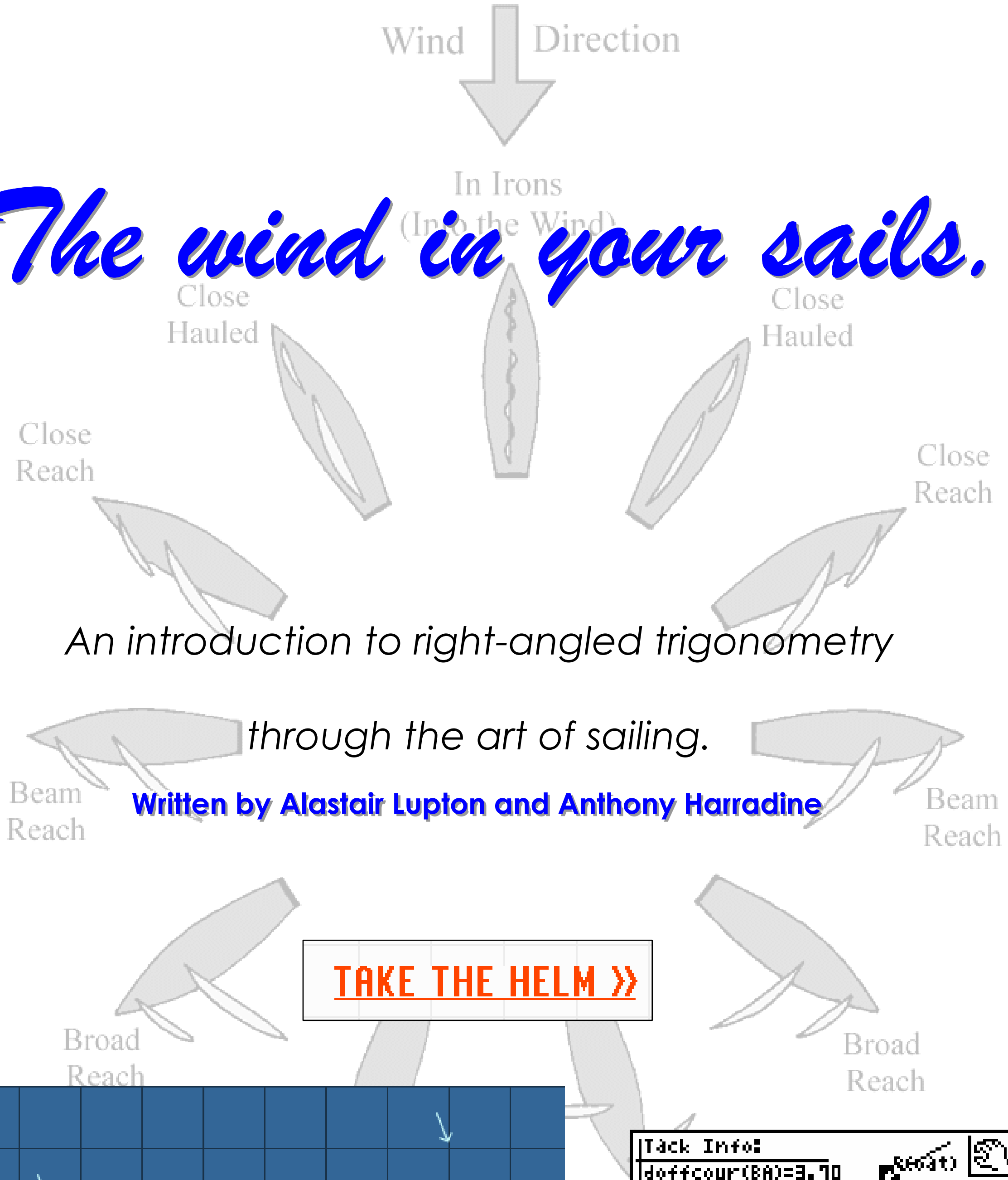
The wind in your sails.

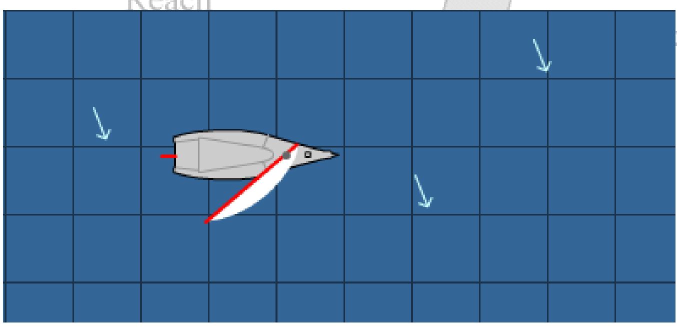


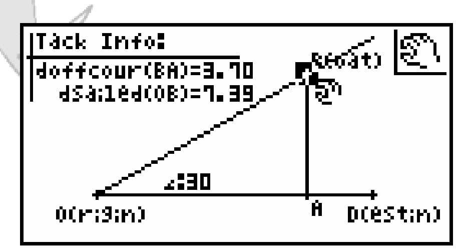
An introduction to right-angled trigonometry

through the art of sailing.

Written by Alastair Lupton and Anthony Harradine

TAKE THE HELM »





Prince Alfred College
Founded 1869

Noel Baker Centre
for School Mathematics

CASIO®

The wind in your sails.

Version 1.02 – April 2008.

Written by Anthony Harradine and Alastair Lupton

Copyright Information.

Index

Section **Page**

Using this resource.

This resource is *not* a text book.

It contains material that is hoped will be covered as a dialogue between students and teacher and/or students and students.

You, as a teacher, must plan carefully 'your performance'. The inclusion of all the 'stuff' is to support:

- you (the teacher) in how to plan your performance – what questions to ask, when and so on,
- the student that may be absent,
- parents or tutors who may be unfamiliar with the way in which this approach unfolds.

Professional development sessions in how to deliver this approach are available.

Please contact

The Noel Baker Centre for School Mathematics
Prince Alfred College,
PO Box 571 Kent Town, Sth Australia, 5071
Ph. +61 8 8334 1807
Email: aharradine@pac.edu.au

Legend.

EAT – Explore And Think.

These provide an opportunity for an insight into an activity from which mathematics will emerge – but don't pre-empt it, just *explore and think!*

At certain points the learning process should have generated some ***burning mathematical questions*** that should be discussed and pondered, and then answered as you learn more!

Time to Formalise.

These notes document the learning that has occurred up to this point, using a degree of formal mathematical language and notation.

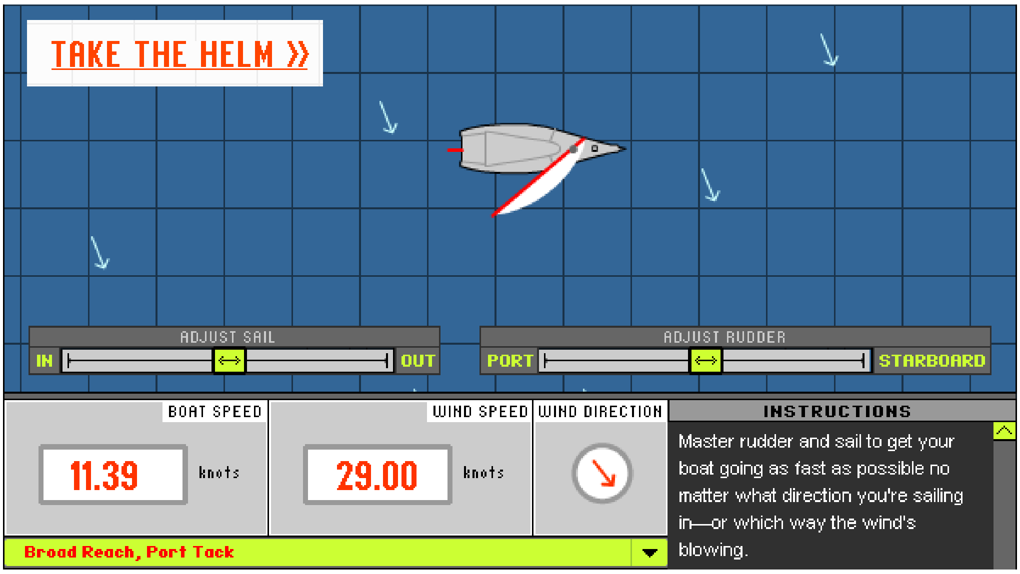
Examples.

Illustrations of the mathematics at hand, used to answer questions.

1. *Take the helm!*

Have you ever sailed in a yacht? Do you know how to sail? If not, don't worry as you are able to gain some ideas about sailing from a simulator. A simple one can be found at:

http://www.nationalgeographic.com/volvooceanrace/interactives/sailing/index.html



1.1 Task 1.

Using the simulator, adjust the rudder and sail controls until you have the yacht sailing *as fast as possible*. **Record your top BOAT SPEED and the WIND SPEED and summarise your sailing strategy**.

1.2 Task 2.

Adjust the rudder controls until you have the yacht sailing *due east*. Now adjust your sail control to maximise your speed.

Record your top BOAT SPEED, the WIND SPEED, the WIND DIRECTION and summarise your sailing strategy.

1.3 Task 3.

Adjust the rudder controls until you have the yacht headed *directly into the wind*. Now adjust your sails to get your yacht travelling at 5 knots or more. If you cannot do this, head your boat slightly out of the wind and adjust the sails so that you can travel at 5 knots.

How '*close to the wind*' can you sail and still achieve a speed of 5 knots?

2. A geometric model of sailing on a 30 degree tack.

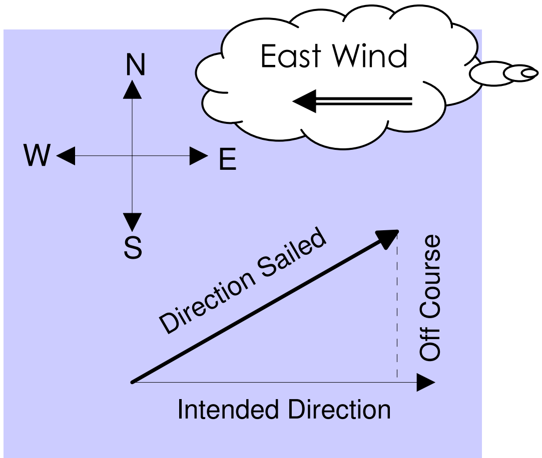
To sail a boat, using wind as the only source of power, is a skilful activity. With the right technique, it is possible to sail in almost any direction, regardless of where the wind is blowing from.

The only limitation is that a boat *cannot* sail directly into the wind, or too close to that direction.[1]

This can mean that, for example, if you want to sail east and the wind is blowing from the east, you are forced to sail slightly off course.

This is referred to as sailing on a <u>tack</u>. The result of a tack is that a boat ends up <u>off course</u>, by a distance that has to be compensated for when the boat changes direction.

The calculation of the distances involved in a tack is important for sailors.



For example, *if a tack of* 30^o *(away from the wind direction) is sailed for given distance, how far off course will the boat be?*

2.1 EAT 1

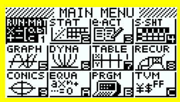
a. Generate a random integer between 1 and 30 (you may wish to use your graphics calculator). This will be the distance of your 30^o tack, in kilometres.

b. Using a ruler and geoliner, draw a <u>scale drawing</u> of your tack on a piece of graph paper. Measure your distance *off course*.

c. Add 2 or 3 more tack lengths (and distances *off course*) to your diagram – generate other random integers or use the values generated by your neighbours.

Pool each class member's result, displaying them in a number of ways.

Will You Look At That! (WYLAT!)



14.1

---

[1] The degree that a boat can sail into the wind depends on the strength of the wind and the characteristics of the boat.

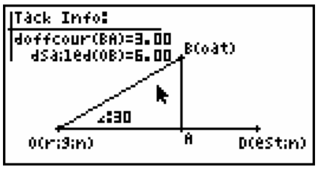
Following EAT 1 it is hoped you have had a few questions pop into your head due to your natural mathematical inquisitiveness. List the questions and share them with your classmates.

2.2 EAT 2

On your graphics calculator, open the geometry file called `ATACMAN`. This is a geometric model of a boat sailing. Select the diagonal line representing *Dist. Sailed*. Open the measurement box (press VARS) and enter four values, in turn, of your choosing.

Pool each class member's result, displaying them in a number of ways.

Will You Look At That! (WYLAT!)



14.2 and 14.3

As a class we now have numerous accurately measured cases of this interesting phenomenon. *Does this raise any further questions?*

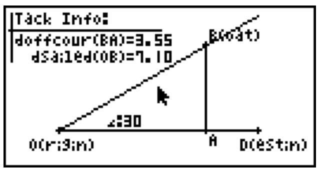
2.3 EAT 3

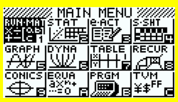
Open the geometry file called `BTACANI`. In this file you can sail the boat manually. To do this, move the cursor to point B and *grab* it (press X,θ,T). Move it to a new location.

Or, you can see the boat in 'full sail' by pressing F6 `Animate` and choose `Go(once)`.

While sailing, the 9860 has been collecting data. Make a table of measurements based on this animation that may help to answer the questions you have.

Store the data as lists in STAT. Use the data to see if you can gain support for your thoughts.





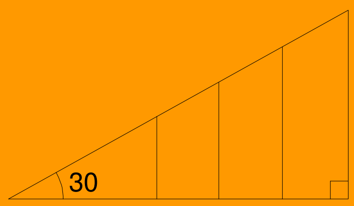
14.4, 14.5 and 14.6

By now you should be fairly confident that:

For a tack of 30^o *of any length* the ratio of the distance *Off Course* to *Dist. Sailed* is <u>constant</u> and has the value 0.5.

We can think about this in a number of ways;

- $\dfrac{Off\ Course}{Dist.Sailed} = \dfrac{1}{2}$
- As a set of 'nested' similar triangles



- $Dist.Sailed = 2 \times Off\ Course$
- *For every 2 unit of distance you sail on a tack of* 30^o*,* *you will be 1 unit of distance off course.*

Clearly, this is a useful piece of information for sailors who are on a tack of 30^o, but ....

3. *A geometric model of sailing on a tack of degrees other than 30.*

In the previous section we gained confidence in a very specific result. It is hoped that you have asked some questions as a result that lead to the investigation of whether the result can be generalised, i.e. does it hold for angles other than 30^o ? Two key questions are:

- Is $\dfrac{Off\ Course}{Dist.Sailed}$ constant for other angle values?
- If it is, what is the ratio for other angle values?

3.1 EAT 4

Investigate the ratio $\dfrac{Off\ Course}{Dist.Sailed}$ for three other angles of your choosing. Be sure to accurately measure many cases before making a conjecture.

Pool your findings with your classmates and display your collective findings in a number of ways.

Note:

If you are using `BTACANI` with a larger tack angle you may have trouble selecting point B in order to tabulate the length OB.

If so, use `SCROLL` (accessed via [F1] and ▶) to move up, then [EXIT] and select B. `SCROLL` can then be used to move back down.

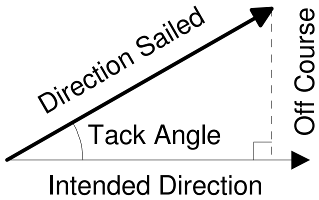
4. Formalising our findings 1 – the sine ratio.

As a result of these investigations you should be confident that the ratio $\frac{Off\ Course}{Dist.Sailed}$ is <u>constant</u> (but different) for all angles.

In other words the fraction $\frac{Off\ Course}{Dist.Sailed}$ has the same value, regardless of the size of the tack, and this value changes depending on the angle of the tack.

This relationship was discovered and proven to be true many years ago. It is an important aspect of *Trigonometry*, the mathematical study of triangles. This relationship has very wide applications.

Our tacks can be thought of as *right angled triangles*, because the intended direction and the distance off course are always *perpendicular*.



Right-Angled Trigonometry focuses on the link between one of the angles in a right angled triangle (not the 90^o angle) and the ratio of pairs of triangle side lengths.

In Trigonometry the following is commonly used language:

theta (greek letter θ) – the angle being studied/known,

hypotenuse (hyp.) – the longest side, the side opposite the right angle.

opposite side (opp.) – the side opposite the angle being studied/known θ

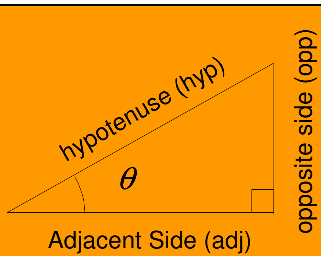
adjacent side (adj.) – the side forming the angle θ (but not the hypotenuse)

We are now confident that, *for a tack of* 30^o, $\frac{Off\ Course}{Dist.Sailed} = \frac{1}{2}$.

This ratio is called as the *sine* of 30^o (i.e. $\sin 30^o = \frac{1}{2}$).

More generally we say:

$$\sin\theta = \frac{opposite\ side}{hypotenuse}$$

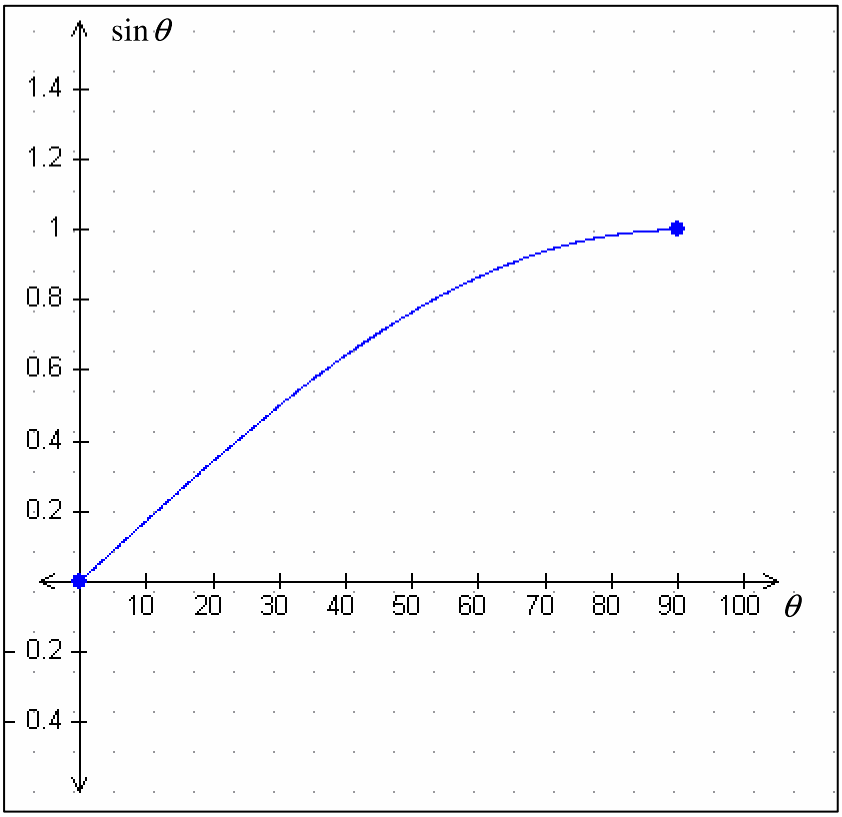


(*sine* can be written *sin* but is always pronounced 'sign')

Traditionally, sine values for any angle (to a set degree of accuracy) were published in *table form* and used by sailors, amongst others. This is an excerpt from one such a table,

θ	0	10	20	30	40	50	60	70	80	90
$\sin\theta$	0	0.174	0.342	0.5	0.643	0.766	0.866	0.940	0.985	1

While there may appear to be no particular pattern to the sine values, drawing a graph of sine of θ vs. θ reveals a clear pattern/trend.

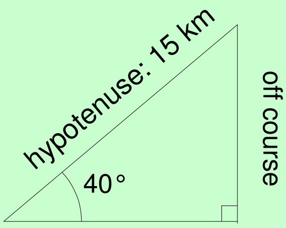


Though discussion with your classmates, decide upon an agreed description of how the value of $\sin\theta$ changes as the value of θ changes.

The sine ratio can be used to compute unknown distances. We must know the value of one angle in a right-angled triangle and the length of either the opposite side or the hypotenuse. Two examples of this are given for you to consider.

Example 1

Suppose a boat sailed on a tack of 40^o to its intended direction for 15 km. How far will it have sailed off course?



Using the relationship $\sin\theta = \dfrac{opposite\ side}{hypotenuse}$ in the triangle,

and the fact that $\sin 40^o = 0.643$ we can say:

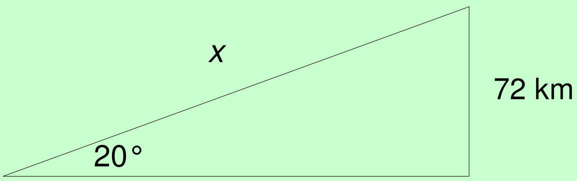
$0.643 = \dfrac{off\ course}{15\,km}$ and so $off\ course = 0.643 \times 15\,km$

Therefore the boat will be $9.645\,km$ off course.

Example 2

If a boat on a tack of 20^o is off course by 72 km how far has it sailed (to the nearest km)?

As $\theta = 20^o$, the opposite side is $72\,km$ long, and if we let x represent the unknown distance sailed, then we have:



$$\sin 20^o = \frac{72}{x}$$

$$\Rightarrow \quad 0.342 = \frac{72}{x}$$

$$\Rightarrow \quad x \times 0.342 = 72$$

$$\Rightarrow \quad x = \frac{72}{0.342}$$

Hence, the boat will have sailed 211 km.

4.1 Can you use the knowledge? 1

Use the table on page 11 to determine <u>how far off course</u> a boat is if it sails on

1. a tack of 20^o to the intended direction for a distance of 4 km?
2. a tack of 70^o to the intended direction for a distance of 1.88 km?
3. a tack of 50^o to the intended direction for a distance of 400 m?

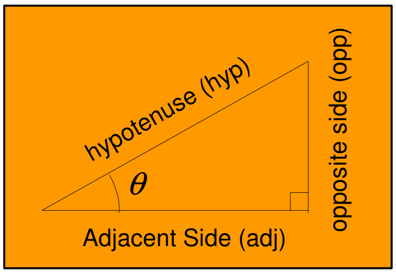
Use the table on page 11 to determine <u>how far a boat has sailed</u> if it on

4. a tack of 80^o to the intended direction and is 73.9 km off course?
5. a tack of 40^o to the intended direction and is 386 m off course?
6. a tack of 60^o to the intended direction and is 32.2 km off course?

5. Automating our new-found knowledge 1 – the sine ratio and unknown sides.

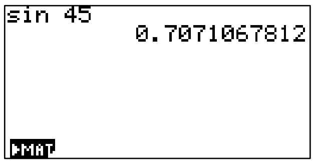
So that you are able work with the sine ratio efficiently and so that you can use it in broader uses in later work, it is important that you automate the process of using the sine ratio. To help you reach a high level of automation, consider the following input and examples and then complete the 'automation sets'.

As we have seen, $\sin\theta = \dfrac{\textit{opposite side}}{\textit{hypotenuse}}$



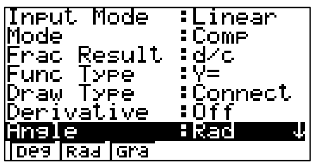
This is a 'formula' into which values can be substituted and for which unknowns can be calculated.

To compute the sine ratio for a given angle, rather than use the tables (which is a tad 'empty') we can use any scientific calculator as the 'full' table is available on them. To compute sin 45^o on the Casio 9860 launch the RUN (RUN-MAT) application and press [sin] [4] [5] [EXE].



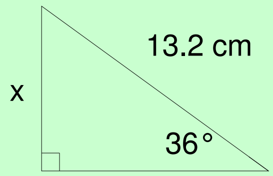
If your calculator returns a different value then your calculator is set to expect angles to be entered in Radians rather than degrees. (Radians is the default setting).

To change the `Angle` setting, press [SHIFT] and [MENU] to enter the `SET UP`. Arrow down to `Angle` and press `Deg` [F1].



Unknown sides of right-angled triangles can be found as follows:

Example 3

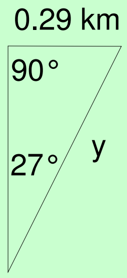


$$\sin 36^o = \frac{x}{13.2\,cm}$$

$$\therefore\ x = \sin 36^o \times 13.2\,cm$$

$$\therefore\ x = 7.76\,cm$$

Example 4



$$\sin 27^o = \frac{0.29\,km}{y}$$

$$\therefore\ y \times \sin 27^o = 0.29\,km$$

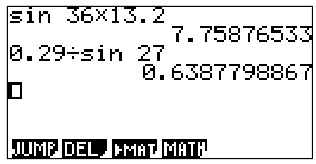
$$\therefore\ y = \frac{0.29 km}{\sin 27^o}$$

$$= 0.639\,km$$

Note: *The value of the sine ratio for the angle does not necessarily need to be written down. It can be 'called up' in the calculator in the last step when the answer is evaluated.*

In this way 15 significant figure accuracy is used (with 10 figures displayed).

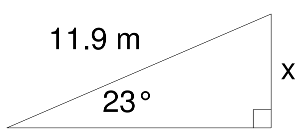
Rounding of the answers to 3 significant figures was used in the above examples.



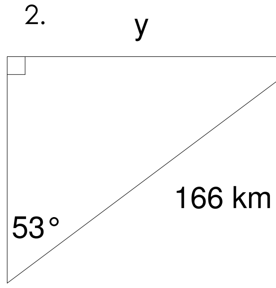
5.1 Automation Set 1

Find the unknown side lengths correct to 3 significant figures.

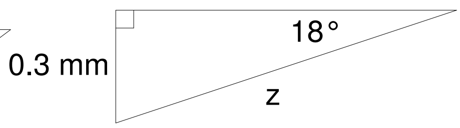
1.



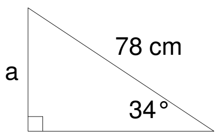
2.



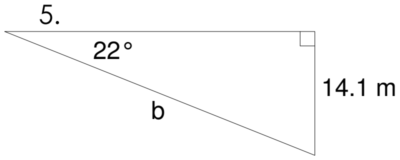
3.



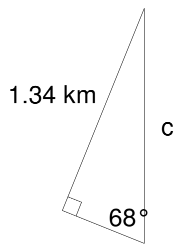
4.



5.



6.



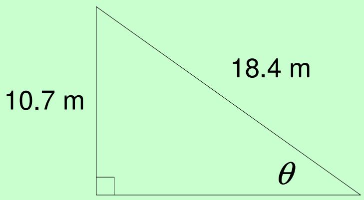
6. Automating our new-found knowledge 2 – the sine ratio and unknown angles.

The sine relationship $\sin\theta = \dfrac{opp}{hyp}$ contains three values *opp*, *hyp* and θ.
If any two of these values are known the third one can be found.

We have found the lengths of the *opp* (and *hyp*) already, when θ and the 'other' side were known. So if the *opp* side and the *hyp* were known, we should be able to find the angle θ.
But how?

Example 5

Find the size of the angled marked as θ.



$$\sin\theta = \frac{10.7}{18.4}$$

$$\Rightarrow \quad \theta = \textit{the angle with a } \sin \textit{ of } \frac{10.7}{18.4}$$

$$\Rightarrow \quad \theta = ???\text{(Read on below).}$$

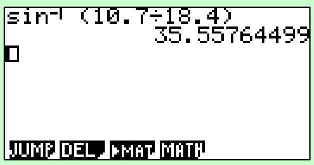
"*The angle with a sine of* $\dfrac{10.7}{18.4}$" can be found by computing the 'inverse' sine of the ratio $\dfrac{10.7}{18.4}$.

This is more formally written as $\sin^{-1}\left(\dfrac{10.7}{18.4}\right)$, which can be computed using your calculator, as seen below (continuing from above).

$$\sin\theta = \frac{10.7}{18.4}$$

$$\therefore \theta = \sin^{-1}\left(\frac{10.7}{18.4}\right)$$

(press SHIFT and sin to obtain $\sin^{-1}$)



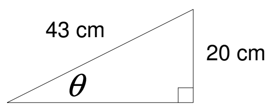
$$\therefore \theta = 35.6^{o} \text{ correct to 3 sig. fig.}$$

Note that, the inverse sine is also called the *arcsin*.

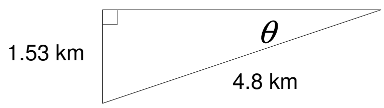
6.1 Automation Set 2

1. Find the value of θ, the unknown angle, to the nearest tenth of a degree.

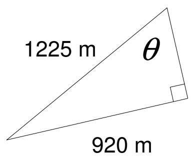
a.



b.

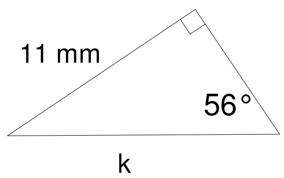


c.

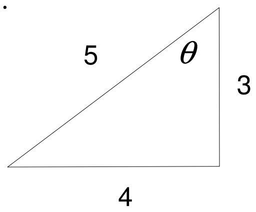


2. Find the value of the unknowns to three significant figures.

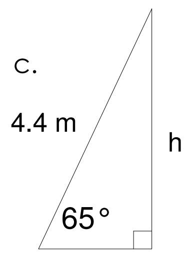
a.



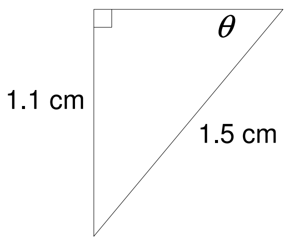
b.



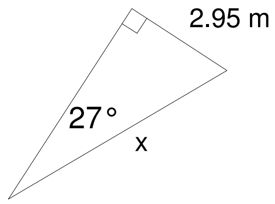
c.



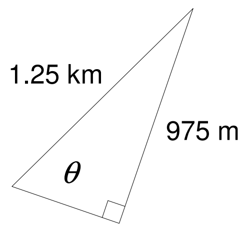
d.



e.



f.



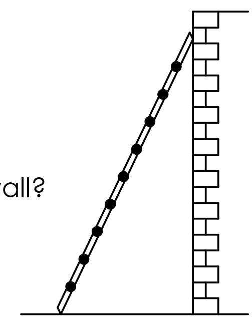
3. The following is entered in a ships log:

Tack 1: 15.5 degrees for 4.8 km – 1.28 km off course
Tack 2: 37.2 degrees for 1.7 km – 1.28 km off course
Tack 3: 20.5 degrees for 16.4 km – 6.93 km off course

The captain thinks errors have been made – is the captain correct?

4. The manual of an extension ladder says

for safe operation this ladder must not be used at an angle of greater than 70^o *to the ground*



a. Will a 3.2 m ladder safely reach a point 3 m up a wall?

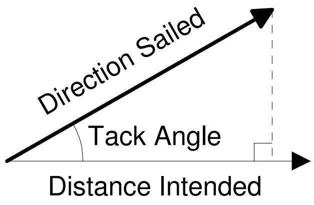
b. How long a ladder is needed to reach a point 5.5 m up a wall?

c. Is a 2.4 m ladder reaching 2.1 m up a wall being safely operated?

d. What assumption is necessary for these calculations to be made?

7. *Tacking – an efficiency rating.*

Another way to quantify tacking is to focus on the distance travelled in the intended direction (the *Distance Intended*) of the boat during a tack.



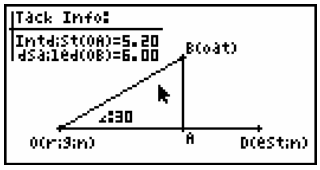
Information about this is also of use in boat navigation.

7.1 EAT 5

Launch the geometry application on your graphics calculator and open the geometry file `CTACMEFF`. For a tack of 30^o select the line (OB) representing *Dist. Sailed*. Open the measurement box (press VARS) and enter four values, in turn, of your choosing.

For each distance sailed, record the appropriate information.

Pool your findings with your classmates and display your collective findings in a number of ways.

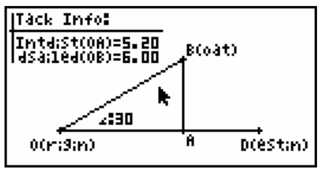


What question(s) does this EAT activity raise?

7.2 EAT 6

Use geometry file (`DTACAEFF`) and/or other methods to investigate the relationship between *Dist. Sailed* and *Distance Intended* for a variety of different tack angles (including 60^o amongst others).

Pool your findings with your classmates and display your collective findings in a number of ways.



8. Formalising our findings 2 – the cosine ratio.

As a result of the previous EATs, you should have seen that the ratio $\frac{distance\ intended}{distance\ sailed}$ is constant for a given tack angle, and for each angle this ratio takes a different value.

You may also have noticed that this ratio works like an efficiency rating,

- Starting at 1 - if you were to sail on a tack of 0^o
  (100% in your intended direction – complete efficiency)
- Ranging down to 0 if you were to sail on a tack of 90^o
  (perpendicular to your intended direction – no progress)

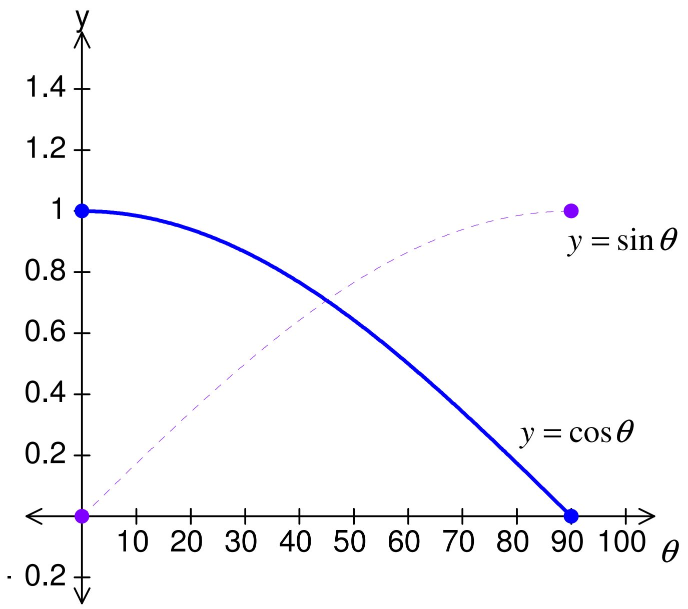
A table of sine values and our new ratio is given below.

Study this table, what do you notice?

θ	0	10	20	30	40	50	60	70	80	90
sin θ	0	0.174	0.342	0.5	0.643	0.766	0.866	0.940	0.985	1
$\frac{distance\ intended}{distance\ sailed}$	1	0.985	0.940	0.866	0.766	0.643	0.5	0.342	0.174	0

You should see that the ratio of $\frac{distance\ intended}{distance\ sailed}$ for a given angle θ is the same the sine ratio ($\frac{off\ course}{dist.sailed}$) for the complementary angle $90-\theta$. It is for this reason that the ratio of $\frac{distance\ intended}{distance\ sailed}$ is known as the *complement of sine* or *cosine* for short.

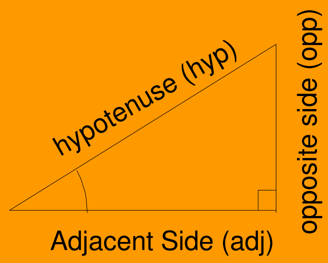
A graph of $\cos\theta$ vs θ can be seen below (solid curve).



We say that:

$$\cos\theta = \frac{adjacent\ side}{hypotenuse}$$

(cosine can be written as the abbreviation 'cos')



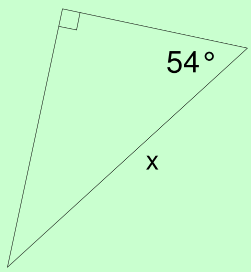
The cosine ratio can be used in a similar way as the sine ratio to find the length of unknown sides or angles in a right-angled triangle, given we know some other information.

When the cosine of an angle is required, just call up the required value using the COS key on your calculator.

Consider the following examples, the first where an unknown side is found and the second where an unknown angle is found.

Example 6

Find the length of the side defined as x.



$$\cos 54^o = \frac{11.4}{x}$$

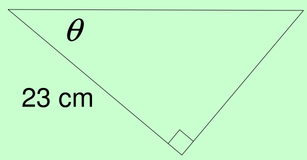
$$\Rightarrow \quad x \times \cos 54^o = 11.4$$

$$\Rightarrow \quad x = \frac{11.4}{\cos 54^o}$$

$$\Rightarrow \quad x = 19.4\,km$$

Example 7

Find the size of the angle defined as θ.



$$\cos\theta = \frac{23}{31}$$

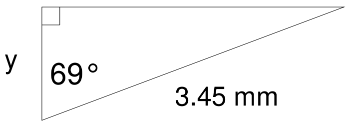
$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{23}{31}\right)$$

$$\Rightarrow \quad \theta = 42.1^o$$

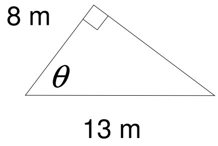
8.1 Automation Set 3

1. Find the value of the unknowns to 3 significant figures.

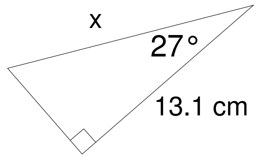
a.



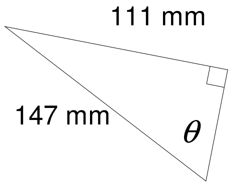
b.



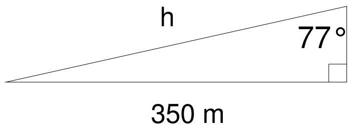
c.



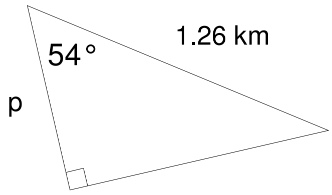
d.



e.



f.



2. Answer parts d and e *without* the use of sine.

3. The cosine ratio for 25^o is 0.906. Interpret this value in terms of a boat sailing on a tack of 25^o

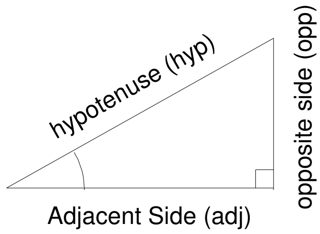
9. Extending our knowledge – the tangent ratio.

So far we have arrived at the two following ratios:

$$\cos\theta = \frac{adj\ side}{hyp} \qquad \sin\theta = \frac{opp}{hyp}$$

Look closely at the composition of the ratios. They both contain a divisor of the length of the hypotenuse. What does this suggest might be possible?

Consider the following:



$$\frac{\sin\theta}{\cos\theta} = \frac{\dfrac{opp.}{hyp}}{\dfrac{adj.}{hyp}} = \frac{opp}{adj}$$

Dividing returns a new *derived* ratio that is free of the hypotenuse length. This could be very useful for problems where we have no immediate knowledge of the hypotenuse.

This is called the *tangent ratio*, commonly shortened to *tan*.

It is defined as $\tan\theta = \dfrac{opposite\ side}{adjacent\ side}$

9.1 Can you use the knowledge? 2

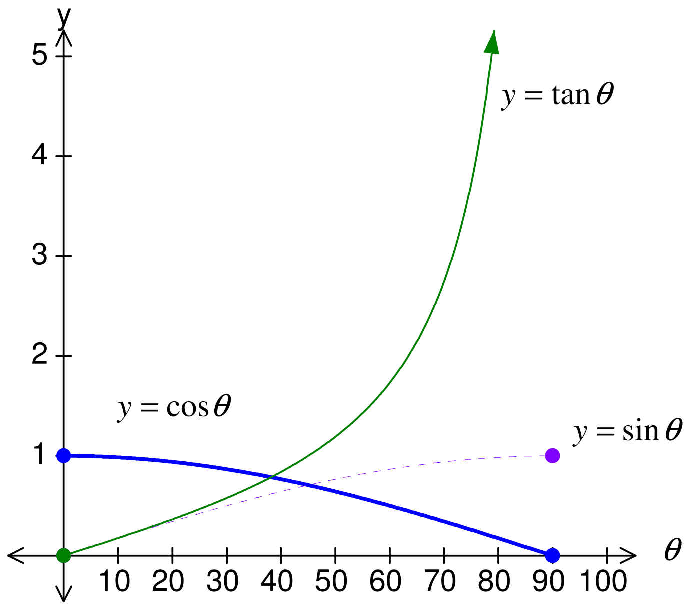
Use the above definition and what you already know about sine and cosine to answer the following questions

1. What is the value of $\tan 0$?
2. When will $\tan\theta = 1$?
3. What will happen to $\tan\theta$ as θ increases?
4. Discuss the value of $\tan 90$.
5. Complete the table below,

θ	0	10	20	30	40	50	60	70	80	90
$\sin\theta$	0	0.174	0.342	0.5	0.643	0.766	0.866	0.940	0.985	1
$\cos\theta$	1	0.985	0.940	0.866	0.766	0.643	0.5	0.342	0.174	0
$\tan\theta$										

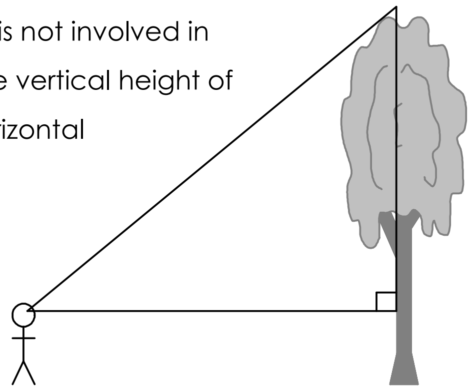
A graph of all three ratios vs θ is given below.

Note that the tangent ratio for 90^o is *undefined*.



What problems might this third ratio help to solve?

The tangent ratio can be used whenever the hypotenuse is not involved in calculations. One common application is to determine the vertical height of objects that are hard to directly measure, but where a horizontal distance and angle of elevation can be measured.



This can be something like the tree shown.

For example, if the angle of elevation is 40^o when a person with an eye height of 1.5 metres is standing 10 metres from the base of the tree then, using the following tan calculation,

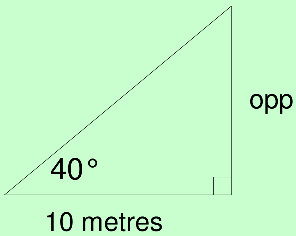
Example 8

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 40^o = \frac{\text{opp}}{10}$$

$$\Rightarrow \quad opp = 10 \times \tan 40^o$$

$$\Rightarrow \quad opp = 8.4\,metres$$

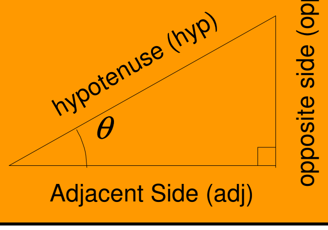


Based on this, the trees height is 9.9 metres (when the eye height of the observer is taken into account).

10. *The three major trigonometric ratios.*

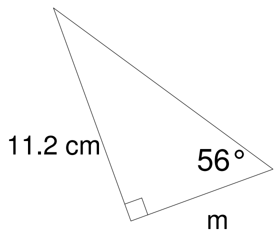
We can now summarise the three major trigonometric ratios.

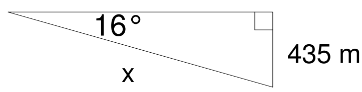
$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

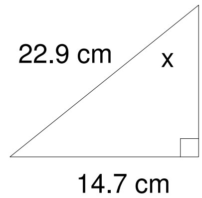


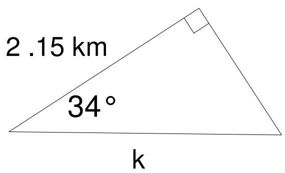
10.1 Automation Set 4

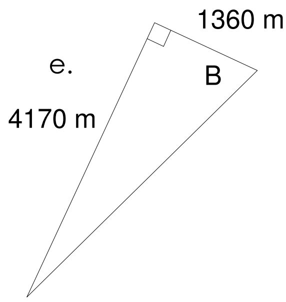
1. Find the value of the unknown lengths/angles to 3 sig. fig's.

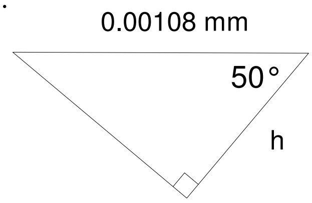
a.


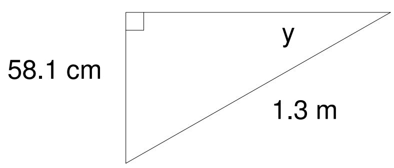
b.


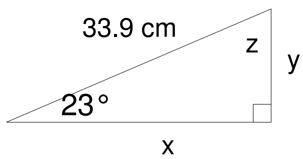
c.


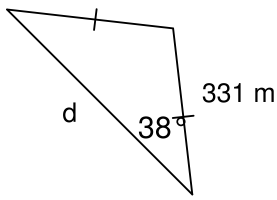
d.


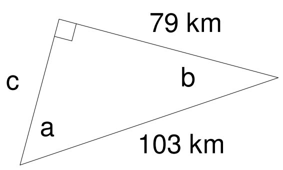
e.


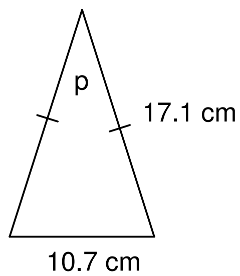
f.


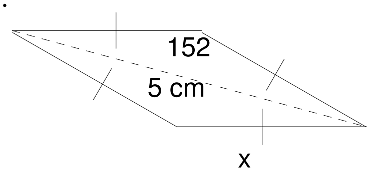
g.


h.


i.


j.


k.


l.


2. A diagonal brace joins opposite corners of a rectangular gate measuring 900 mm by 1250 mm.
   What angle does this brace make with the long side of the gate?

3. A boat sails 14 km east then 5 km north. It then sails straight back to its starting position.
   What is the distance and bearing of the return journey?

4. Find the area of the right angled triangle with a hypotenuse 50 cm that contains an angle of 42^o.

11. Application tasks

11.1 Application Task 1

Using a method similar to the one discussed on page 23, determine the height of something near you, like a flag pole, tree or building. You will need measuring equipment.

Compare your answer with others in the class who have 'measured' this same thing. Discuss any discrepancies and assumptions in your work.

In the last activity you may have experienced discrepancies between different 'measurements' of the same thing and might have wondered about their origin. This sort of calculation relies on two assumptions

- The right angled triangle i.e. the measured object is actually vertical and the ground is level and horizontal.
- The measurements were accurate.

It is this second point that causes the discrepancies you saw.

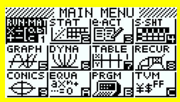
So how big is the effect of measurement error?

11.2 Application Task 2

As a class decide on an estimate for the greatest error in your measurement of the horizontal distance. Hence calculate a *range* of height values - based on the range of possible horizontal distances. Repeat this process for the angle measurement.

Which measurement seems to cause greater discrepancies?

See *eTech Support* for a possible approach to this activity.



14.7

Why do we care so much about the height of trees?

Forestry plantations are a valuable and carefully managed resource. To maximise the profits made the time of harvest is crucial. To make this decision calculations about the volume of wood in the plantation are needed. These calculations rest upon accurate values for the width and height of the trees in the plantation.

12. Contextual questions.

By now we hope you have automated the process of applying the three trigonometric ratios to simple triangles. It is also important to be able to solve questions posed about contexts. The following set of questions will provide you with the opportunity to develop this skill.

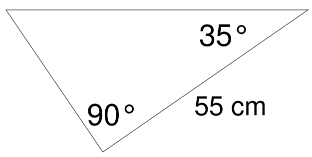
We suggest that you:

- Familiarise yourself with a clearly drawn diagram.
- Look for useful geometric features, including right angles.
- Identify the unknown(s) that you wish to find values for.
- Identify the known information, marking it on the diagram if possible.
- Work systematically from what you know to what you want to find out.

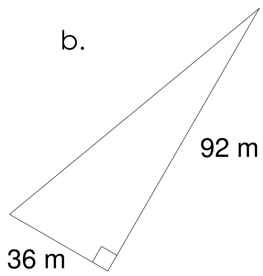
12.1 Contextual Set 1

1. Find the value of *all* unknown sides and angles in the following triangles to the nearest whole unit.

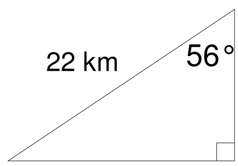
a.



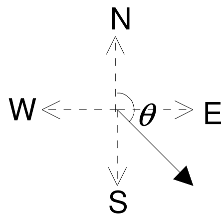
b.



c.



Can you get your bearings?



Bearings (or *true bearings*) are a way of measuring directions as **an angle clockwise from north**.

A bearing is quoted as … *degrees true*. This diagram shows a direction of $135^o T$. This idea fits well with N – E – S – W and trigonometry.

2. <u>Use a diagram to help you answer these questions.</u>

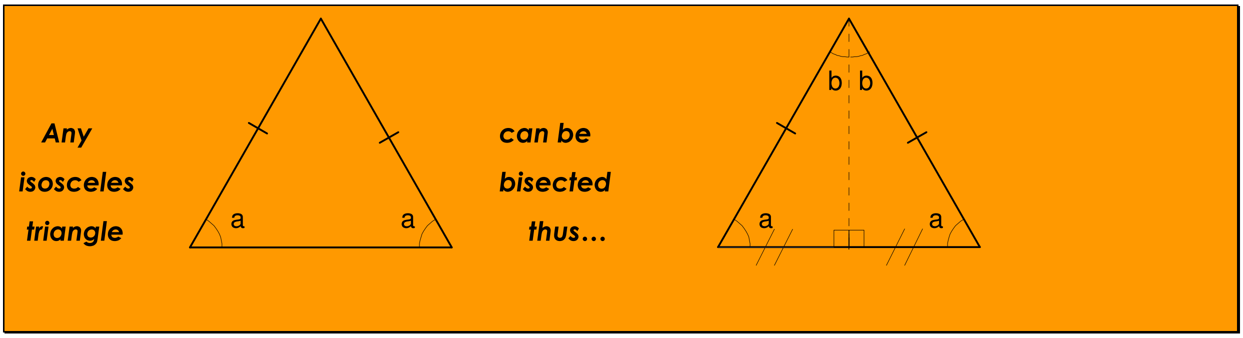
a. A plane flies on a bearing of $37^o T$ for 1750 m. How far north has it flown?

b. A hiker walks 1.2 km south then 1.9 km west. What is the bearing of their new position from their starting point?

c. Wishing to sail west, a boat sails on a tack of $306^o T$ for 2.7 km before sailing on a course of $228^o T$ for 2.4 km. How far west has the boat sailed and how far off course is it?

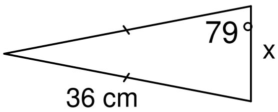
d. An expedition travels on a bearing of $110^o T$ for 45 km before heading on a bearing of $20^o T$ for 72 km. Find the distance and bearing of the return leg of their trip.

Does this look familiar… ?

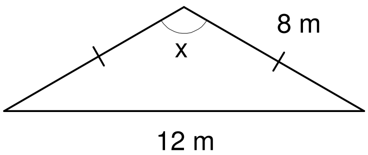


3. Find the value of x in these isosceles triangles to 3 sig. fig's.

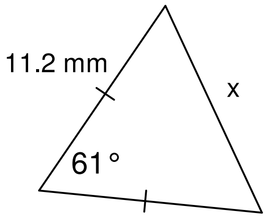
a.



b.



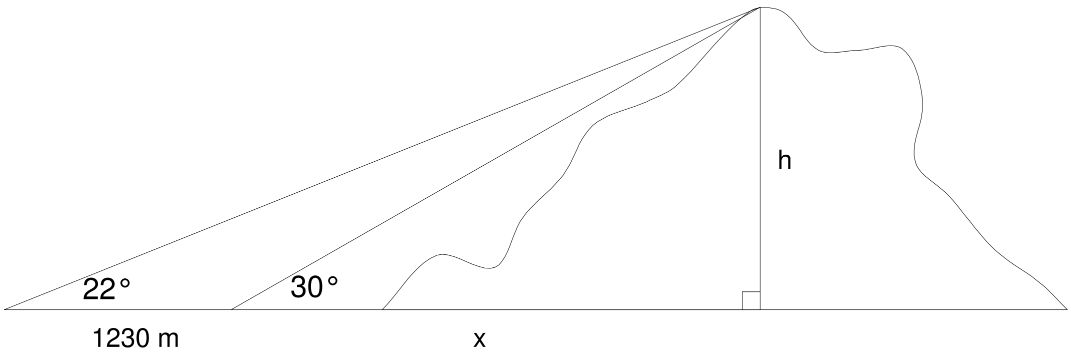
c.



4. An isosceles triangle has side lengths of 2, 2, and 3 cm. Find its smallest angle.

5. Find the area of an isosceles triangle with a repeated side length of 14 cm and an apex angle of 100^o.

6. *How high is the mountain…measure one <u>without</u> climbing it?*



To find the height of this mountain an angle of elevation of 22^o is taken of the mountains peak. Then, from 1230 metres directly closer, and at the same altitude, another angle of elevation is taken, this time of 30^o.

a. Use the first elevation angle to write down a trigonometric ratio.

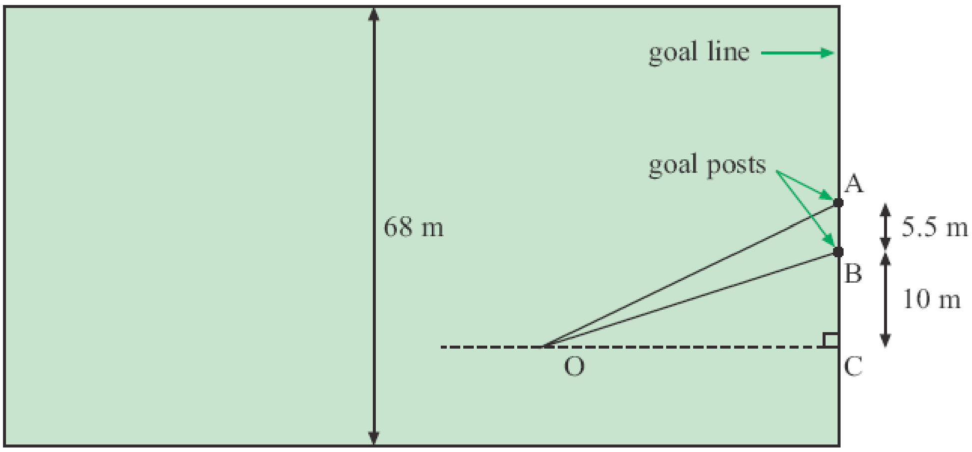
b. Do similarly using the second elevation angle.

c. Re-arrange these equations into the form $h = \ldots$

d. Find the height of the mountain by finding the simultaneous solution of these two equations.

e. Why might this height differ from the value quoted as the 'actual' height of this mountain.

7. A try is scored in a game of Rugby (at point C below). A kick at goal is taken from *somewhere* along the line from C that perpendicular to the goal line. The goal kicker *chooses* the point (O) that he kicks from. A diagram of a rugby pitch is shown below,



Imagine that a try is scored 10 metres from the right hand goal post.

The margin for error in kicking can be thought of as the angle of view AOB.

So, how does the choice of point O affect this margin for error?

a. If possible, go to a nearby sports field and make your own choice for point O.

b. Alternatively, open the geometry file RUGBYKCK and use this geometic simulation to get a 'feel' for the optimal position for O.

c. Find the margin for error AOB in the case where the kick is taken from point O when it is 5 metres from the goal line.

d. Repeat part c when the distance of O from the goal line is

i. 10 metres ii. 15 metres iii. 20 metres.

e. Describe what seems to happen to the margin for error as the distance of O from the goal line increases.

f. Determine from where the goal kicker should kick the ball.

What factors will influence this discussion? Discuss with your class.

13. *Approximate or exact?*

Did you notice that …

$\sin 30^o = \frac{1}{2}$ (nice and neat) but $\cos 30^o = 0.866...$ (a messy decimal)

…and vice versa for $\sin 60^o$?

13.1 EAT 7

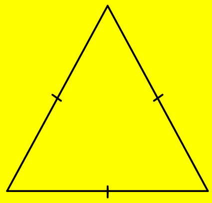
Is there a neater value for $\cos 30^o$?

Consider an *equilateral triangle* with side lengths of 1 unit.

Bisect it to create a right angled triangle (or two).

Use this construction to help you find exact values for

$\cos 30^o$ (and therefore also for $\sin 60^o$).



13.2 EAT 8

What about $\sin 45^o$?

Consider bisecting a *square* with sides of 1 unit to find an exact value for $\sin 45^o$ (and therefore also for $\cos 45^o$).

Use these constructions (plus a little extra thought) to copy and complete this table of *exact* trigonometric values.

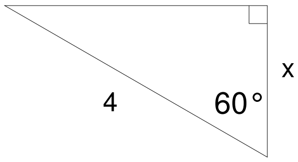
θ	0^o	30^o	45^o	60^o	90^o
$\sin\theta$					
$\cos\theta$					
$\tan\theta$					

It is possible to obtain exact trigonometric values for other angles (like 15^o). The table above includes all the widely used exact trigonometric values. They are worth remembering if possible as they can come in quite useful in future work.

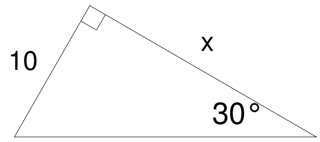
13.3 *Automation Set 5*

Use these exact trigonometric values to find the exact value of x in the following shapes.

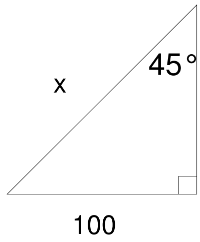
a.



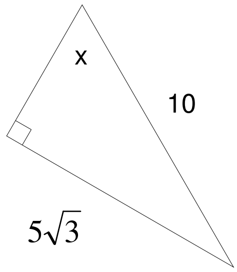
b.



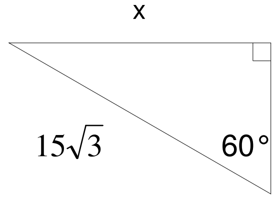
c.



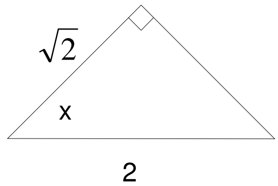
d.



e.



f.



14. eTech Support.

14.1 Generating random numbers.

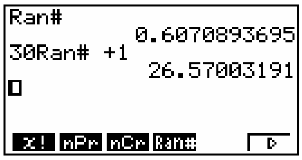
Random numbers between 0 and 0.999… are generated by the command `Ran#`.

This can be accessed by pressing

OPTN then ▷ F6, PROB F3, Ran# F4.

Multiplying this command by a constant (like 30) ‘stretches’ the range of these random numbers from 0 to 29.999… .

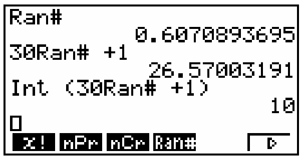
By adding 1, values from 1 to 30.999… are obtained.



To return just the *integer* part of this, the command needs to be prefaced by the command `Int`.

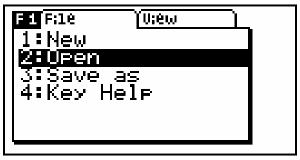
This can be found by pressing

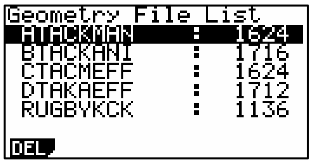
OPTN then ▷ F6, NUM F4, Int F2.



14.2 Opening a geometry file.

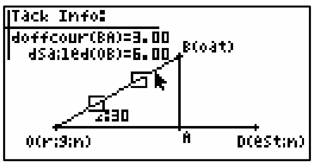
Enter GEOM in your CASIO *fx-9860G AU*. Press F1 and, using the arrow pad and EXE choose `2:Open` and select `ATACMAN`.

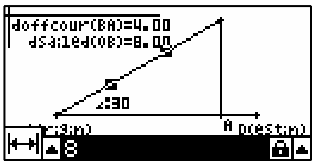




14.3 Selecting and altering the size of geometric features.

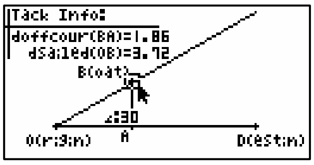
Move the arrow, using arrow pad and, when the line segment is highlighted, press EXE to select it. Press VARS to open the measurement box. Enter a new length for the line segment and press EXE.

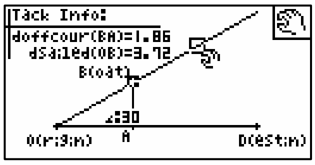




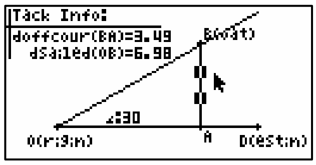
14.4 Grabbing and dragging geometric features.

Move the arrow, using arrow pad and, when the point B is highlighted, press `Grab` [X,θ,T]. Move the Grab icon to a new position for the point B (using the arrow pad) and press [EXE].





14.5 Running a geometry animation and tabulating.

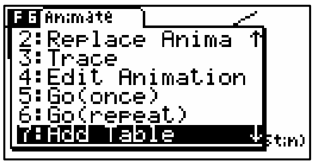


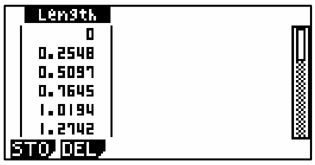
Press [F6] (`Animate`) and select `5:Go(once)`. Watch and enjoy!

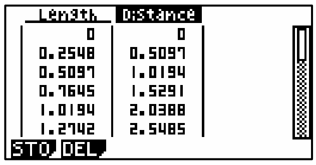
The values used in an animation (i.e. the distances at each step along the way) can be tabulated in the following way:

Move the cursor to the line segment representing *Off Course* and select it by pressing [EXE].

Now press [F6] (`Animate`) and go down and select `7:Add Table`







Press [EXIT] to return to the geometry window.

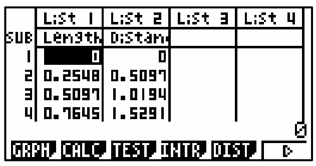
Deselect this line segment by pressing [AC/ON].

Now repeat the process for the distance OB (*Dist. Travelled*). You will need to select the points O and B. Your table should now look like this,

14.6 Storing and manipulating tables as lists.

To store this table in [STAT] choose the column representing distance *Off Course* with the arrow pad and press `STO` [F1] then `LIST` [F1] [1].

Repeat this, storing the column representing *Dist. Sailed* in `List 2`.

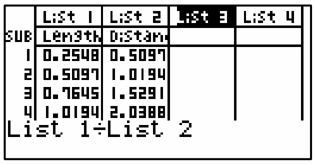


Moving now to [STAT] we should see, (left).

If your conclusion was that the length *Off Course* is half of the length *Dist. Travelled* then this can be checked by dividing List 1 by List 2.

Before this is can be done the zeros at the top of the Lists need to be deleted with the [DEL] key.

Perform the calculation in List 3 by moving the cursor into the heading row of List 3 and entering the calculation `List 1` [÷] `List 2`. (To obtain `List` press [SHIFT] then [1]).



14.7 Entering an error function.

(Based on a horizontal distance of 10 metres and a greatest error estimate of ± 0.2 metres)

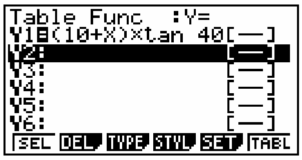
The height calculation of $adj \times \tan\theta$ could be evaluated for

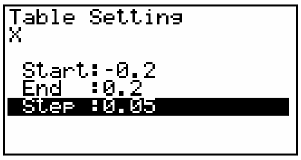
$adj = 9.8, 9.85, 9.9, ..., 10.15, 10.2.$ (using a fixed angle of elevation of 40^o)

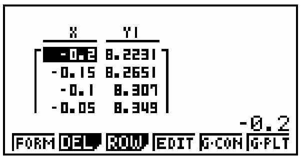
in [TABLE] mode by entering the height as a function (use [X,θ,T] for the variable `X`).

Press `SET` [F5] and give `X` the range of – 0.2 to 0.2 in steps of 0.05.

Press [EXE] then `TABL` [F6].

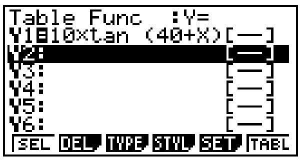


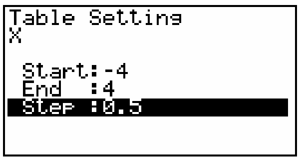


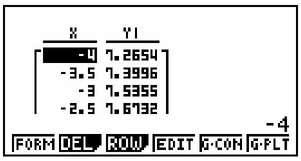


Exploration of this table shows variation in calculated height of less than 40 cm due to this degree of error in horizontal distance.

A similar approach to possible error involving the angle of elevation would yield something like







This table shows that angle errors of 4 degrees or less can cause the calculated height to vary by nearly two and a half metres!

15. Answers.

Can you ….1.

1. 1.368 km
2. 1.7672 km
3. 306.4 m
4. 75.025 km
5. 600.3 m
6. 37.18 km

Auto. Set 1

1. 4.65 m
2. 133 km
3. 0.971 mm
4. 43.6 cm
5. 37.6 m
6. 1.45 km

Auto. Set 2

1.

a. 27.7^o

b. 18.6^o

c. 48.7^o

2.

a. 13.2 mm

b. 53.1^o

c. 3.99 m

d. 47.2^o

e. 6.50 m

f. 51.3^o

3.

Tack 1 right

2 & 3 wrong

4.

a. yes (just)

b. 5.85 m

c. yes

d. Some are;
ground flat,
wall straight,
ground and wall are perpendicular.

Auto. Set 3

1.

a. 1.23 mm

b. 52.0^o

c. 14.7 cm

d. 49.0^o

e. 359 m

f. 0.741 km

3.

For every 1 unit of distance travelled, 0.906 units of progress is made toward your destination.

Can you ….2.

1. 0
2. when $\theta = 45^o$
3. It increases, at a greater and greater rate.
4. As $\theta \to 90$, $\tan\theta$ gets very large. At 90^o its exact value is undefined.

Auto. Set 4

1.

a. 7.55 m

b. 1580 m

c. 39.9^o

d. 2.59 km

e. 71.9^o

f. 0.000694

g. 26.5^o

h. x=31.2 cm
y=13.3 cm
z=67^o

i. 522 m

j. a=50.1^o
b=39.9^o
c=66.1 km

k. 36.5^o

l. 2.58 cm

2. 35.8^o

3. 14.9 km, 250.3^o T

4. 623.1 cm^2

Context. Set 1.

1.

a. 55^o, 39 cm 67 cm

b. 21^o, 69^o, 99 m

c. 34^o, 18 & 12 km

2.

a. 1398 m

b. 238^o T

c.

west 3.96 km

off course 0.02 km

d. 85 km, 232^o T

3.

a. 13.74 cm

b. 97.2^o

c. 11.36 mm

4. 41.4^o

5. 96.3 cm^2

6.

a. $\tan 22^o = \dfrac{h}{x+1230}$

b. $\tan 30^o = \dfrac{h}{x}$

c. $h = (x+1230)\tan 22^o$
$h = x\tan 30^o$

d. 1655 m

7.

c. 8.69^o

d.

i. 12.17^o

ii. 12.25^o

iii. 11.21^o

e. It seems to increase then decrease.

Auto. Set 5

a. 2

b. $10\sqrt{3}$

c. $100\sqrt{2}$

d. 60^o

e. 22.5

f. 45^o