Describing an additive pattern (linear)

The balance of an investment account grows over time. There is usually a pattern of some kind to such growth. It is important to understand the connection between simple number patterns, their algebraic description and the way an investment account may grow.

A simple type of number pattern is formed by *adding* a constant amount to the previous value in the pattern. We are only going to consider patterns that start with zero. For example:

$$0, \quad 0+5, \quad 0+5+5, \quad 0+5+5+5, \quad 0+5+5+5+5, \quad \ldots$$

or

$$0, \quad 5, \quad 10, \quad 15, \quad 20, \quad \ldots$$

This is an example of a *linear pattern*.

To help describe this pattern we assign a number to each term in the pattern so we know that number's *position* in the pattern. For ease of description we have chosen to start with position zero.

This is best displayed in a table. Let the position number be p and the value of the number in the pattern be v.

position (p)	0	1	2	3	4
value (v)	0	5	10	15	20

Note that as well as the *adding by 5* pattern that can be seen in a horizontal manner, a vertical pattern also exists. The value (v) is simply *5 times* the position number (p). This allows us to very simply describe this pattern with the following rule:

$$v = 5p, \text{ where } p \text{ is an integer}$$

Note that the multiplier, 5 in this case, is the same as the increase in V each time.

Is this a coincidence?

Interaction C

1. Consider the following number pattern

position (p)	0	1	2	3	4
value (v)	0	3	6	9	12

   a) how much is added to each of the previous terms in the pattern?
   b) How many times the position number is the value of each term?
   c) Write down a rule that describes this pattern. Check that your rule works.

2. Consider the following number pattern

position (p)	0	1	2	3	4
value (v)	0	120	240	360	480

   a) how much is added to each of the previous terms in the pattern?
   b) How many times the position number is the value of each term?
   c) Write down a rule that describes this pattern. Check that your rule works.

3. Consider the following number pattern

position (p)	0	1	2	3	4
value (v)	0	-10	-20	-30	-40

   a) how much is added to each of the previous terms in the pattern?
   b) How many times the position number is the value of each term?
   c) Write down a rule that describes this pattern. Check that your rule works.

4. Consider the following number pattern

position (p)	0	1	2	3	4
value (v)	0	a	2a	3a	4a

   a) how much is added to each of the previous terms in the pattern?
   b) How many times the position number is the value of each term?
   c) Write down a rule that describes this pattern.

5. Recall we noted in our first example that the number of times the value is of the position number, is the same as the amount that was added to each previous term in the pattern. Did this happen in each of the questions in this interaction? Explain why this is no coincidence and will happen in any *additive* pattern in which the first term has a value of zero.

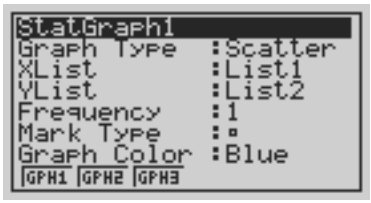
A graphical display of a pattern can be produced on your calculator and following this we can check that the rule we have generated is correct.

Enter STAT mode. Use SET UP (SHIFT then MENU) to ensure that Stat Wind is set to Auto. This way the calculator will automatically choose appropriate scales for the axes of the graph we will draw.

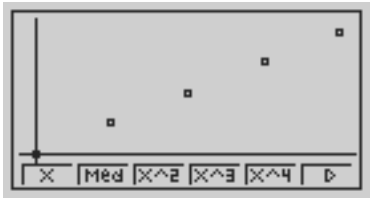
Press EXIT.

Lists are available for the entering of data. If data already exits in the lists and you wish to delete them, press F6 (the continuation key) and then with the cursor in the appropriate column use DEL·A (F4) to delete all of the data in that list. If you do not want to delete the data, simply move the cursor to an empty list.

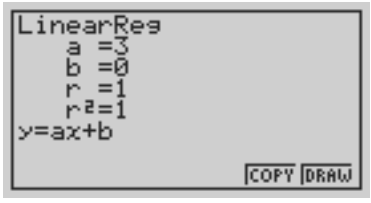
Enter the position and value data from question 1 of Interaction C using the number keys and pressing EXE after each number is entered.



Press F6 (if necessary) and then use GRPH (F1) and then SET (F6) to set up StatGraph1. Ensure that each setting is as shown opposite. We want to produce a scatterplot of v by p (or List 2 by List 1).



Press EXIT and then use GPH1 (F1) to draw the scatterplot.



Note that the data points fall in what appears to be a straight line. We should have been able to predict this from the structure of the table of values. To check that the rule we produced for this data is correct press X (F1). This will fit a *straight line of best fit* to the data of form $y = ax + b$.

If the data can be modeled *exactly* by a linear rule, then the r^2 value reported by the calculator will be equal to 1, as it is in this case. If it is not 1, then the points do not fall in a perfectly straight line. You will learn more about the r^2 value next year.

The a value is the slope or gradient of the line of best fit – three in this case. This is equivalent to value that is added to form the next number in the pattern. The b value is the vertical intercept and is zero in this case. So the answer we should have arrived at in question 1 of Interaction C is $v = 3p$.

Interaction D

1. Use the technique illustrated above to check you answers to question 2 and 3 in Interaction C.
2. Use the calculator to verify that a linear rule for the following pattern is $v = 7p+5$.

position (p)	0	1	2	3	4
value (v)	5	12	19	26	33

3. Explain why the rule in question 2 does not have zero as the value of b.