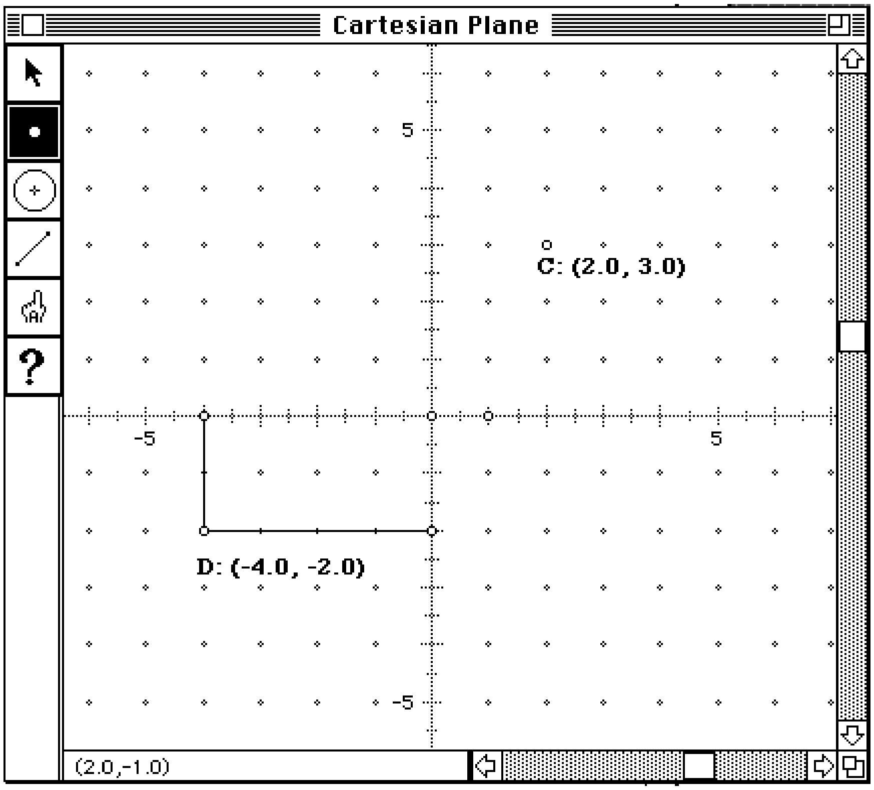
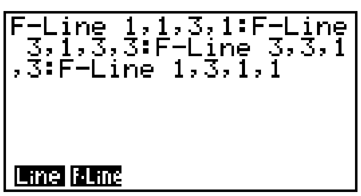
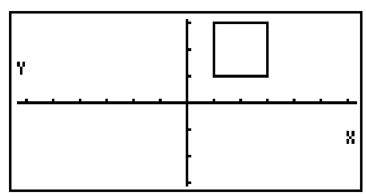
Interacting with those families

A unique learning experience – Student Learning Booklet







A product of the Noel Baker Centre for School Mathematics

WIP (Work in progress)

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Acknowledgements.

The ‘approach’ within is that developed by Anthony Harradine. The process of development has occurred over a number of years and has been tested on students at Prince Alfred College. Thanks to all those students.

Pre-requisite knowledge.

It is expected that students will have studied the ‘families’ unit and the ‘patterning’ unit (both written by Anthony Harradine, Baker Centre) prior to studying this unit. The trio of units form a pathway to the destination to a strong understanding and competence in algebra.

Daily Quiz.

It is suggested that a daily quiz take place that incorporates the concepts and techniques learned throughout the three units discussed above.

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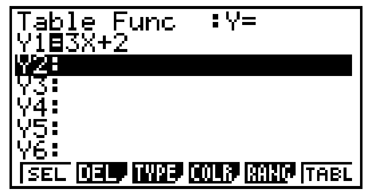
 (WIP, Draft - Friday, 27 August 2004)

1. Non-integer generations?

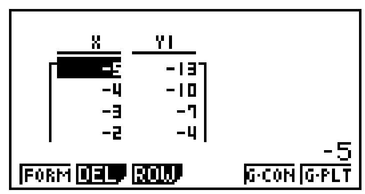
In the last unit you studied on families we learned to understand and display number families. For example: $y = 3x + 2$. You may or may not have noted that we only looked at values of *x* (generations) that had integer values. Unlike human families, number families can have non-integer generations.

So, if *x*=1.5, *y*=6.5, if *x*=2.2, *y*=8.6 and so on. Are you wondering how this changes the *graphical representation (Kodak snap)* of the family?

Let's use some electronic technology to speed up the process. Using Table mode on your graphic calculator, enter the family $y = 3x + 2$.

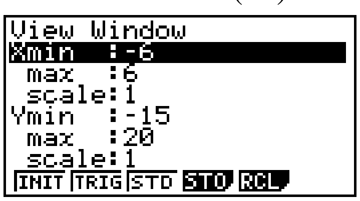


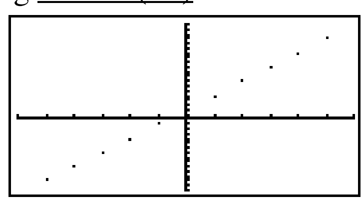
Make a table for integer generations only by setting the Range (F5) to have a whole number pitch.



Have a good look at your table so that you can set the View Window correctly.

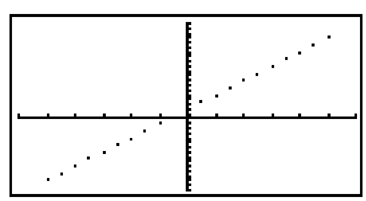
Using the V-Window command (Shift then F3) to set the axis scales of your graph. Press Exit and then make a table (F6) and then a graph using **<u>G-Plot (F6)</u>**.



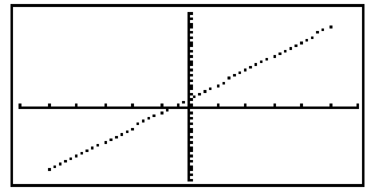


Now to include some non-integer values of *x* (generations) press EXIT and use RANG (F5) to change the pitch to be 0.5. This way we will have 'half generations' being shown.

Produce a graph as before. Mine looks like that seen opposite.



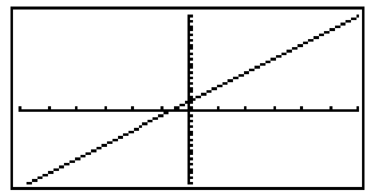
Now go back and change the pitch to show the ‘0.2 generations’



As you can see if appears as though a straight line is beginning to appear. Do you trust the technology?

<u>How could you argue it is in fact a straight line – if it is? Give it a go.</u>

Using the G-CON option to produce a graph essentially plots values of the family for many, many values of *x*.



When you use GRAPH mode on your graphic calculator it works the same as the G-CON option in table mode.

2. Sometimes when you are looking for ‘one thing’ it is easier to consider the infinitely many cases first!

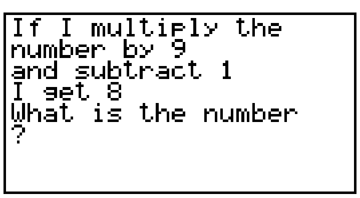
2.1 TINK2

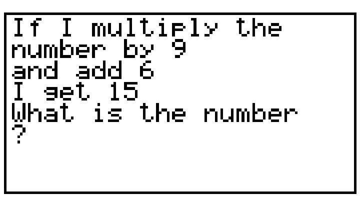
Use your graphic calculator to run TINK2.
Share your experiences with the rest of your classmates.

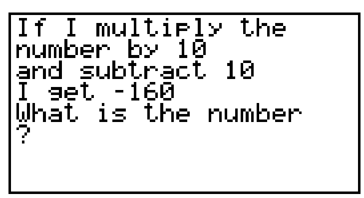
Some room for some important notes.

Recall using TINK1 on your graphic calculator in Section 4.3 of the ‘Families’ unit.
TINK3 is a slightly modified version – run it now.

Here are three examples from TINK3:







Share your approach to solving the little mysteries with your classmates.

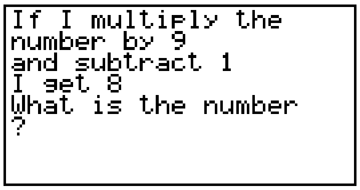
 (WIP, Draft - Friday, 27 August 2004)

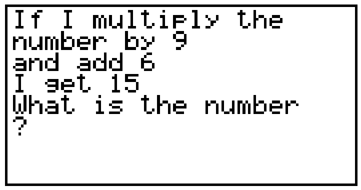
2.2 TINK3 **Solving Equations**

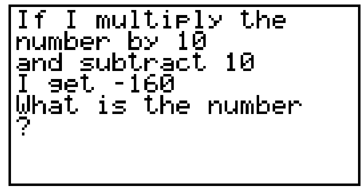
Sometimes when you are looking for one thing, it is easier to consider the infinitely many cases first! How can this be?

Play TINK3 on the Casio 9850 GB Plus. Have ten shots and record your score.

Each screen below (from TINK3) displays information about a family.







The first family is 9x-1. TINK3 asks you to find the family member called 8.

We could write down this quest in the following (mathematical) way:

Find x if 9x-1 = 8.

9x-1 = 8 is called an algebraic equation

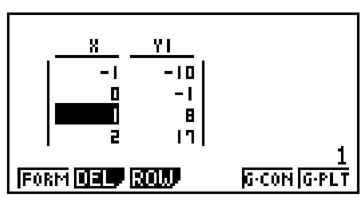
The process of finding the value of x is called ***solving the equation***.

The correct value of x makes the left hand side (LHS) of the equation equal the right hand side (RHS).

There are a number of methods that can be employed when ***solving*** equations. Some methods work no matter what type of family we are working with; others do not.

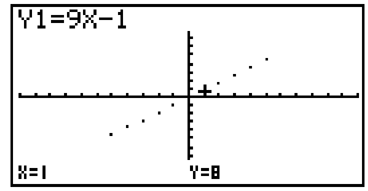
2.2.1 Method One – A hunting we will go.

We could use our calculator as a hunter. If asked to solve 9x-1 = 8, we could define a family in the **Table** mode and then make a table and search for the member we want. You can see below in this case, x = 1.

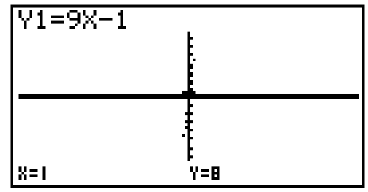


1 is called the ***solution*** to this equation. It is the same as *generation*!

We could of course see the *Kodak happy snap* of this as well. First ensure the *view window* of you calculator is set up correctly (for the table you have) and then make the table again and press G-PLT (F6). You can trace the points using the Trace (F1) key.



It is unlikely that this graph will be *square* – that is it is unlikely that a movement of 1 unit in the *x* direction is the same actual distance on the screen as one unit in the *y* direction. You can make this so by using ZOOM (F2), DES (F6) and then SQR (F2) to get the image opposite. You can see that sometimes a square set up is hard to view – just like it would be if you drew it by hand.



Exercise One

Use the hunting method to solve the following equations:

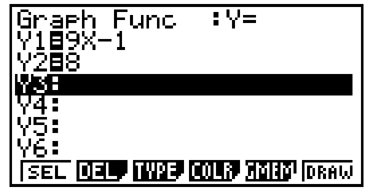
1. $4x$-1 = 15
2. $3x$-1 = 10
3. $5x$+3 = 18
4. $2x$+6 = -30
5. -$2x$-1 = 9
6. x+16 = 30
7. $2x$-8 = -15
8. $4x$+3 = 8
9. $3x$-1 = 10
10. $5x$-1 = 10

Exercise Two
Were you able to predict or know any of the solutions to these equations before hunting for them on the calculator? If so explain how you did so.

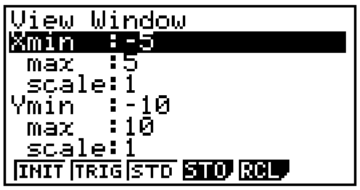
Exercise Three
What do you think the ***value*** of the solutions to Exercise One part 9 is in ***fractional form***– can you be sure of your answer?

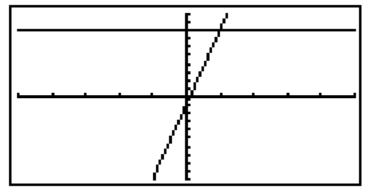
2.2.2 Method Two – A graphing we will go.

We could use our calculator in a slightly different way. We could define a family in the **Graph** mode and then define a second family with only one member! Like seen opposite.

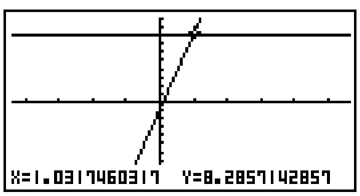


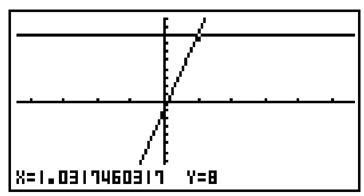
Choosing an appropriate view window (F3) (seen below) and then pressing DRAW (F6) we see a different view of what the 'hunting process' does.





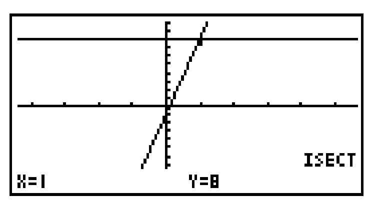
The intersection of the two lines providing us with the information we want.
We could now Trace (F1) to find the values of *x* and *y* at the point of intersection. To be sure we are at the point of intersection, you can press the up and down arrows, this will move the trace marker from one line to the other, if the results are the same then you will be on it!





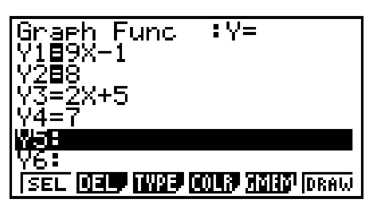
As you should have experienced the trace increments are 'not nice' and in this case we cannot get to the point of intersection. This can be fixed, but is often not worth the effort. Instead we can use a feature of the machine that 'homes in' on the intersection point for us.

With the graph drawn, use G-Solv (F5) and then ISCT (F5). Watch the top right hand corner of the screen and a little blue dot appears to tell us the machine is working. When it is done, it reveals the image seen opposite – reporting the information we are interesting in.



So the solution to the equation is $x = 1$.

Note
If you define a family in the **Table** mode it will also appear in the **Graph** mode. If you have some families in there you do not need to use, you could delete them, but we suggest you just **de-select** them.



When selected the equal sign has a dark rectangle on it, when not selected it does not and will not be involved in the graph or table. To select and de-select, use the arrow key to highlight it and then use F1 to select or de-select. You can see the results of this opposite.

Exercise One

Use the graphing method to solve the following equations:

1. $3x-3 = 15$
2. $7x-5 = 9$
3. $3x+7 = 25$
4. $6x+5 = -31$
5. $-4x-1 = -5$
6. $x+11 = 19$
7. $2x-8 = -17$
8. $3x+3 = 19$
9. $6x-1 = 12$
10. $9x-1 = 18$

Exercise Two

What do you think the ***value*** of the solutions to Exercise One parts 8, 9 and 10 are in ***fractional form***– can you be sure of your answer?

2.2.3 Method Three – The power of the human mind and proof beyond any doubt.

What did you think the ***exact solution*** to $9x$-1 = 18 was?

Standard graphic calculators do not return answers like those we were interested in fractional form, only decimal form. This is due to the algorithm the calculator is programmed to use.

We can, however, use our minds and its ability to be logical, to solve equations. Such a technique will return answers in fractional form. Such a form is know as an exact form and is often and important form to have an answer in.

Consider the following **logical argument that solves the equation.**

$$9x - 1 = 8$$
$$\Rightarrow 9x = 9$$
$$\Rightarrow x = 1$$

Some room for important notes:

Ask your self the question, "What must $9x$ be if $9x$ subtract one is 8?"

Then ask yourself the question, "What must x be if 9 times x is 9?"

Now try this one together.

$$2x - 4 = 18$$
$$\Rightarrow$$
$$\Rightarrow$$

Some room for important notes:

Now try this one together.

$-5x+3=18$

$\Rightarrow$

$\Rightarrow$

Some room for important notes:

Now try this one together.

$2x+3=18$

$\Rightarrow$

$\Rightarrow$

Some room for important notes:

Exercise One

Use the power of your mind to solve the following equations. Show each step of your logic and give solutions in fractional form where necessary:

1.	$4x-1 = 15$	11.	$x+16 = 30$
2.	$3x-1 = 10$	12.	$2x-8 = -15$
3.	$5x+3 = 18$	13.	$4x+3 = 8$
4.	$2x+6 = -30$	14.	$3x-1 = 10$
5.	$-2x-1 = 9$	15.	$5x-1 = 10$
6.	$3x-3 = 15$	16.	$x+11 = 19$
7.	$7x-5 = 9$	17.	$2x-8 = -17$
8.	$3x+7 = 25$	18.	$3x+3 = 19$
9.	$6x+5 = -31$	19.	$6x-1 = 12$
10.	$-4x-1 = -5$	20.	$9x-1 = 18$

Exercise Two

Which of the three methods used to solve equations in this section do you prefer? Why?

2.2.4 Summing Up

Equations that are of the form $ax + b = c$, where a,b and c are numbers can be solved simply by using the logical process above.

There are many other forms of equations, but you will see that many can be reduced to this form, and so the technique you have just learned is critical to your progress.

2.2.5 Lap running

Not good enough at this skill yet? Well it is time to run some more laps then. Ask your teacher for some 'laps'.

3. Describing a position (and multiple positions) in the plane.

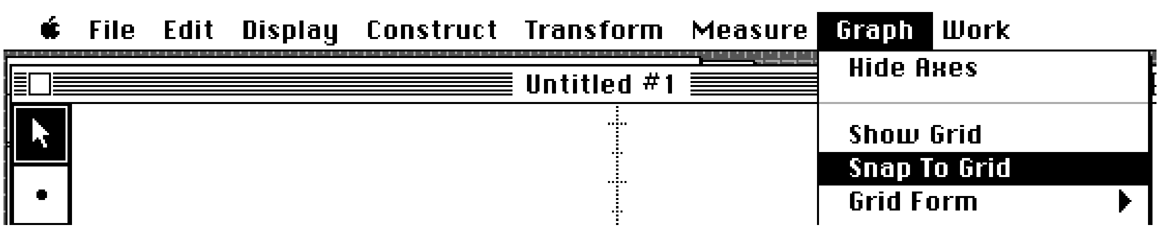
3.1 What is the Cartesian Plane?

We have chosen to look at how we might describe the location of any point on a section of the computer screen you use so frequently. We could have just as simply used the white-board, the floor or any other plane.

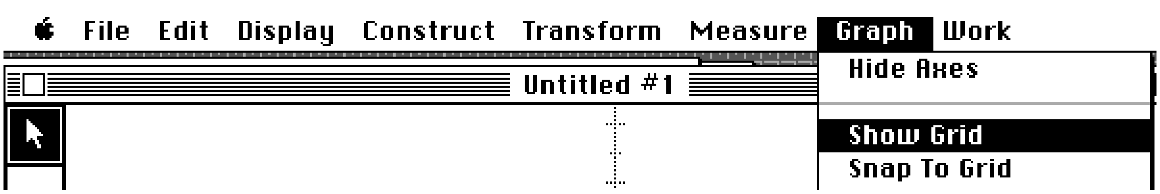
The following section supplies you some instructions for using The Geometer’s Sketchpad that will help you to learn how points in 2-d space, a plane can be located.

From the '**Graph**' menu, select '**Create Axes**' as shown below.

From the '**Graph**' menu, select '**Snap To Grid**' as shown below.



From the '**Graph**' menu, select '**Show Grid**' as shown below.



These two moves should produce a screen similar to that shown overleaf (without the annotation).

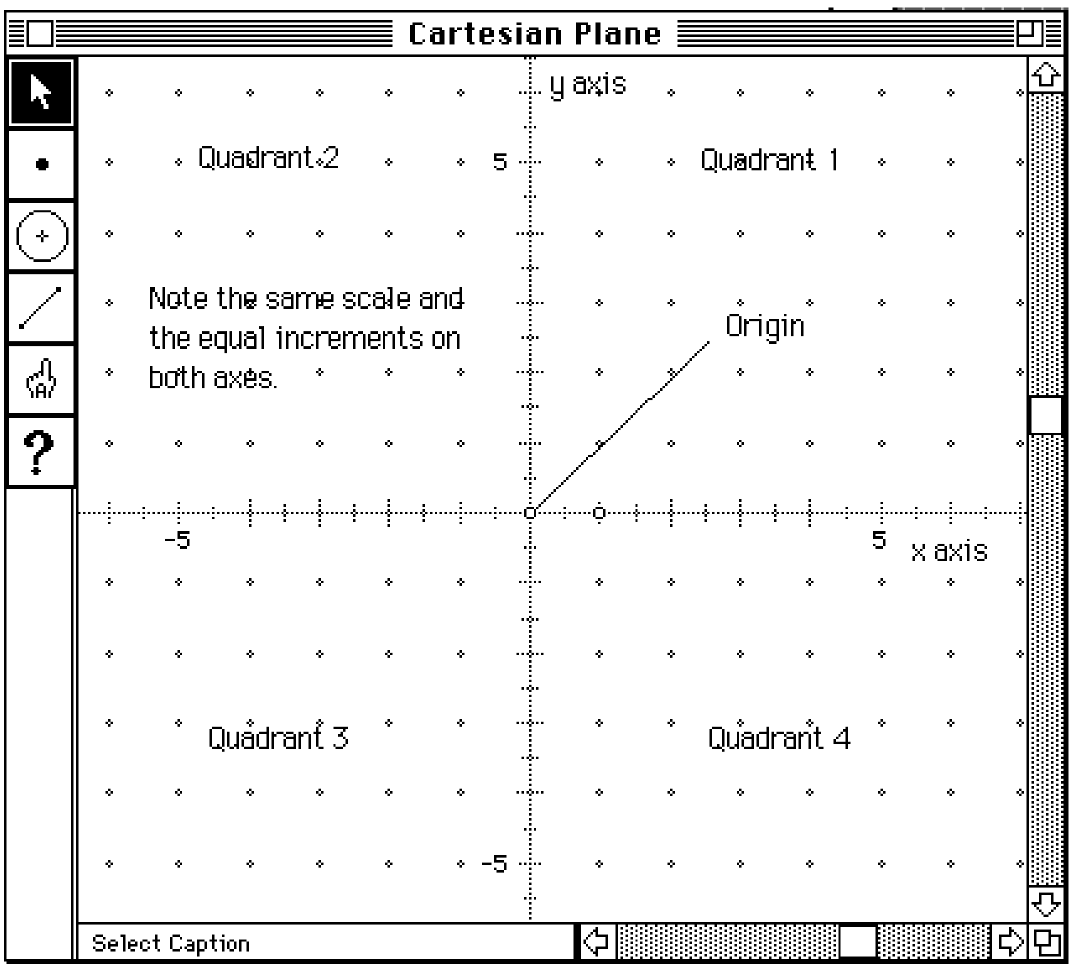
The two lines that are **perpendicular** to each other divide the **plane** into 4 regions. The lines continue in the four directions forever. Each line is called an axis, and the **plane**, when divided by such a set of axes is called the **Cartesian Plane** - named after the person who invented this system, French mathematician Rene des Carte.
The four regions are called **QUADRANTS 1 to 4**, as shown below.

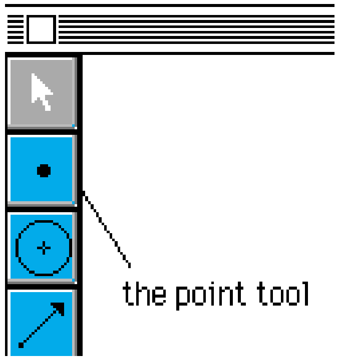
Note that each axis is just like a ruler. It has both positive and negative measurements. It is important, for the type of work we are going to be doing, that both the horizontal and vertical axes have the **same scales** – a square set up as mentioned earlier.

Also, as in a ruler, the axes are **equally incremented (ie each division is worth the same number of units)**.

The point where the two axes meet is called the **ORIGIN**. Each axis has a name. The **horizontal** is called the **'x axis'** and the **vertical** is called the **y axis**.

One purpose of the Cartesian Plane is to allow us to give a '***grid reference***' to any point in the PLANE. It also allows us to pictorially represent the values of algebraic expressions (families) - like 4x+5, among other things.





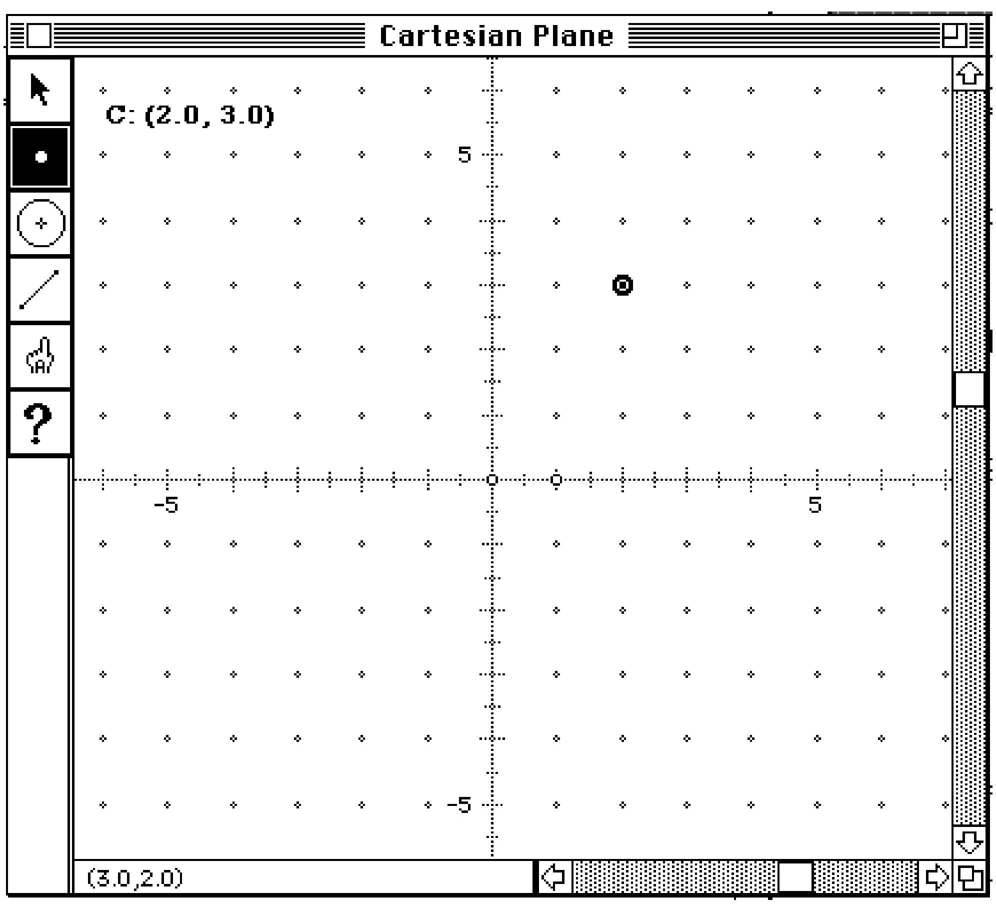
Select the point tool, shown opposite, by clicking on it once.

This will allow you to plot a point on the Cartesian Plane. Remember that you selected '**Snap To Grid**' earlier, this will force any point you place to go to a position on the plane grid reference by integer values.

Place your cursor somewhere on the Cartesian Plane, watch the "status bar" at the bottom of the window, and click once. It will produce a point that will be 'selected'. This is indicated by its black colour.

Ensure the point is selected and then go to the '**Measure**' menu and select '**Coordinates**'.

You should achieve something similar to the screen below.



Note that upon measuring, **C:(2.0,3.0)** appears.

These are the **co-ordinates** of the point you have plotted. Note also that the status bar reads a different co ordinate. This is where the cursor was when I recorded the screen dump.

This is the standard way that we name a point on a Cartesian Plane. We use a capital letter, together with a pair of coordinates. The first is the *x* coordinate - in this case 2, and the second is the *y* coordinate - in this case 3.

The x (horizontal) coordinate is always reported first, and the y (vertical) second:

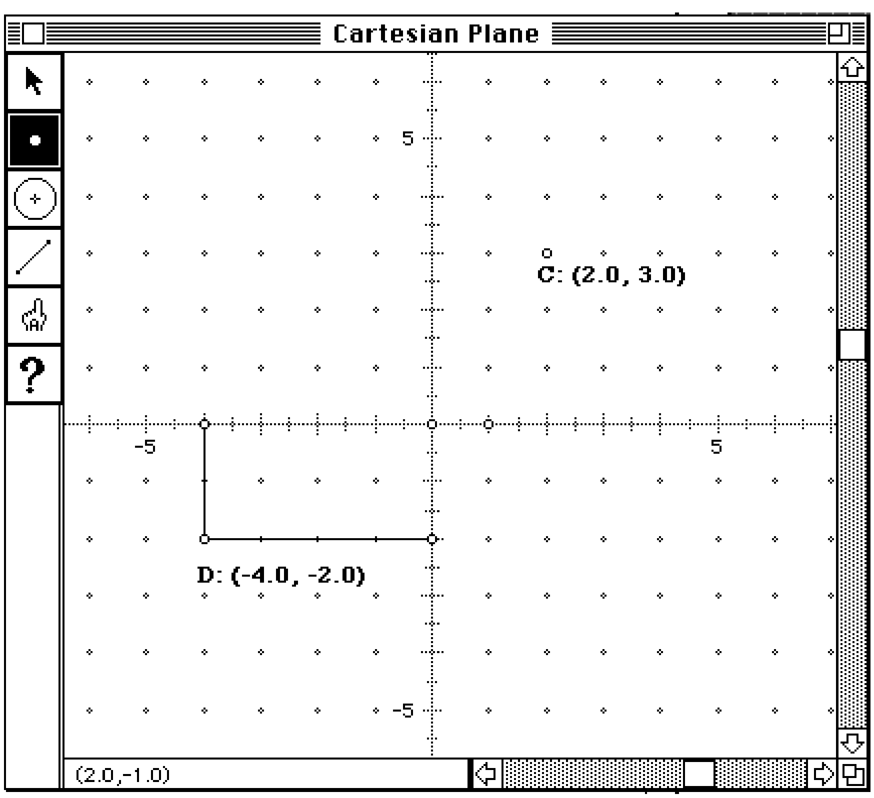
P (*x*,*y*)

The *x* value is called the **abscissa** and the *y* value is called the **ordinate**. Together they are called a **coordinate** or an **ordered pair**.

This system of describing the location of a point is called a **'rectangular coordinate system'**.

To determine the coordinates of a point you draw an imaginary line perpendicular to each axis, as shown below. The values on each axis are the x and y coordinates respectively. You need not draw these lines.

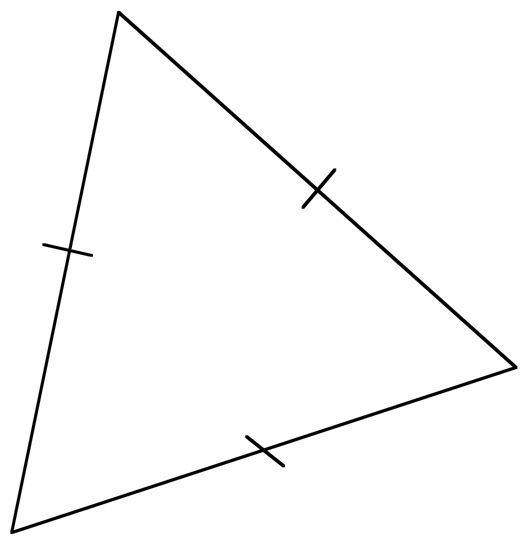
Note that at the bottom left hand corner of the window, if you have the point tool selected, the coordinates of the cursor are shown.

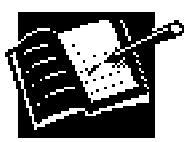


3.2 Working on the Cartesian Plane.

Task One: Producing an equilateral triangle

An equilateral triangle is one that has all three of its sides equal in length.





RESEARCH

Start a new page in your problem book - title it
'Task One : Producing an equilateral triangle' and write down the defining characteristic of the equilateral triangle printed above.

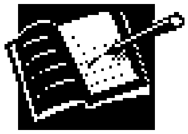
R1.1 Write down all other properties of an equilateral triangle you know or can find out through your own research.

From the '**File**' menu, choose '**New Sketch**'.

From the '**Graph**' menu, choose '**Show Grid**'. This automatically place the axes on the plane, turns on '**Snap To Grid**' and show a Grid, just like graph paper. It will help you to plot points.

Plot the points **A:(4.0,1.0)** and **B:(-2.0,-1.0).**
You must now plot a third point, so that a triangle is formed by joining the three points, it is an *equilateral* triangle.

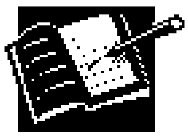
Remember that to join points with a line segment, you must highlight two points (use your shift key) and choose segment from the construct menu. To check whether or not the triangle is equilateral, click on each segment in turn and measure their length.



RECORD

Record each of the points that you plot in your attempt to produce an equilateral triangle.

If you are not successful after TEN ATTEMPTS, stop. Highlight your best attempt in your workbook.



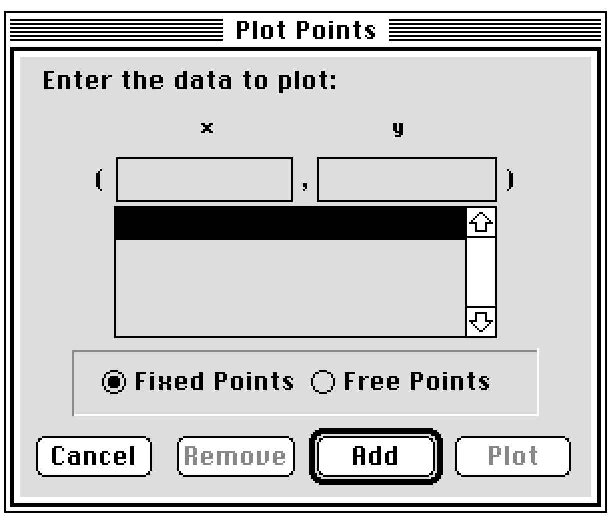
THOUGHTS

T1.1 Is it possible to produce an Equilateral Triangle with the present set up? Explain.

Delete everything from your sketchpad.

From the '**Graph**' menu, turn the '**Snap To Grid**' off. This will allow you to plot a point anywhere and move it around anywhere.

From the '**Graph**' menu, select '**Plot Points . . .**', it will give you a dialogue box like the following:

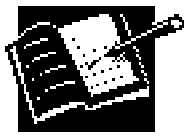


We will use this to re-plot the points **A:(4.0,1.0)** and **B:(-2.0,-1.0),** but this time so that these points are fixed at the coordinates we give them. Enter the x and y values for **A.** Click **Add** and then enter the x and y values for **B.** Be sure the '**Fixed Points'** is checked. Click '**Plot'**. Try to move the points from their position, the whole Cartesian Plane should move!!

Now select the point tool from the toolbox and plot a point where you think it needs to be to produce an equilateral triangle.

You may like to select '**Preferences'** from the '**Display'** menu and change the precision of the distances etc.

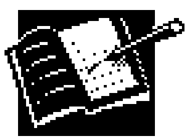
Join the points, measure their coordinates, measure the side lengths of the triangle formed and then move your third point around until you make an equilateral triangle.



RECORD

Record the points that you plot which produce an equilateral triangle.

Draw (using hand, pencil and paper technology) an accurate representation of what you have produced on the screen.



THOUGHTS

T1.2 How many equilateral triangles could you produce using points A and B? Why?

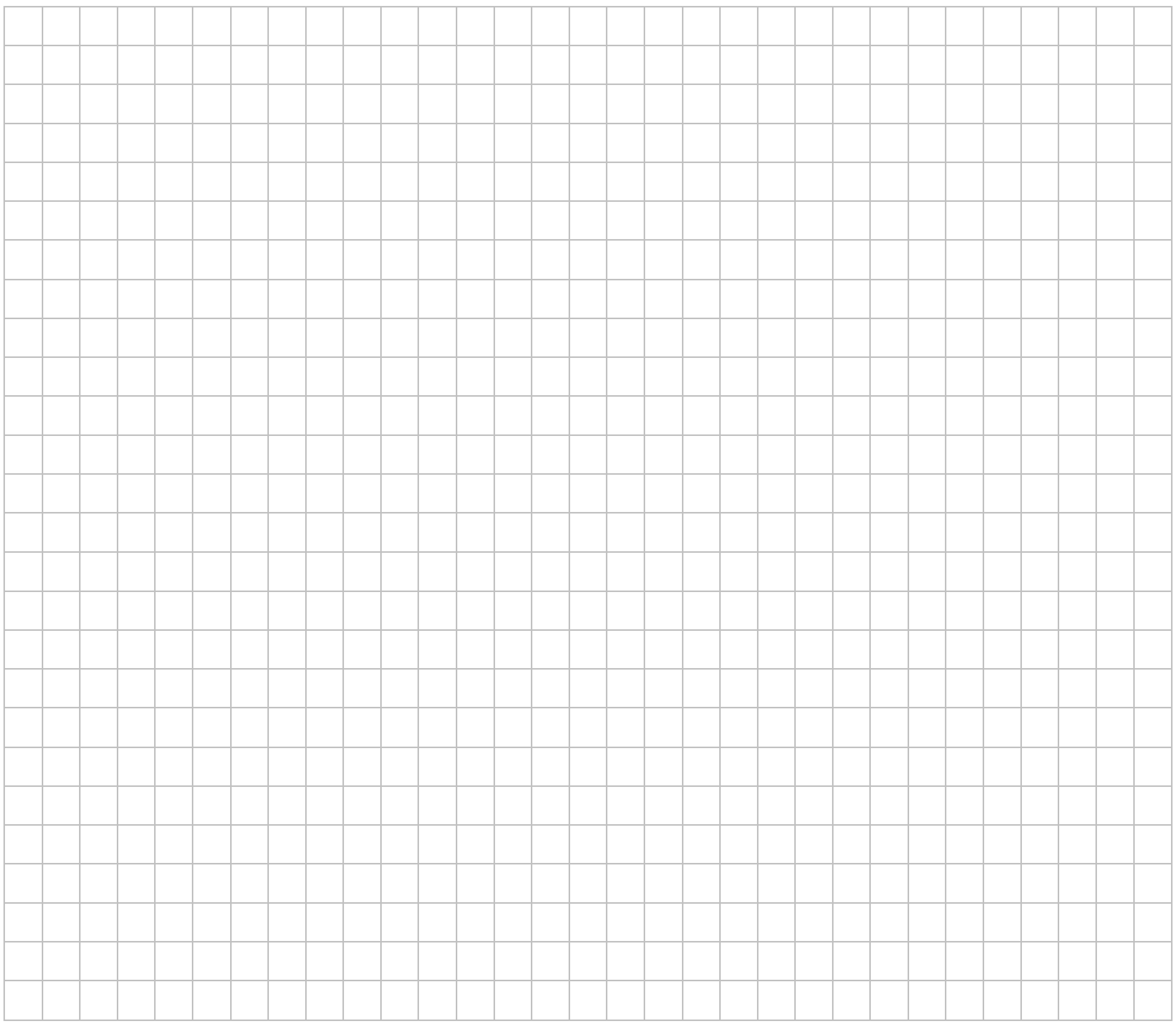
3.3 Drawing a Cartesian Plane 'by hand'.

When drawing a Cartesian Plane in your book you should make it look exactly like the ones on GSP, except you must add two things:

- Label each axis *x* or *y*
- Arrow the end of each axis to show it continues on forever in both directions.

You must also always use ***graph paper***. Anything else is **unacceptable**.

It should resemble the following (copy what your teacher does on the white-board):



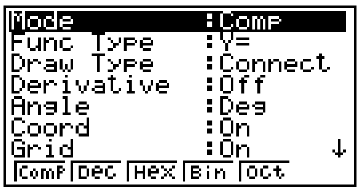
For the work we are going to do now, you may change the scales that you use, so long as the scale on the *x*-axis is the same as that you use for they.

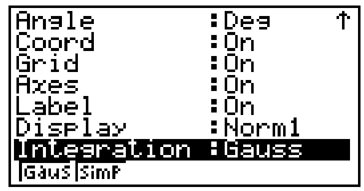
On GSP you can change the scale by dragging the point, (1,0), that is near the ORIGIN when you first construct the axes. Try it out now.

3.4 Making the screen of the Casio CFX-9850GB Plus into a Cartesian Plane.

Your graphics calculator can be used as an electronic Cartesian Plane.

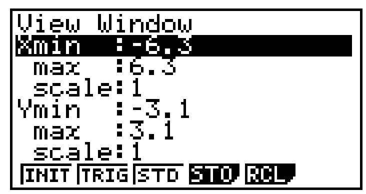
Enter the RUN mode of the calculator. Use SET UP (Shift and then MENU) to set up the preferences for this mode as shown below – study each setting carefully. Press EXIT.





The critical settings are the Axes, Grid and Label settings.

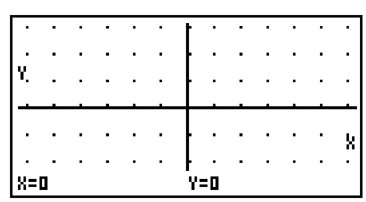
Before creating the Cartesian Plane we must tell the calculator how we wish the axes to be scaled. The View Window function allows us to do this. Use SHIFT and then V.WIN to reveal the view window settings. The settings will be whatever they were set at last time the calculator was used. For now we are going to use the INIT (initial) settings that are stored in the calculator. Press INIT (F1). These values may look a little strange – more on that later, for now these will do.



Press EXIT and then use Shift to reveal four options at the base of the screen. Use SKTCH (F4), then DES (F6) and PLOT (F1) to reveal the screen seen opposite.

Now use the Plot (F1) (note the case difference to the previous PLOT command) to insert the command Plot into the screen of the calculator.

Press EXE and you will find the screen is converted into a Cartesian Plane – minus the arrows and plus rather silly positions for the *x* and *y* labels. You will see, on your screen, the red cursor flashing and ready for action.



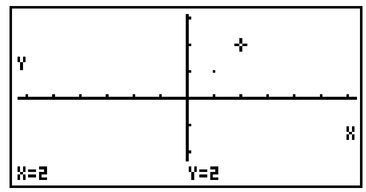
You can then use the arrow keys to move around the Cartesian Plane and plot points by pressing EXE when you arrive at the desired position.

Note that the scaling of the axes is such that the distance along the x-axis that represents 1 unit is the same as the distance along the y-axis that represents one unit.

This gives what could be called a '*square'* Cartesian Plane. It is important to have a square Cartesian Plane if you wish to draw figures, for example a square and expect it to look like a square.

GSP's axes are locked in a square form.

Note that the points you plot look identical to the grid points – what a shame. **You might like to turn the grid off in the set up menu.** I have done this and you can now see I have plotted one point (1,1) and am ready to plot another.

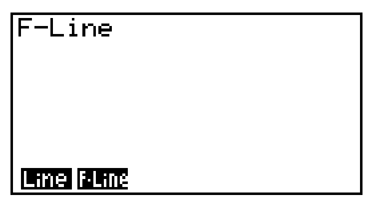


Note: Unlike the 'real' Cartesian Plane, this electronic version has a finite area, you can not plot 'outside' the screen. Note also how the cursor moves. It moves in steps of 0.1. This is dependent on the how you set the view window, ie how you told the calculator to scale the axes.

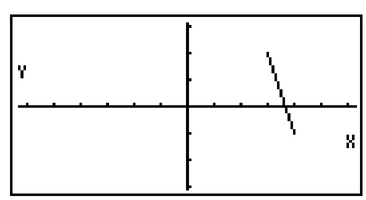
To plot a line that joins two points you can use the F-Line command.

To clear all the other plotting you have done, use SHIFT and then SKTCH (F4) to reveal the Cls command, select Cls (F1) and then press EXE. All other plotting will have been cleared.

Now use the continuation key (F6) to and then use LINE (F2) and then select F-Line (F2).



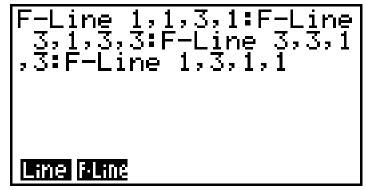
To tell the calculator which two points you would like the line to be drawn between enter something like 3,2,4,-1 after the F-Line command and then press EXE. This will result in a line being drawn between (3,2) and (4,-1).



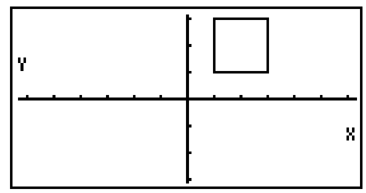
3.5 Square Graphs

We commented before on the importance of a ‘square’ Cartesian Plane if we wanted to draw a geometric figure.

With the calculator set with the view window on the initial (INIT) setting enter the commands seen opposite in run mode. It asks the calculator to draw a square.
The : is found in the PRGM (SHIFT then VARS) menu after pressing the continuation key (F6).



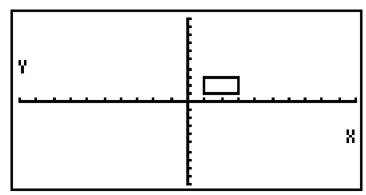
Press EXE and the result is a square that looks like a square.



Access the view window settings (SHIFT and F3). Select the STD (standard)(F3) settings – be sure to look at what they are.

Press EXIT and then the up arrow key.

The code you entered for a square should return. Press EXE and the square will be re-drawn with the new scaling of the axes – eek it not a square – or is it?



You **cannot** use ZOOM (F2) and then DES (F6) and then SQR (F2) in this area of the calculator.

Exercise One
As a class, discuss possible reasons why the INIT settings give a square but the STD does not.

Exercise Two
Find *all* view window settings that will result in a square looking like a square.

Exercise Three
What actually happens when you use the ZOOM/SQR option? Go back and try it Table or Graph mode.

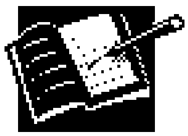
Exercise Four
Experiment by drawing some of your own shapes – document your work in your workbook.

You may now proceed with any one of HP&P technology, graphic calculator technology or GSP. Choose the most appropriate.

3.6 Working on the Cartesian Plane.

Exercise One - Producing a Square.

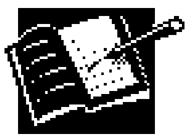
A square is a four sided plane figure in which each of the four sides are equal in length and each interior angle measures 90 degrees.



RESEARCH

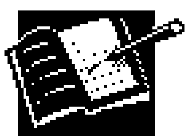
Start a new page in your problem book - title it **'Task Two: Producing a Square'**. Write down the defining characteristics of the square (seen above).

R1.4 Write down all the properties of a square you know or can find out by your own research.



SOLVE

S1.2 By following similar steps to those in the last investigation, produce a SQUARE, using the same two points, **A:(4.0,1.0)** and **B:(-2.0,-1.0)** used for the equilateral triangle.



THOUGHTS

T1.5 Record the points that you plotted that form a square.

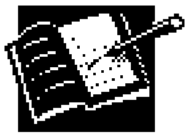
T1.6 Describe the process you used to achieve a square.

T1.7 How many squares could you produce using points A and B? Why?

T1.8 At home tonight, draw an accurate representation of what you have produced on the screen.

Exercise Two - Producing a right-angled triangle

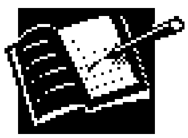
A right-angled triangle is one in which one of its interior angles measures exactly 90 degrees.



RESEARCH

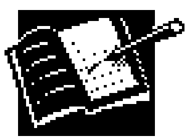
Start a new page in your problem book - title it
'Task Two : Producing a right-angled triangle'

R1.2 Write down all other properties of a right-angled triangle you know or can find out through your own research.



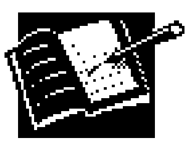
SOLVE

S1.1 Use what you have learned to produce a right-angled triangle.



RECORD

Record all of your work accurately and neatly in your workbook..

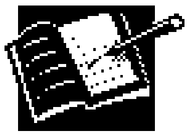


THOUGHTS

T1.3 How many right-angled triangles could you produce using points A(2,3), B(5,3) and another of your choice? Why?

Exercise Three - Producing an Isosceles Triangle

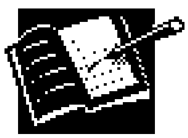
An isosceles triangle is one that has at least two of its sides equal in length.



RESEARCH

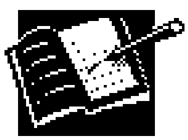
Start a new page in your problem book - title it **'Task Two : Producing an Isosceles Triangle'**. Write the defining characteristic of an isosceles triangle (seen above) in your book.

R1.3 Write down all other properties of an isosceles triangle you know or can find out through your own research.



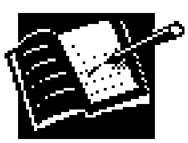
SOLVE

S1.2 Use what you have learned to produce an isosceles triangle.



RECORD

Record all of your work accurately and neatly in your workbook.



THOUGHTS

T1.4 How many isosceles triangles could you produce using points A(2,3), B(5,3) and another of your choice? Why?

3.7 Viewing only selected parts of a family.

Consider the family described by $y = 2x+1$. If we only wanted to see the members of the family for the –2th generation to the 5th generation we could write the following in mathematical symbols:

$$y = 2x+1 \text{ for } -2 \le x \le 5,\ x \in R$$

The $-2 \le x \le 5,\ x \in R$ says that we want to see **all** the values of y for **all** the values of x that are real numbers between and inclusive of –2 and 5.

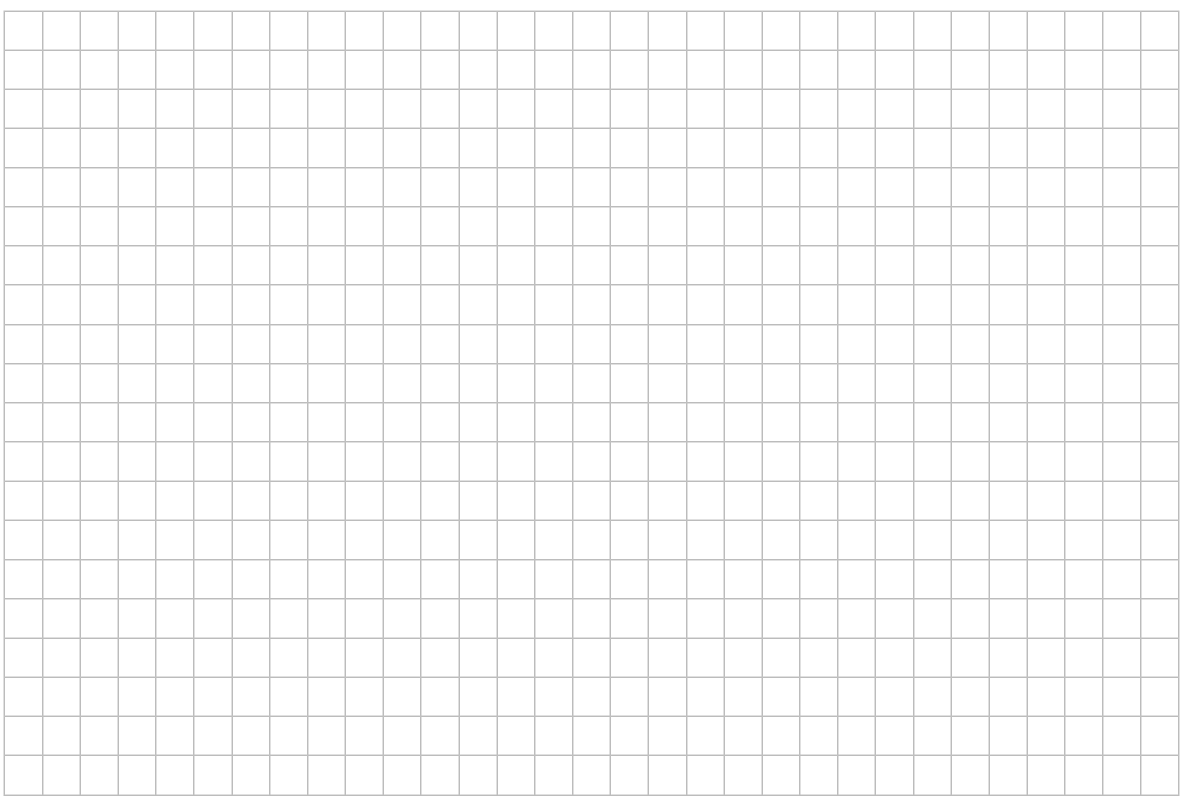
If we only wanted to see the values of y for the integer values of x between and inclusive of –2 and 5, then we could write:

$$y = 2x+1 \text{ for } -2 \le x \le 5,\ x \in Z$$

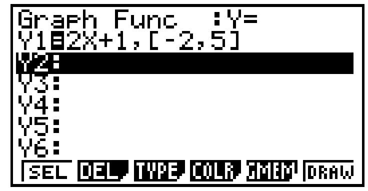
If we only wanted to see the values of y for the integer values of x between and **not** inclusive of –2 and 5, then we could write:

$$y = 2x+1 \text{ for } -2 < x < 5,\ x \in Z$$

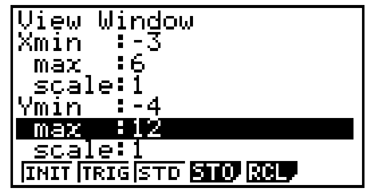
By now you should expect that $y = 2x+1$ for $-2 \le x \le 5,\ x \in R$ might look like a **line segment** if represented graphically.



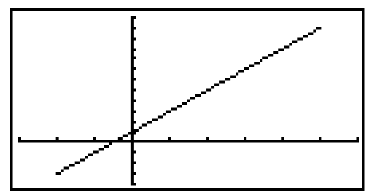
To display this on your graphic calculator we can do the following.
Enter the family using the syntax shown opposite.



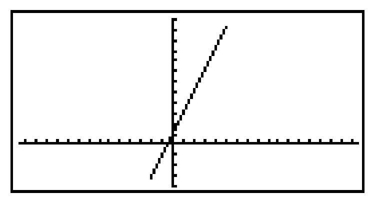
Consider how the view window should be set so you see all of this part of the family. Ensure you add a bit of space either side of the end points in each direction.



Then press EXIT and use DRAW (F6) to produce the graph



Now remember this will NOT necessarily be a square window, and so now press ZOOM (F2), then DES (F6) and then SQR (F2) to give you what you produced by hand.



Exercise 1

(Write your answers to these questions in your 'Problem Book')

Question 1 (Use your calculator as a check of your work only, that is, use your head and hands first.)

a) On one Cartesian Plane draw the lines segments described by $y=2x+3$ for $-4<x<6$ and $y=-2x+13$ for $3\leq x\leq 5$.

b) On one Cartesian Plane draw the lines segments described by $y=\frac{1}{2}x+3$ for $2\leq x<6$ and $y=-2x+13$ for $3\leq x\leq 5$.

c) On one Cartesian Plane draw the lines segments described by $y=\frac{1}{3}x+3$ for $2\leq x\leq 6$ and $y=-3x+13$ for $-1<x\leq 6$.

d) On one Cartesian Plane draw the lines segments described by $y=\frac{1}{5}x+4$ for $2<x<6$ and $y=5x+1$ for $3\leq x<8$.

Question 2

a) Look closely at the line segments produced in Question 1 and see if you can make a conjecture (or two!) about the appearance of the line and the coefficient of the *x* term in the rule.

b) Test your conjecture.

Question 3

a) On one Cartesian Plane draw the lines segments described by $y=\frac{1}{2}x+3$ for $2\leq x\leq 6$ and $y=-2x+13$ for $3\leq x\leq 5$.

b) Join each end point with a straight line. What sort of quadrilateral is produced?

c) State as many properties of this figure as you know or can find out by your own research.

Question 4

a) On one Cartesian Plane draw the lines segments described by $y=x+2$ for $1 \le x \le 4$ and $y=-x+7$ for $0 \le x \le 5$.

b) Join each end point with a straight line. What sort of quadrilateral is produced?

c) State as many properties of this figure as you know or can find out by your own research.

Question 5

a) On one Cartesian Plane draw the lines segments described by $y=x+2$ for $1 \le x \le 4$ and $y=-\frac{3}{5}x+6$ for $0 \le x \le 5$.

b) Join each end point with a straight line. What sort of quadrilateral is produced?

c) State as many properties of this figure as you know or can find out by your own research.

Question 6

a) On one Cartesian Plane draw the lines segments described by $y=x+2$ for $1 \le x \le 4$ and $y=-\frac{1}{2}x+5$ for $0 \le x \le 6$.

b) Join each end point with a straight line. What sort of quadrilateral is produced?

c) State as many properties of this figure as you know or can find out by your own research.

MATHEMATICAL REFLECTIONS

Mathematical reflections must be answered in sentences, in your exercise book.

Describe how two straight lines must be placed such that they become the diagonals of a

- Square
- Rhombus
- Parallelogram
- Trapezium

Exercise 2

(Write your answers to these questions in your 'Problem Book')

Question 1

On one Cartesian Plane draw a square and determine the rule for each of its diagonals.

Question 2

On one Cartesian Plane draw a rhombus and determine the rule for each of its diagonals.

Question 3

On one Cartesian Plane draw a parallelogram and determine the rule for each of its diagonals.

Question 4

On one Cartesian Plane draw a trapezoid and determine the rule for each of its diagonals

MATHEMATICAL REFLECTIONS

Mathematical reflections must be answered in sentences, in your exercise book.

Parallel lines are lines that never cross. You have seen many pairs of parallel lines in the last two exercises. Discuss how you can be sure that two lines are parallel.

MATHEMATICAL EXPLORATION

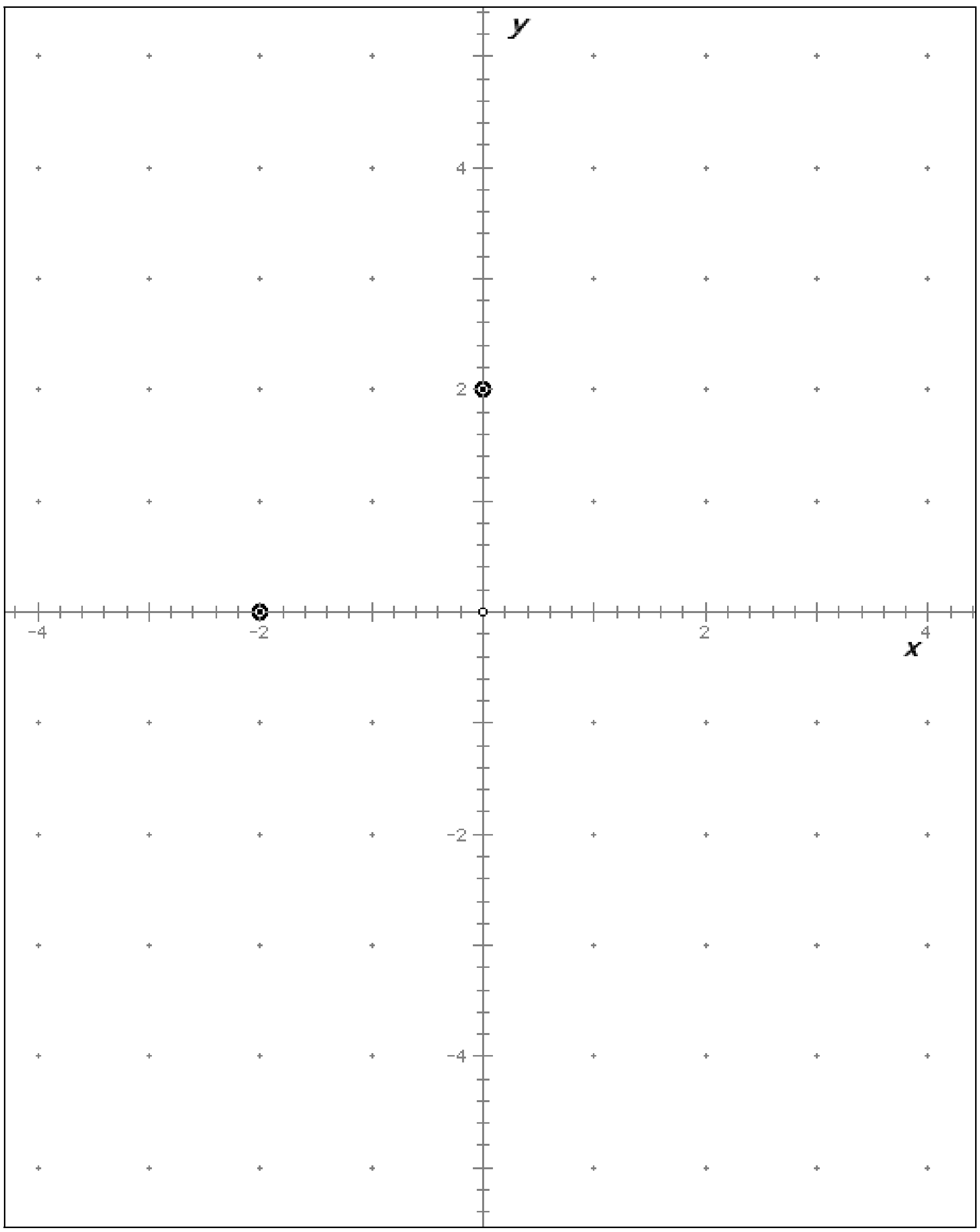
Have you ever wondered how the calculator is ‘told’ to make that pretty opening screen appear? How do all the colours get turned on and off in the correct spots? Well if you would like to learn how to do these sorts of things, ask you teacher for the unit called ‘**Application of Location in the Plane**’ – have fun with it, it is a hoot.

4. Changing names – or not?

4.1 Where are all the points that are ……

Locate 10 points on the plane below that are the same distant (equidistant) from the points (0 , 2) and (-2 , 0).

How many points could you mark on? Are they a family? What is the family's surname? How could you display this family on you graphic calculator?



Room for some important notes.

Exercise 1

(Write your answers to these questions in your 'Problem Book')

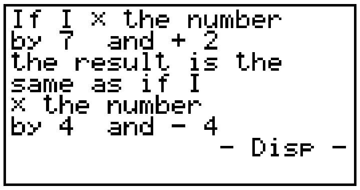
Question 1

a)

Some questions are needed here on other families and dual names that allow students to rearrange and so on. Some simple formula rearranging.

5. Solving slightly more involved equations.

5.1 But what about TINK2?



How do we cope with this?

Remember that you are able to solve equations like
$6x - 2 = 10$. Be sure to solve the problem in a number of ways – including the use of electronic technology.

Room for some important notes.

Exercise One

Use at least two methods to solve the following equations. Show each step of your logic and give solutions in fractional form where necessary:

1. $4x - 11 = 2x - 1$
2. $2x - 5 = x - 1$
3. $7x + 20 = 3x - 4$
4. $-5x - 11 = 2x + 3$
5. $-x + 11 = 2x - 1$
6. $4x - 10 = 2x - 1$
7. $4x - 4 = x + 1$
8. $7x + 15 = 2x - 1$
9. $10x - 6 = x - 1$
10. $8x - 12 = 2x - 1$

Exercise Two

How might you be sure beyond any doubt of your solutions to the above equations – especially what the exact values are of the solutions that seem to be non-integer.

5.3 Solving beyond any doubt.

How might we be sure beyond any doubt of the solution to $8x - 12 = 2x - 1$

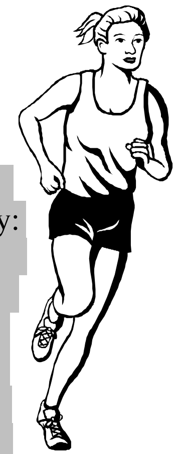
Using electronic technology to solve this we find $x = 1.83333333333\ldots$

What do you think the exact fractional value might be?

How might we find out?

Room for some important notes.

5.3 Lap running



Exercise One

Use at least two methods to solve the following equations. Show each step of your logic and give solutions in fractional form where necessary:

1.	$4x - 11 = 2x - 1$	11.	
2.	$2x - 5 = x - 1$	12.	
3.		13.	
4.		14.	
5.		15.	
6.		16.	
7.		17.	
8.		18.	
9.		19.	
10.		20.	

Exercise Two (Meta-Cognition)
Which of the methods used to solve equations in this section do you prefer? Why?

Exercise Three (Meta-Cognition)
Can you just ‘see’ the answer to these sorts of equations, or are you using a process without knowing it, or are you overtly using a process?

Exercise Four
Skills not good enough yet? Find some more of this type of equation to solve. Do it until you are very, very good at it.

6 What about that MREADER program – how does it work?

Remember that MREADER program? Worked out how it reads your mind yet?

Go on give it another go.

7 Some problems to finish with – where only integers apply

Problem One
At my school, four-fifiths of the students occupy five-sixths of all desks if every student is present at school.

How many students attend my school?

Problem Two
Find all integer values of x that have a corresponding integer value of y if $5y - 3x = 1$.

Problem Three
Show that, if a, b and c are integers there will always be integer pairs (x, y) that satisfy $ay + bx = c$.

Problem Four
Do a WWW search on ‘school level Diophantine Equations’ and ‘Archimedes Cattle problem’. Investigate what you find.