

10th planet – fact or fiction?

- An application involving the verification of an algebraic model -

Introduction

The planets of our solar system are more than just well-known aspects of astronomy. They are a part of popular culture. However, memory aids such as

My Very Easy Mnemonic Just Summed Up Nine Planets

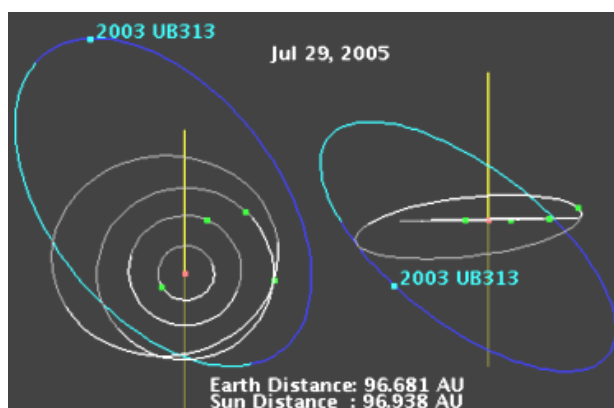
may need to be revised, with the recent discovery of what has been claimed to be the 10th planet of our solar system.¹



2003UB₃₁₃ (artists impression) with the Sun in the background

This new discovery is currently called 2003UB₃₁₃ and has been nick-named *Xena* while it waits for its official name. At present it is roughly 15 billion kilometres from the Sun, 100 times more distant than the Earth. It has a very elliptical 560 year orbit, which is inclined at nearly 45° to the orbit of the other planets, as shown here.

This unusual orbit suggests the possibility that this new 'planet' may not conform to the laws of planetary motion, as known to humankind since the 16th century.



Kepler's Laws of Planetary Motion

The three fundamental laws that describe planetary motion were determined by Johannes Kepler, born in Germany in 1571. His Third Law, when simplifiedⁱⁱ says that, if R is the average radius of a planet's orbit, measured in Astronomical Units (AU) and P is the period (length of time) of a planet's orbit in earth years, then

$$R^3 = P^2$$

So, how well does 21st century information about the nine planets fit this 400 year old law, given that three of these planets were unknown at the time it was formulated? How well does Kepler's law describe 2003UB₃₁₃ with its unusual orbit?

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Activity 1: Viewing the data - theory vs practice

The table below contains data relating to the orbit of the nine established planets of the solar system

Planet	Orbital Period P(earth years)	Orbital Radius – average R (AU)
Mercury	0.241	0.387
Venus	0.615	0.723
Earth	1	1
Mars	1.88	1.524
Jupiter	11.86	5.203
Saturn	29.46	9.539
Uranus	84	19.18
Neptune	164.8	30.06
Pluto	247.7	39.53

Before we view the data in scatter plot form we need to decide which variable we will represent horizontally and which we will represent vertically. As there is no obvious independent or dependant variable, the choice is somewhat arbitrary.

Whilst either representation will suffice, we might choose to allocate the more likely 'input variable' to the horizontal axis. The period is more easily measured than the radius, as it can be calculated based on movement through the sky. This makes it more likely to be 'input', so we will allocate it to the horizontal axis.



A. Draw a scatter plot of Period verses Radius, using a *CASIO 9860G AU*.

To see how well Kepler's third law fits this data we would like to define and graph an algebraic model upon the scatter plot.

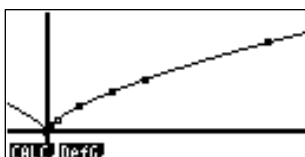
To be graphed on the *CASIO 9860G AU*, Kepler's third law needs to be written in the form $P = \dots$. By taking the cube root of both sides of $R^3 = P^2$ we obtain

$$R = \sqrt[3]{P^2} \text{ which simplifies to } R = P^{\frac{2}{3}}.$$



B. Graph this theoretical model on your scatter plot

- Press the *Define Graph* key DefG **[F2]**.
- In the Graph window that appears enter $R = P^{\frac{2}{3}}$. Press DRAW **[F6]**



Checkpoint



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Activity 2: Looking closer – how well does the theory fit the practice?

By inspecting the graph on the previous page we can see that Kepler's law appears to fit the data on planetary orbits very well, but what if we look more closely?



- Press **SHIFT** then TRACE **F1**
- Press **◀** and **▶** to move through StatGraph1 – your scatter plot of the data.
- Press **▲** and **▼** to move between the data values and those values predicted by Kepler's law.

1. For which planet's orbit does Kepler's third law fit *least* well?
2. How can this discrepancy between the observed data and the model's prediction be best represented?

Activity 3: What about 2003UB₃₁₃ ?

Given that 2003UB₃₁₃ has an orbital period of 560 years and an average orbit radius of 67.5 AU

1. Add 2003UB₃₁₃ to the data on the other 9 planets.
2. Redraw the scatter plot to include 2003UB₃₁₃.
3. Draw Kepler's third law on your new scatter plot.
4. Look at how well Kepler's law describes the data on the orbit of 2003UB₃₁₃.
5. Does Kepler's law work as well for 2003UB₃₁₃ as it does for the other nine planets? Explain your answer in detail.

Checkpoint



ⁱWhat constitutes a planet is a surprisingly complex question. Whilst the International Astronomical Union is yet to make a ruling, the finders of 2003UB₃₁₃ consider that *anything larger than Pluto* should be considered a planet. On that basis 2003UB₃₁₃, which is approximately 5% bigger than Pluto, is the Sun's 10th planet.

ⁱⁱ Kepler's third law states that the ratio of the *square* of a planet's orbital period P to the *cube* of its orbital radius R is constant so, for two planets *a* and *b*, $\frac{R_a^3}{P_a^2} = \frac{R_b^3}{P_b^2}$. If we take earth as one of the two planets, with an orbit of 1 earth year and an orbital radius of 1 AU (where AU – the Astronomical Unit - is defined as the orbital radius of the Earth) then the law simplifies to $\frac{R_a^3}{P_a^2} = \frac{1}{1}$ implying $R_a^3 = P_a^2$.