligned} & T = 2.2^{f-1} - 1 \\ \Rightarrow\quad & T = 2.2^{4-1} - 1 \\ \Rightarrow\quad & T = 2.2^{3} - 1 \\ \Rightarrow\quad & T = 2.8 - 1 \\ \Rightarrow\quad & T = 16 - 1 \\ \Rightarrow\quad & T = 15 \end{aligned}$$

and yes, they generate the same correct result.

Note that it is not good enough to now simply say that $T = 2.2^{f-1} - 1$ is an EQUIVALENT RULE to $T = 2^{f-1} + 2^{f-1} - 1$. We have only shown this to be true for two cases.

MATHEMATICAL REFLECTIONS

Mathematical reflections must be answered in sentences, in your exercise book.

It is possible to write down, in English, a logical argument that will prove $T = 2.2^{f-1} - 1$ is an equivalent rule to $T = 2^{f-1} + 2^{f-1} - 1$. This is not hard and is based on your knowledge of operations. Try it.

BACK TO THOSE HOPPING FROGS

Now recall the rule you generated for the 'hopping frogs'.

Most students will have noticed that

'the minimum number of moves is found by multiplying the number of pairs of frogs by 2 more than the number of pairs of frogs'.

or algebraically, $M = p(p+2)$.

Now I will pose the conjecture that an equivalent rule to this is $M = p^2 + 2p$.

Let us try to show that these rules are equivalent for p=1 and p=50.

for p = 1

$$\begin{aligned} & T = p(p+2) \\ \Rightarrow\; & T = 1(1+2) \\ \Rightarrow\; & T = 1.3 \\ \Rightarrow\; & T = 3 \end{aligned} \qquad \begin{aligned} & T = p^2 + 2p \\ \Rightarrow\; & T = 1^2 + 2.1 \\ \Rightarrow\; & T = 1 + 2 \\ \Rightarrow\; & T = 3 \end{aligned}$$

so it is fine for p = 1

if p = 50

$$\begin{aligned} & T = p(p+2) \\ \Rightarrow\; & T = 50(50+2) \\ \Rightarrow\; & T = 50.52 \\ \Rightarrow\; & T = 2600 \end{aligned} \qquad \begin{aligned} & T = p^2 + 2p \\ \Rightarrow\; & T = 50^2 + 2.50 \\ \Rightarrow\; & T = 2500 + 100 \\ \Rightarrow\; & T = 2600 \end{aligned}$$

so it is fine for p = 50

So it **seems** that "adding two to a number and multiplying it by itself **may** be the same as multiplying a number by two and multiplying it by its square".

It is common practice to try two or three cases for small values, say p = 1, 2 and 3 and then two larger values say p = 50 and 400. This way we have tested a reasonable breadth of cases. Remember though that there is an infinite number of cases - we can not try them all and as a result can not prove, using this method, that these rules are equivalent.

Here we are using the process of **INDUCTIVE LOGIC**, that is we are observing that the conjecture of euqivalence works for a few cases so we induce that it may work for all.

MATHEMATICAL REFLECTIONS

Mathematical reflections must be answered in sentences, in your exercise book.

In the last reflections you actually used **DEDUCTIVE LOGIC** to reason why the two rules were the same. That is you worked from things you knew were true to deduce the truth of something you were unsure about. It is possible to deduce that $M = p(p+2)$ and $M = p^2 + 2p$. are equivalent rules. You need to think about the operations involved again (multiplication just means 'lots of') and may need the help of a diagram to make you argument more clear. You may also like to consider the rule $M = (p+1)^2 - 1$ and show it is equivalent to the other two. Write, in words, what the equivalence of these rules means.

P Exercise 1D

(Write your answers to these questions in your 'Problem Book')

1. a) Show that the pairs of rules below are equivalent rules using p = 1, 2, 40 and 400

   i) K = p(p+3) and K = $p^2 + 3p$
   ii) K = p(p+5) and K = $p^2 + 5p$
   iii) K = p(p+10) and K = $p^2 + 10p$

   b) Use the Casio 9850GB PLUS to show that the rules below are equivalent for p = -3,-2.5,-2,-1.5, _ _ _, 30

   i) K = p(p+3) and K = $p^2 + 3p$
   ii) K = p(p+5) and K = $p^2 + 5p$
   iii) K = p(p+10) and K = $p^2 + 10p$

   c) Use what you have found in a) to induce what you think will be the equivalent rule for K = p(p+q)

2. a) Show that the pairs of rules below are equivalent rules using p = 1, 2, 40 and 400

   i) K = $2^{p+3}.2^4 - 1$ and K = $2^{p+7} - 1$
   ii) K = $2^{p+3}.2^p - p$ and K = $2^{2p+3} - p$
   iii) K = $2^{2p-1}.2^{1-p} - p^2$ and K = $2^p - p^2$

   b) Use the Casio 9850GB PLUS to show that the rules below are equivalent for p = -10,-9.5,-9,-8.5, _ _ _, 20

   i) K = $2^{p+3}.2^4 - 1$ and K = $2^{p+7} - 1$
   ii) K = $2^{p+3}.2^p - p$ and K = $2^{2p+3} - p$
   iii) K = $2^{2p-1}.2^{1-p} - p^2$ and K = $2^p - p^2$

   c) Use what you have found in a) to induce what you think will be the equivalent rule for K = $2^a.2^b - v$

3. a) Determine which of the of rules below is equivalent to the rule K = $(p+4)(p-4)$ using p = 1, 2 and 40

   i) K = $p - 4$
   ii) K = $p^2 - 16$
   iii) K = $p^2 - 4$

   b) For how many values of p would you have to show a rule does not match with its counterpart to conclude that it is not identical. Why?

4. Consider the little problem below called '**regions in circles**'.

a) If n is the letter to denote the number of chords and R is the letter to denote the number of regions, construct a table that shows the number of regions for 1 to 5 chords inclusive.

b) Show that the rule R = $\frac{1}{2}n^2 + \frac{1}{2}n + 1$ generates the correct R values for n = 1 - 3 inclusive.

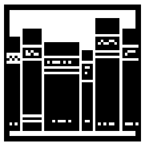
c) Show that an equivalent rule to the one used in b) is R = $\frac{n(n+1)}{2} + 1$, using n = 1- 3 inclusive and n = 100.

5. Consider the little problem below called '**triangles in a triangle**'.

a) If n is the letter to denote the number of units on a side of the triangle and T is the letter to denote the number of different equilatral triangles in the figure.

b) Show that the rule T = $\frac{1}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{3}n$ generates the correct T values for n = 1 - 3 inclusive.

c) Show that an equivalent rule to the one used in b) is T = $\frac{n(n+1)(n+2)}{6}$, using n = 1- 3 inclusive and n = 100.

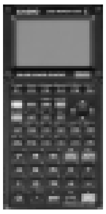


TEXT-BOOK QUESTIONS

Text book questions must be done in your 'problem book', including diagrams and working out.

Haese et al, **Mathematics for Year 8,** Chapter 5

- Exercise 5C Questions 1 - 4
- Exercise 5E Questions 1 - 3



SUMMING THE TERMS OF A SEQUENCE (with the help of the CASIO 9850GB PLUS)

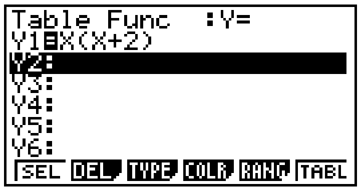
Return to the hopping frog puzzle. Suppose we want to know how many moves a player would have to make in order to play the game using from 1 to 100 pairs of frogs inclusive.

This means we would have to calculate the following SUM:

$$3 + 8 + 15 + 24 + \_\ \_\ \_\ \_\ \_ + 10200$$

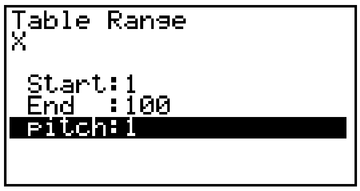
Follow the instructions below to see how the 9850GB PLUS will help to calculate this rather long SUM.

Enter the TABLE module and enter the link rule y = x(x+2).



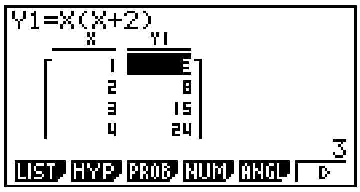
Access RANG (F5) and set the table range values as shown opposite.

Press the EXIT key.



Access TABL (F6)

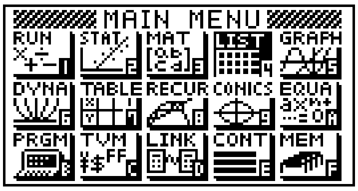
With the value highlighted asshown opposite press the OPTN key (stands for option) to reveal the menu seen opposite.



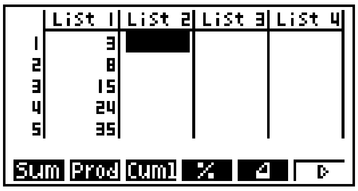
Access LIST, then LMEM and then List1.

This process will have copied the Y1 column into List 1.

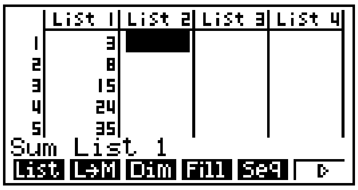
Now press the MENU key and select the LIST module and press the blue EXE key.



Press the OPTN key, access LIST (F1) and then access the arrow twice (F6 twice) to revela the menu seen opposite.

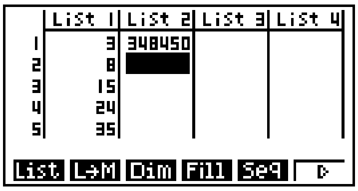


Place the cursor as shown and
access Sum (F1),
then access the arrow (F6),
then List (F1) and
then press 1.



This will input the command as shown

Press the blue EXE key and the sum of the list will be shown.



Hence it would take a person 348450 moves to play the hopping frog puzzle with 1 to 100 pairs of frogs.

PROBLEMS TO SOLVE

(Write your attempts and final solutions to these problems in your 'Problem Book')

1. Justin and Laura are two members of a 20 strong youth group. If the group are asked to stand in a straight line, determine in how many positions Justin and Laura stand so that there are three or more people between them.

2. a) Find the sum of the first 2 odd numbers
   b) Find the sum of the first 3 odd numbers
   c) Find the sum of the first 4 odd numbers
   d) Find the sum of the first 5 odd numbers
   e) Use the CASIO 9850GB Plus to find the sum of the first 50 odd numbers
   f) Write down a formua for the sum of the first n odd numbers

3. a) Write down the angle sum of a triangle, quadrilateral, pentagon and hexagon.

   b) Find the angle sum of a convex 50-a-gon.

   c) The smallest angle in a convex polygon is 125 degrees. The next largest angle is 5 degrees larger and next is 5 degress larger and so on. Find the number of sides in this polygon. Is there more than one solution?

4. Last night my door bell rang 80 times. The first time it rang 4 guests arrived, the second time 10 guests arrived, then 18 then 28 and so on. How many guests did I receive?

5. Consider the sequence 1, 3, 6, 10, _ _ _ _. This is called a triangular sequence, why?.

   a) Find the 20th term in this sequence.
   b) Find the 200th term in this sequence.

   c) Given a set of n points in a plane, no three which are collinear, determine how many line segments could be formed.

   d) Given a set of 80 points in a plane, no three which are collinear, determine how many line segments could be formed.

6. The king of Nanastam decided to have a reception for the marriage of his daugther Hulu to the Prince of the Northland. Many invitations were sent out far and wide, in all 10 000 guest were invited. When the quests arrived for the grand occasion they did so in a strange way. The first lowering of the draw bridge saw three people cross it. The next time eight people entered, the next time thirteen entered and so on.

   When the drawbridge had been lowered fifty times, the king asked the bridgeman how many guests were still to come. What should have his answer been?

   If all of the 10 000 guests attended and entry continued as explained, determine if the number of guests in last group to pass the draw bridge violated the pattern.