

1. Home Screen Calculations (MM1-4&SM3/4)

Many simple expressions are built into the graphics calculator and can be accessed directly from the keypad or from the appropriate menu. The size of the display area gives a graphics calculator a distinct advantage over a scientific calculator for arithmetic calculations.

Arithmetic calculations

1. Evaluate $2/3 + 1/4$ and write your answer as a fraction.
2. Calculate: (i) $\sqrt[3]{5.7}$
(ii) $\sqrt[5]{5.7}$
3. Calculate, to 4 decimal places, log to the base e of the integers from 1 to 5.

Trigonometric calculations

4. Evaluate $\sin 20^\circ$.
5. Evaluate $\tan^{-1} 1.9$ and round your answer to two decimal places.
6. Solve $\sin x = 4/5$ and give your answer in degrees, minutes, and seconds.

Permutations and combinations

7. Evaluate ${}^{10}P_4$ and ${}^{10}C_4$, and verify that ${}^{10}P_4 = 4! \times {}^{10}C_4$.

For more complex expressions, such as a particular term in a binomial expansion, the concatenation feature and variable storage feature of the graphics calculator can be used to advantage.

Binomial expansion

To answer the question 'What is the coefficient of x^5 in the binomial expansion of $(3x + 7)^{11}$?', enter the following expression:

$$11 \rightarrow N: 5 \rightarrow R: 3 \rightarrow A: 7 \rightarrow B: N nCr R *(A)^R*(B)^{(N-R)}$$

Modify this expression to answer the following questions.

8. (a) What is the coefficient of x^3 in the binomial expansion of $(5x - 2)^7$?
(b) What is the coefficient of x^9 in the binomial expansion of $(2x^3 - 2)^{15}$?
(c) What is the coefficient of x^7 in the binomial expansion of $(2x^2 - 3x)^5$?

Note: The purpose of these items is to familiarise the user with a range of options for using home screen calculations. Also shown are some tasks that might make use of such features.

2. Storing Results in Lists (MM1-4&SM3/4)

The use of lists, containing patterned or random data, is an important feature. Storing a series of results in a list is a very useful tool for future analysis.

Entering patterned data into lists

To enter, for example, the squares of the first ten whole numbers into a list, there are at least four distinct methods.

Method 1. *The tedious way:*

- Enter the list and type in the numbers

Method 2. *The half-tedious way:*

- Enter the following expressions from the Home Screen (RUN mode):

0→N

N+1→N:N²→List 1(N)

- Press the EXE key 10 times and check the contents of List 1

Method 3. *The sequence way:*

- Enter the rule as a sequence directly into the list (STAT mode):

List 1=seq(N²,N,1,10,1)

- Check the contents of List 1

Method 4. *The simplest way:*

- Enter {1, 2, 3, ...} into List 1 and set List 2 as a function of List 1 in STAT mode:

List 2=List 1²

- Check the contents of List 2

1. (a) Using Methods 2 and 3, enter the reciprocals of the first ten whole numbers into List 1.
(b) Repeat (a) for the sequence $-9, -7, \dots, 9$.
2. (a) Using method 4, enter the sequence $-9, -7, -5, \dots$ into List 2.
(b) Repeat (a) for the sequence $1 \times 2, 2 \times 3, 3 \times 4, \dots$

Generating and storing random numbers

To generate random integers between 1 and 20 (inclusive), enter the following expression:

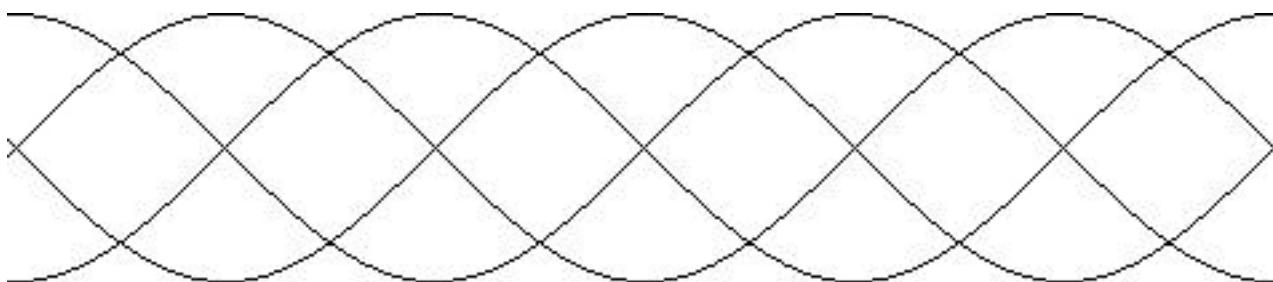
1→A: 20→B: Int((B-A+1)*Ran#+A)

3. (a) Use Method 2 with the above technique to generate ten random integers between 1 and 6 (inclusive) and store them in successive elements of list variable List 1.
(b) Investigate the contents of List 1

Note: The purpose of these items is to familiarise the user with a range of options for generating and storing useful results (including the use of list variables). Also shown are some tasks that might make use of such features.

3. Exploring Sine Functions (MM1/2)

(KP:2.1;4.1;5.1;7.1)



1. Enter and plot the function $Y1 = \sin x$ in the viewing window $[-10,10]$ by $[-10,10]$. If you wish to examine the first cycle of the graph of this function to the right of the y-axis, why might this not be the most appropriate viewing window for examining the graph of this function?
2. Enter more appropriate viewing window parameters and then re-plot the graph.
 $X_{\min} = \underline{\hspace{2cm}}$ $X_{\max} = \underline{\hspace{2cm}}$ $X_{\text{scale}} = \underline{\hspace{2cm}}$ $Y_{\min} = \underline{\hspace{2cm}}$ $Y_{\max} = \underline{\hspace{2cm}}$ $Y_{\text{scale}} = \underline{\hspace{2cm}}$
3. Adjust the function in Y1 so that it now has the rule $Y1 = \sin Ax$. Store the value 2 into the parameter A, and then plot the function. Over what period does Y1 cycle (as a fraction of π)?
4. Experiment with other positive integer values of A. Build a table for the period (as a fraction of π) of the sine function for each value of A.
5. Using your answers from question 5, create a rule which calculates the period of the function in terms of A.

Extension

The water depth, D metres above the sea bed near a particular coral reef, is modelled by the rule

$$D(t) = 3.3 + 1.5 \sin At$$

where t represents the number of hours after midday on 20th May, 1996. Some parts of the coral reef, which has its highest peaks 3 metres above the sea bed, are exposed at low tide. The successive high tides are 12.5 hours apart.

6. Use the information given to find the value of A.
7. How long will some part of the reef be exposed during low tide?
8. How long will some part of the reef be exposed in the 24 hours following midday 20th May, 1996?

Note: The purpose of this task is to illustrate one method in which graphics calculators can be used to quickly link the symbolic and graphical representations of functions. Students are encouraged to discover the links. The spirit of this task may be reproduced for other types of functions.

4. The Steamboat Problem (MM1/2)

(KP:2.1;5.1;5.3)

Sigrid sat on the deck of a Murray River steamboat. As the paddlewheel turned, a point on the paddle blade moved in such a way that its displacement, d metres, from the water's surface was a sinusoidal function of time, t seconds. When her stopwatch read 4 seconds, the point was at its highest, 5 metres above the water's surface. The wheel's diameter was 6 metres, and it completed a revolution every 10 seconds.

1. Write the equation of the sinusoid, expressing d in terms of t .
2. Use your graphics calculator to enter and plot a graph of the sinusoid, showing d against t .
3. How far above the surface was the point when Sigrid's stopwatch read 6 seconds?
4. How long was the point below the water's surface during a full revolution of the paddlewheel?

Note: The purpose of this task is to present an in-context example of a circular function. Using graphics calculators does not diminish the need for students to formulate carefully. This task requires that student has a clear understanding of how period and amplitude can be represented in a problem. The graphics calculator permits the user to construct suitable functions and test the reasonableness of such constructions. Also introduced here is the notion that there may exist more than one solution which is 'reasonable'.

5. Transformation Creations (MM3/4)

(KP:2.2;4.5;5.3)

The creations in this section are based on families of reciprocal functions. Use your graphics calculator to create the following designs. The creations do not need to be exactly the same as the ones shown, but the picture you make should look similar. It will help if you set the viewing window for each creation before you start. You should use the form $y = \frac{a}{x-h} + k$ and experiment with the values of a, b, h, and k to make these creations.

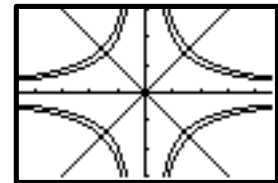
1. Parliament House

Adjust the viewing window so that you are looking at the region

$[-5,5]$ by $[-3,3]$. This design is loosely based on a bird's eye view of the Australian Parliament House. Create a design like this.

Record the functions you have used in your workbook.

What do you notice about the functions that create this design?



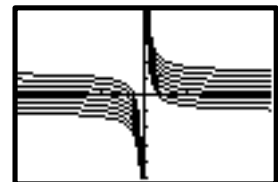
2. Waterfall

Adjust the viewing window so that you are looking at the region

$[-3,3]$ by $[-15,15]$. The waterfalls in this design all drop around the line $x = 0$.

Create a design like this. Record the functions you have used in your workbook.

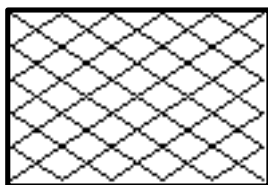
What do you notice about the functions that create this design?



After experimenting with the graphs of reciprocal functions, you will have noticed that constants such as a, h, and k have a marked affect on the shape of the graph. Now create your own designs by experimenting with various values of these constants. *Problem Reference - "Graphic Algebra - Explorations with a graphics calculator" by Asp, Dowsey, Stacey, Tynan (Key Curriculum Press)*

Extensions: Using Lists With Creations

Lattice

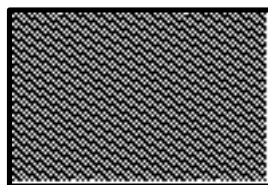


$$Y1 = x + k \text{ \& } Y2 = -x + k$$

$$k = \{-5, -4, -3, \dots, 3, 4, 5\}$$

$$[-5,5] \times [-5,5]$$

Carpet

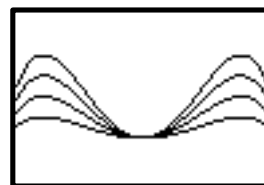


$$Y1 = x + k$$

$$k = \{-40, -39, \dots, 39, 40\}$$

$$[-20,20] \times [-20,20]$$

Spider 1

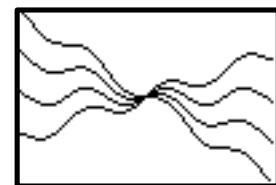


$$Y1 = k * x^2 (1 - x^2)$$

$$k = \{1, 2, 3, 4\}$$

$$[-0.9, 0.9] \times [-0.5, 1.5]$$

Spider 2

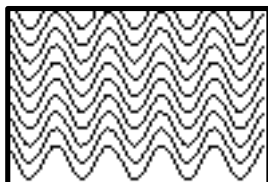


$$Y1 = \sin x + k * x$$

$$k = \{-1, -0.5, 0, 0.5\}$$

$$[-10,10] \times [-10,10]$$

Crinkle-Cut



$$Y1 = \sin x + k$$

$$k = \{1, 2, \dots, 10\}$$

$$[-10,20] \times [0,10]$$

Splash!

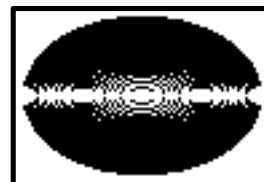


$$Y1 = \left| \frac{3}{x-k} \right|$$

$$k = \{1, 2, \dots, 10\}$$

$$[-5, 15] \times [0, 10]$$

Football

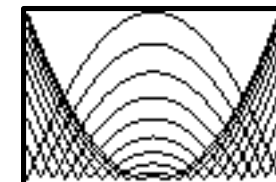


$$Y1 = +\sqrt{k - x^2} \quad Y2 = -\sqrt{k - x^2}$$

$$k = \{1, 3, 5, \dots, 91\}$$

$$[-10,10] \times [-10,10]$$

String Art



$$Y1 = |(x-k)(x+k)|$$

$$k = \{1, 2, \dots, 10\}$$

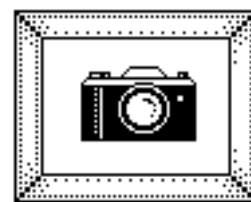
$$[-10,10] \times [0,100]$$

Note: An understanding of the way in which families of functions behave is an important goal of learning algebra and an important objective of this task. Graphics calculators let students easily experiment with a variety of examples from which they can build an intuitive understanding of functional behaviour.

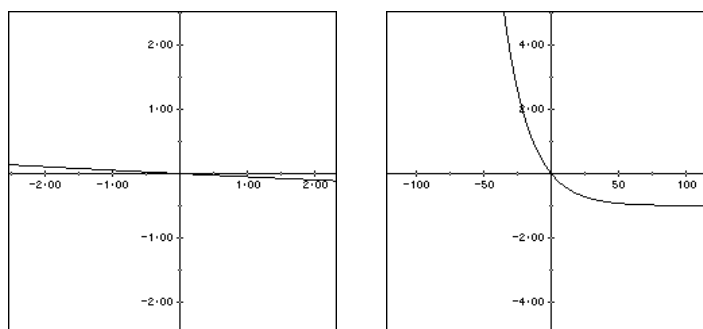
6. The Big Picture (MM1/2)

(KP:2.1;5.2;5.3;6.1-9;)

Many function graphs can be deceptive when displayed in a particular viewing window. For example it is possible to miss features such as turning points and intercepts. In previous examples of graphs plotted on a graphics calculator, you may have noticed how much adjusting of the scaling and viewing window options was sometimes needed to bring the relevant graph portion into view.



The graph of $f(x) = 0.95^x - 1$, for instance looks like a straight line when viewed in the viewing window shown on the left (see diagram below), but if you use the viewing window on the right, it looks like a typical exponential graph. In the second view, we have the 'big picture' because all the essential features are shown.



Two views of the of the function $f(x) = 0.95^x - 1$ Which view shows the big picture?

In the following questions:

- start each one on the standard viewing window of your graphics calculator, then enter and plot the functions;
- zoom as necessary to find the 'big picture';
- find any turning points;
- find any intercepts with the axes;
- sketch the function in your workbook, showing all the key features.

Polynomial Functions

1. (a) $f(x) = (x^2 - 2)(1 - 0.05x)$ (b) $f(x) = x^2(1 - x^2) - 12$

Reciprocal & Exponential Functions

2. (a) $f(x) = 25 - 0.2(1.5)^x$ (b) $f(x) = \frac{10 - x}{10 + x}$

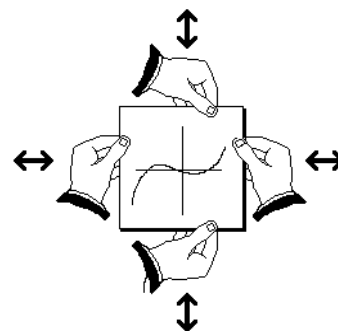
Problem Reference - 'Graphic Algebra - Explorations with a graphics calculator' by Asp, Dowsey, Stacey, Tynan (Key Curriculum Press)

Note: Many students, particularly when using graphics calculators, misinterpret graphs by not appreciating that there are many "views" of a graph, and that zooming both in and out is sometimes necessary to get a complete picture of the behaviour of a function. To solve problems with graphs of functions, students need to have a good idea of the big picture of a graph, not just what can be seen through any one viewing window.

7. The Rubber Sheet (MM1/2)

(KP:2.1;5.3;6.1-9)

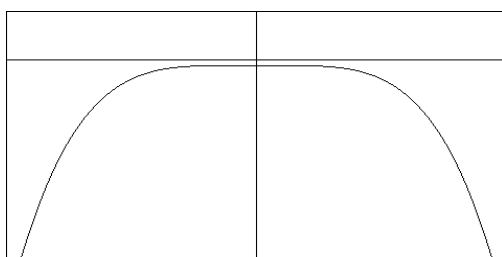
We often have a picture in our mind about how a function looks. For example, we imagine that a quadratic function will be either an inverted or upright 'u' shape. However, the shape of a function graph is strongly influenced by both the viewing window through which we are looking, and the respective scales used on each of the two axes.



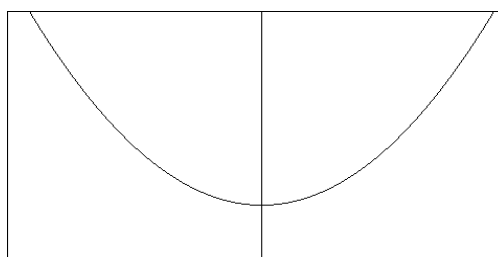
The scale on the axes are critical. Think of a square section of the X-Y plane as being an extremely elastic 'rubber sheet'. Then consider scaling as the process of stretching and shrinking the rubber sheet parallel with either or both the X and Y axes.

1. Enter and plot the graph of the function $f(x) = x^2(1 - x^2) - 12$ in the standard viewing window. By adjusting the viewing window, make your graph look like the following graphs. Record the dimensions of the viewing window in your workbook.

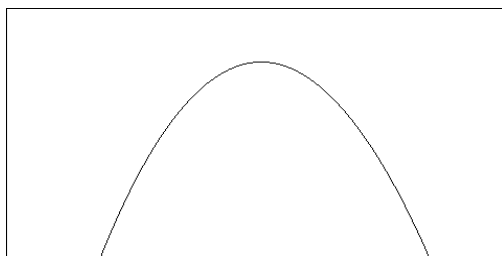
(a)



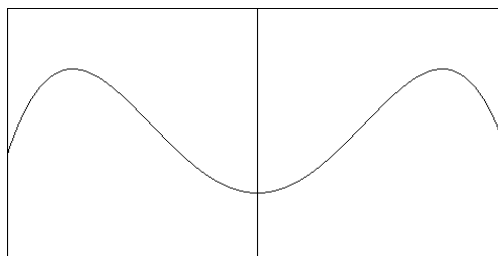
(b)



(c)



(d)



2. (a) Make up a function of your own with four quite different views. Record the function rule, each view and the dimensions of each viewing window.
- (b) Exchange your rule and your four views with another student and challenge them to reproduce each view.

Problem Reference - 'Graphic Algebra - Explorations with a graphics calculator' by Asp, Dowsey, Stacey, Tynan (Key Curriculum Press)

Note: Many students, particularly when using graphics calculators, misinterpret graphs by not appreciating that there are many "views" of a graph, and that zooming both in and out is sometimes necessary to get a complete picture of the behaviour of a function. Giving students tasks that challenge them to create various views, confronts their conceptions about what constitutes the 'correct' shape.

8. Operations on Functions (MM&SM3/4)

(KP:2.2-4;4.3;5.1;5.3;7.1)

1. Enter and plot the function $Y1 = x^2 - 3$ in the standard viewing window.
2. Enter the function $Y2 = 1 \div Y1$. In your workbook, write a rule for $Y2$ in terms of x . Using your graphics calculator, plot both $Y1$ and $Y2$ in the standard viewing window. Why might this viewing window be unsuitable for visualising the behaviour of $Y2$?
3. Change the viewing window to $[-5,5]$ by $[-3,3]$ and replot the functions $Y1$ and $Y2$. What do the two apparently vertical lines in this viewing window represent?
4. Enter $Y3 = 1$ and $Y4 = -1$. In the context of the reciprocal function in $Y2$, what is significant about these horizontal lines? Where does the graph of $Y1$ intersect the function $Y2$?
5. Turn off the functions in $Y2$ to $Y4$, and enter a new rule $Y5 = Y1^2$. $Y5$ represents the square of $Y1$. In your workbook, write a rule for $Y6$ in terms of x . What type of function is $Y5$?
6. Locate the minimum value of the function $Y1$. What will be the corresponding value of the squared function $Y5$ in this case? Use this information to help find an appropriate viewing window to view both $Y1$ and $Y5$.
7. For what values of x is $Y5 < Y1$?

Note: Sometimes functions are defined in terms of other function (for example reciprocal functions). It is useful to be able to write the rule succinctly (for example $1/Y1$), so that students are encouraged to think of functions as objects that can be manipulated, as well as processes that can be applied to numbers. This activity also explores connections between the original function and the transformed function.

9. Modelling Beats (MM3/4)

(KP:2.2;4.3;5.1;5.3)

Beats are a phenomenon in music when a person hears two 'pure' notes simultaneously that differ in frequency by a small amount. The beat is perceived audibly as a low frequency sound. The beat frequency is one half the difference of the two frequencies. This phenomena is due to constructive and destructive interference. It is used by musicians to tune instruments.



1. To model beats, graph two sine functions, each with amplitude 1, but with different frequencies of 8 and 10 cycles per second. View them in the window $[0,1]$ by $[-2,2]$
2. Enter and plot a function, Y_3 , which is the sum of the sine functions in Question 1. Note that where the graphs are in phase there is constructive interference, and where they are out of phase there is destructive interference.
3. Find two functions, Y_4 and Y_5 , which envelop the graph of Y_3 .

(Note - In general, the envelope of the sum of 2 sine curves with amp = A and freq. f_1 and f_2 where $f_2 > f_1$ is a cosine curve with amplitude $2A$ and beat frequency $(f_2 - f_1)/2$.)

Note: This task provides a context for examining a phenomenon in which the addition of ordinates technique is a key feature. In addition, the task introduces the notion of an 'envelope' curve.

10. The Last Lap (MM&SM3/4)

(KP:2.2;4.3;5.1;5.3;7.1)

In the 1994 VCE Specialist Mathematics Problem Solving CAT, one problem asked students to consider the relative performances of two runners Carli and Jade, whose distance-time functions for the last lap (400 metres) of a 3000 metre race could be modelled by

Jade: $Y_1 = x_j(t) = -0.01t^2 + 6t - 16$

Carli: $Y_2 = x_c(t) = -4 \cos(t/4) + 5t + 4$

One question asked students to establish how many times the lead changed between Carli and Jade in the last lap of the race.



1. Enter appropriate values for a viewing window to view the distance functions for Jade and Carli (think of typical performance times for this distance).

Xmin = _____ Xmax = _____ Xscl = _____ Ymin = _____ Ymax = _____ Yscl = _____
2. Plot Y_1 and Y_2 and comment on the difficulty of determining all points of intersection visually in this viewing window.
3. Construct a function $Y_3 = 400$ to mark the finishing line. Who actually wins the race?
4. Using the Trace feature (or otherwise), estimate the maximum distance between Carli and Jade in the last lap. Be careful to examine where each runner is at $t = 0$, as well as the maximum distance apart after they have both entered the bell lap.
5. Construct a new function in Y_4 for the number of metres that Carli is in front of Jade after Carli begins the last lap. Using your answer to question 4, create an appropriate viewing window in which to examine how the lead changes in the last lap.

Note: This task provides a context for examining how the difference of two functions and its graph can be used to facilitate problem analysis.

11. Function Smorgasbord (MM&SM3/4)

(KP:2.2;4.6;5.3)

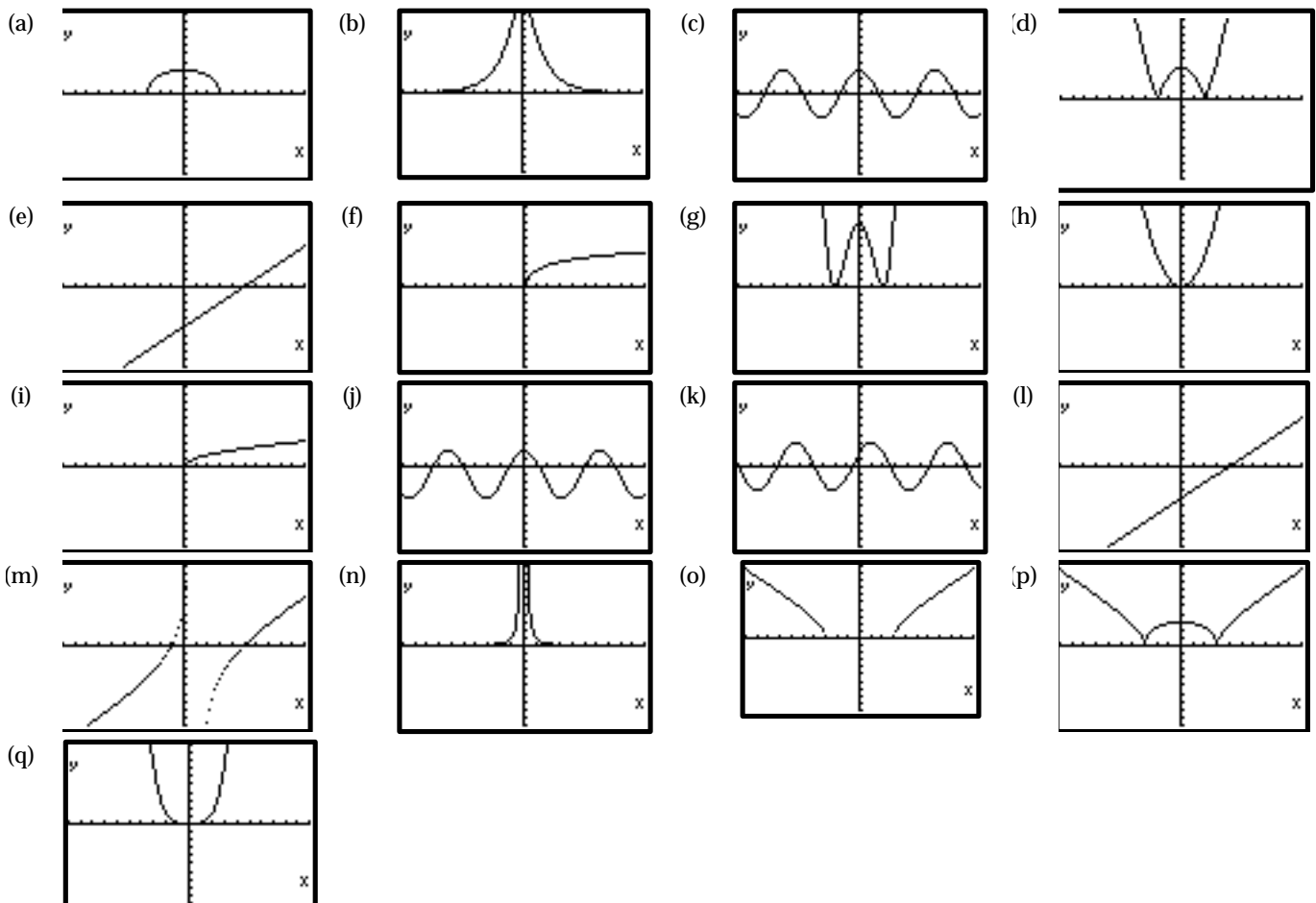
Try to match the function rule with its graph. The windows shown have dimensions $[-10,10]$ by $[-10,10]$.

- Class to be organised (at random) into groups of three, and assign roles as scribe, checker, and researcher.
- Match the function with the corresponding graph (insert graph 'letter' next to symbolic rule).
- First attempt to get agreement on a match, and have the scribe record any answers that are not in dispute.
- For any matches that are in dispute, mark with an asterisk (these will be checked with the aid of the calculators in the last ten minutes)

Symbolic Representation

- | | | | | | |
|--|-----|--------------------------------------|-----|---|-----|
| 1. $y = \sqrt{x}$ | () | 2. $y = \ln x + 2$ | () | 3. $y = \begin{cases} 4 - x^2 & \text{for } -2 \leq x \leq 2 \\ x^2 - 4 & \text{for } x < -2 \text{ \& } x > 2 \end{cases}$ | () |
| 4. $y = 0.5(x - 2)^2(x + 2)^2$ | () | 5. $y = \frac{1}{10}x^4$ | () | 6. $y = x^2$ | () |
| 7. $y = \begin{cases} 2^{x+4} & \text{for } x < 0 \\ 2^{4-x} & \text{for } x > 0 \end{cases}$ | () | 8. $y = \sqrt{9 - x^2}$ | () | 9. $y = \sqrt{x^2 - 9}$ | () |
| 10. $y = 3 \cos(x - 1)$ | () | 11. $y = 3 \cos(-x)$ | () | 12. $y = x - 4$ | () |
| 13. $y = \begin{cases} \sqrt{9 - x^2} & \text{for } -3 < x < 3 \\ \sqrt{x^2 - 9} & \text{for } x < -3 \text{ \& } x > 3 \end{cases}$ | () | 14. $y = \frac{1}{x^2}$ | () | 15. $y = 3 \cos x - 1$ | () |
| 16. $y = \frac{x^2 - 4x - 5}{x - 1}$ | () | 17. $y = \frac{x^2 - 4x - 5}{x + 1}$ | () | | |

Graphical Representation

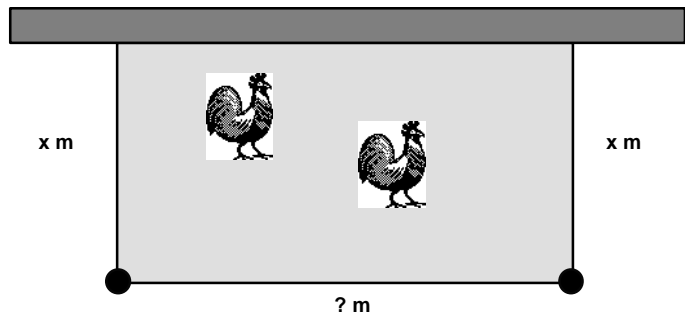


Note: The purpose of this task is to assist students, working collaboratively, to match the symbolic and graphical representation of functions. Such a task can also be varied to concentrate on one function family. This task also encourages students to work cooperatively in groups, and justify a position.

12. Extending the Fencing Problem (MM1/2)

(KP:2.2;5.3;7.1;9.1;9.3;9.4)

Imagine that you have to construct a 'run' for your friend's chickens. You have 20 metres of wire netting with which to construct 3 sides of this rectangular 'run' (see diagram right), and the 4th side will be the fence line of the property. The depth of the run is x metres.



- Investigate different possible dimensions for this run, and construct a table of the dimensions and area $A \text{ m}^2$ enclosed by each of these possibilities. Make an initial estimate of the maximum area that the netting might enclose.

Depth x (m)	2				
Width (m)	16				
Area A (m^2)	32				

- Write an expression for the width of the run in terms of x , and hence construct an expression for A in terms of x .
- On your graphics calculator, draw a graph of how the area of the run depends on x , and use the graph to find the maximum area. Verify your answer by an analytic method.
- Using a variation of the above function, find the maximum area that could be enclosed if you had the following amounts of wire netting available:
 - 30 metres;
 - 40 metres;
 - 50 metres;
 - 60 metres.
- Construct a table of your answers to question 4.

Wire Netting available (m)	20	30	40	50	60
Depth x (m) for maximum area					
Width (m) for maximum area					
Maximum Area A (m^2)					

- Plot the graphs of the Area functions (in Y1 to Y5) for each of the wire netting lengths listed in the table above. Find a suitable viewing window to view the maximum values for each scenario.
- Plot each of the maximum points. Using the table, the graph, or by other means, find a relationship between the maximum areas and the corresponding depths.
- Create a rule for the maximum area that can be enclosed for a given length of wire netting. Verify your answer by an analytic method.

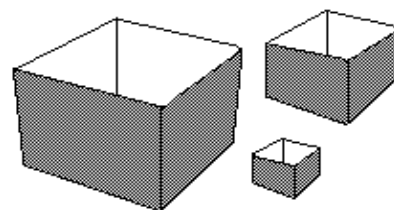
Note: This is an old problem in a new guise. Using the graphics calculator, students solve the problem of maximum area, but verify their answer analytically. A feature of the extension is to use a curve-fitting approach to generalise their solution to the original problem.

13. Extending the Open Box Problem (MM3/4)

(KP:2.2;5.3;7.1;9.1;9.3;9.4)

An open box is constructed by cutting equal size squares from the corners of a rectangular sheet of cardboard and folding up the sides. Your initial task is to make the cuts so that the box has maximum volume.

Then you will experiment with various sized sheets of cardboard, in which the breadth is always twice the width, to see how the maximum volume varies with the width.



1. Start with a cardboard sheet with dimensions 5 cm by 10 cm. Construct a function which will give the volume V (cm^3) of the box as function of the length x (cm) of the side of the square cut.
2. Use the graph of the function in question 1 to approximate the maximum volume to the nearest cubic mm. Also give the approximate length of the side of the square cut which produces that volume.
3. In order to investigate what effect changes in the dimensions have, repeat questions 1 and 2 for cardboard sheets which are 10 cm by 20 cm, 15 cm by 30 cm, and 20 cm by 40 cm. Collect your results for all 4 in a table that displays breadth of the sheet, width of the sheet, approximate maximum volume and corresponding length of square cut.
4. How does the size of the cut in each case seem to compare to the width of the cardboard sheet? State the comparison verbally and as a mathematical statement.
5. Experiment to see if the maximum volume is directly proportional to the width, the square of the width, or the cube of the width of the cardboard sheet. If you find a relationship, express it verbally and as a mathematical statement.
6. Justify your answer to question 5 using a calculus approach.

Note: Like the fencing problem, this task, extends a standard maximum/minimum problem to consider a possible generalisation. Using the graphics calculator, students solve the problem of maximum volume, and apply a curve-fitting approach to generalise their solution.

14. Looking at Derivatives & Integrals (MM1-4)

(KP:2.2;4.3;4.4;5.1;7.1; 7.2)

1. Enter and plot the function $f(x) = 0.1x(x - 6)(x + 6)$ in the viewing window $[-10,10]$ by $[-10,10]$.
2. Locate the local maximum of this function. If possible, verify that the derivative is zero at this point by drawing a tangent at this point.
3. On a piece of paper, draw a quick sketch of the graph of $f(x)$. With a different coloured pen, mark tangents to the curve of $f(x)$ at unit intervals in the domain $[-10,10]$. From these marks, superimpose a rough sketch of the gradient function $f'(x)$.
4. Use your graphics calculator to draw some tangents to the curve near the local maximum. What does this suggest about the gradient function $f'(x)$ near a local maximum?
5. Define the derivative function $f'(x)$ in Y2 and define the second derivative function $f''(x)$ in Y3.
6. Plot the functions of $f(x)$, $f'(x)$, and $f''(x)$ in turn.
 - (a) Pause the plot of $f'(x)$ near the x coordinate that corresponds to the local maximum of $f(x)$. What do you notice about the value of $f'(x)$ here?
 - (b) Pause the plot of $f'(x)$ near where $f(x)$ intersects the origin. What do you notice about the value of $f'(x)$ here?
 - (c) Pause the plot of $f'(x)$ near the x coordinate that corresponds to the local minimum of $f(x)$. What do you notice about the value of $f'(x)$ here?
 - (d) Pause the plot of $f''(x)$ near where $f(x)$ intersects the origin. What do you notice about the value of $f''(x)$ here? What does this tell you about the behaviour of $f(x)$ near the origin?
7. Adjust the Xmin to -12.6 and the Xmax to 12.6 (this provides better x -increments for tracing along the curves). Check the value of the new x -increment by tracing along the curve.

Note: the x -increment for the TRACE cursor is calculated by the rule

$$x\text{-increment} = \frac{X \text{ max} - X \text{ min}}{\text{No. of pixel intervals horizontally}} = \frac{X \text{ max} - X \text{ min}}{126}$$
8. Find the integral of $f(x)$ between -6 and 0:
 - (a) interactively (that is, via the Graph Screen);
 - (b) on the Home Screen.

What does this integral represent?
9. Repeat question 8 to find the integral of $f(x)$ between 0 and 6.
10. Find the integral of $f(x)$ between -6 and 6 interactively. What is the area enclosed by the graph of $y = f(x)$ and the x -axis between $x = -6$ and $x = 6$.

Note: In this task, the power of the graphics calculator to numerically calculate the derivative is used to explore connections between a function and its gradient function. Also, the numeric integration feature is used to explore connections between the area under a curve and the numeric value of the integral. The technique of 'freezing' the plotting to ask a critical question is also illustrated in this task.

15. Areas Under Curves (MM&SM3/4)

(KP 10.1)

The program below:

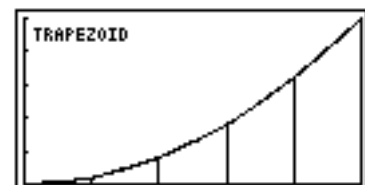
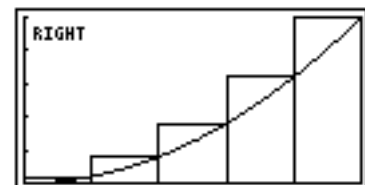
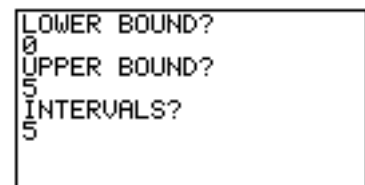
- assumes a suitable function in Y1 and created an appropriate viewing window (note that if the function is strictly increasing, the left/right areas correspond to the lower/upper areas respectively);
- prompts for the left (A) and right (B) x-values, and for the number of intervals (N);
- draws the region bounded by the left rectangles and calculates its area using the values of A, B, and N;
- repeats the process with right rectangles and the midpoint and trapezoid methods.

1. Investigate for relationships between the four methods. How do they compare for accuracy?

```

Filename: INTAREA
Cls.
ClrText.
"LOWER BOUND"?A.
"UPPER BOUND"?B.
"INTERVALS"?N.
(B-A)N:H.
H2:D.
OJ.
OM.
OR.
DrawGraph.
OJ.
AZ.
ZX.
Lbl 1.
P1+LJ.
P/L-Orange:Plot Z,0:Line:Plot
Z,H,0:Line.
Z+HJ.
ZX.
J+1J.
If J-N:Then Text 5,5,"LEFT"»
Else Goto 1.
IfEnd.
HLJ.
ClrText.
Locate 1,2,"LEFT SUM=".
Locate 10,2,L»
Cls.
DrawGraph.
OJ.
A+HJ.
ZX.
Lbl 2.
P1+RJR.
P1Q.
Plot Z-H,0:Line:Plot Z-H,Q:Line:Plot
Z,Q:Line:Plot Z,0:Line.
Z+HJ.
ZX.
J+1J.
If J-N:Then Text 5,5,"RIGHT"»
Else Goto 2.
IfEnd.
HRJR.
ClrText.
Locate 1,2,"LEFT SUM=".
Locate 10,2,L»
Locate 1,3,"RIGHT SUM=".
Locate 11,3,R»
Cls.
DrawGraph.
OJ.
A+DZ.
ZX.
Lbl 3.
P1+M.
P1Q.
Plot Z-D,0:Line:Plot Z-D,Q:Line:Plot
Z+D,Q:Line.
Z+HJ.
ZX.
J+1J.
If J-N:Then Text 5,5,"MIDPOINT"»
Else Goto 3.
IfEnd.
HM.
ClrText.
Locate 1,2,"LEFT SUM=".
Locate 10,2,L»
Locate 1,3,"RIGHT SUM=".
Locate 11,3,R»
Locate 1,4,"MIDPT SUM=".
Locate 11,4,M»
Cls.
DrawGraph.
OJT.
A+HJ.
ZP.
Lbl 4.
P1X.
2EP1+T.
P+HJ.
If B-P>.001:Then Goto 4.
IfEnd.
AX.
P1F.
Lbl 5.
ZX.
P1Q.
Plot Z-H,F:Line:Plot Z,Q:Line:Plot
Z,0:Line.
Z+HJ.
QF.
ZX.
If B-X<0:Then Text 5,5,"TRAPEZOID"»
Else Goto 5.
IfEnd.
AJ.
P1V.
BJ.
P1W.
(H2)(T+V+W)I.
(P1,A,B)I.
ClrText.
Locate 1,2,"LEFT SUM=":Locate 14,2,L.
Locate 1,3,"RIGHT SUM=":Locate 14,3,R.
Locate 1,4,"MIDPT SUM=":Locate 14,4,M.
Locate 1,5,"TRAP SUM=":Locate 14,5,T.
Locate 1,6,"BEST EST=":Locate 14,6,I»

```



Method	Sum	Disp
LEFT SUM	30	-
RIGHT SUM	55	-
MIDPT SUM	41.25	-
TRAP SUM	42.5	-
BEST EST	41.66666	-

Note: The program above illustrates how numeric methods can be used to calculate areas under a curve. The connections between integration and antidifferentiation can also be explored.

16. Halving a Region (MM3/4)

(KP:2.2,7.4;7.5)

1. Enter and plot the function $Y1 = (x - 1)^2(3 - x)$ in the window $[0,4]$ by $[-.2,2]$
2. Where will a vertical line cut the region bounded by the curve and the x-axis into equal areas? Record an initial estimate, and then use the numeric integral feature to check (and improve if necessary) your estimate.

(One method of achieving this is to find an approximate value for $A = \int_1^3 (x - 1)^2(3 - x) dx$, and then try to find a

value for b such that $\frac{A}{2} = \int_1^b (x - 1)^2(3 - x) dx$. A similar procedure can be used to divide the region into equal thirds.)

Extension 1

A particle is moving with a velocity at time t , described by the rule $x'(t) = t(t - 2)^2(t - 5)^2$, where $0 \leq t \leq 5$. Find the time at which the particle has travelled half of the total distance travelled in the 5 seconds.

Extension 2

Enter and plot the functions $Y1 = 9$ and $Y2 = x^2$. Divide the region enclosed by the line and the parabola into equal halves with a horizontal line.

Note: The appearance of a curve sometimes suggests a symmetry that is not present. In the above task, the region might appear symmetrical, leading students to think that the midpoint of the x-values provides the position for a vertical line to halve the region. The task enables students to examine this assumption and then employ the numeric integration feature to solve the problem.

17. The Days are Getting Longer (MM3/4)

(KP:2.2;4.3;4.4;5.3;7.7;11.1)

The number (y hours) of daylight in Melbourne x days after the longest day of the previous year can be modelled by the rule

$$y = 12.125 + 2.625 \cos \frac{2}{365} x$$



1. Find a value of x for which the length of the day is increasing fastest, and find out how much the length of the day is changing at this time.
2. This model is based on the number of days elapsed since the longest day. If you wanted it to use the number of days elapsed since the start of 1996, how would the function change? (The longest day of the year in 1995 was the 22nd of December).
3. Using the information from your answer to question 2, predict the dates and number of daylight hours of the following significant events:
 - (a) Summer Solstice;
 - (b) Winter Solstice;
 - (c) Spring Equinox;
 - (d) Autumn Equinox.

Check your answers against the dates listed for the above events in the 1996 calendar.

Note: This task provides a context for examining the derivative function and possibly the second derivative function. Often, the graphs of a function and its derivative function are not easy to view simultaneously due to scaling problems. In the task above, the graph of the derivative function can be explored independently from the graph of the original function to tackle the problem at hand. Alternatively, a table of values of the function and its derivative function can be used to locate the points of interest. The second derivative may be a useful additional feature in determining the key points.

18. Introducing Euler's Number, e (MM3/4)

(KP:2.2;4.3;4.4;5.1)

In this task, we explore derivatives of exponential functions, that is, functions of the type $f(x) = b^x$, $b > 0$ and x any real number.

- In Y1, define the function $f(x) = 5^x$.
 - Calculate $\frac{f(2.001) - f(2)}{0.001}$. This is an estimate of the derivative of 5^x at $x = 2$, that is, $f'(2)$.
Calculate $[\frac{f(2.001) - f(2)}{0.001}] / f(2)$. This is an estimate of $\frac{f'(2)}{f(2)}$.
 - Use the same method as (a) to estimate $\frac{f'(1)}{f(1)}$.
 - Use the same method as (a) to estimate $\frac{f'(-2)}{f(-2)}$.
 - The results of (a) – (c) above suggest something about the ratio $\frac{f'(x)}{f(x)}$. Write in words what you believe is true.
- We will now use the graphics calculator to check the results of question 1.
 - Enter the functions $Y1 = f(x) = 5^x$ and $Y2 = f'(x)$ and plot them. Can you say anything about the general shape of the function and its derivative?
 - Now enter the function $Y3 = Y2/Y1$. By tracing along the graph of $Y3$, check the result of question 1(d).
- In Y1, change the rule to $f(x) = b^x$. Repeat question 2 for the function $f(x) = b^x$ for the values of b in the following table, and complete the table:

b	$\frac{f'(x)}{f(x)}$
0.5	
1.0	
1.5	
2.0	
2.5	
3.0	

- Use the results of question 3 to help you find a base b for which the ratio $\frac{f'(x)}{f(x)} = 1$. Try to find this number correct to two decimal places.
- The number you have found in question 4 is Euler's number e : the reason it is so important is that if we let $b=e$, the Euler number, then $f(x) = b^x = e^x$ has derivative $f'(x) = f(x)$ [because $\frac{f'(x)}{f(x)} = 1$]. Verify this by repeating question 2 for the function $f(x) = e^x$.
- The inverse of the function $f(x) = e^x$ is $g(x) = \log_e x$ (sometimes written **In x** meaning the natural log of x). Can you find a relationship between the two columns of numbers in your table in question 3 using either e^x or $\log_e x$? If $f(x) = b^x$, can you write a rule for $f'(x)$?

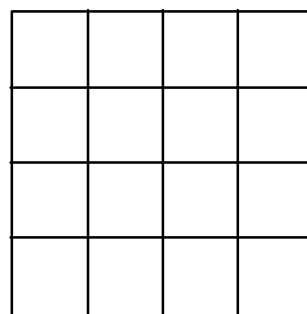
Note: This task uses derivatives of exponential functions to introduce a special number, e .

19. Raindrops Keep Falling On My Head (MM1/2)

A random number can be generated on a spreadsheet using the **Ran#** function. The **Ran#** function will return a random value between 0 and 1. The following formulas can be used to produce more useful random numbers. The **Int** function converts a decimal value to the next lowest integer. **Int** and **Ran#** together can be used to produce random integer values.

- $Ran\#$ Random number between 0 and 0.999999999
- $6 * Ran\#$ Random number between 0 and 5.999999999
- $6 * Ran\# + 1$ Random number between 1 and 6.999999999
- $Int(6 * Ran\# + 1)$ Random integer between 1 and 6 inclusive
- $Int((B - A + 1) * Ran\# + A)$ Random integer between A and B inclusive

1. Four raindrops land on the patio. Using a copy of the figure shown right, show where they might land. Then show another 12 raindrops on your diagram. Make a note of how many blank squares are left on your grid. Also, record the largest number of raindrops on any one square.
2. Now use the random number feature of your graphics calculator to simulate where on the grid the first 16 drops of rain might fall. Again record how many blank squares there are and the largest number of the 16 raindrops to fall in any one square.
3. Compare your class results for questions 1 and 2. Try to explain anything surprising.

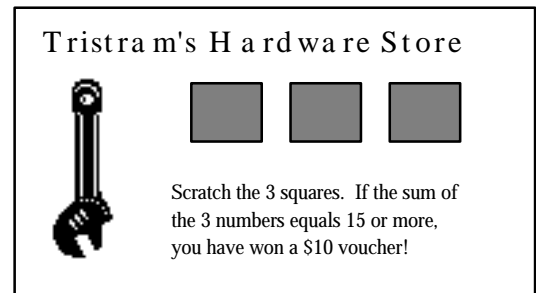


Note: This task introduces the random number generation feature of a graphics calculator and enables students to perform simple simulations. In addition, students' own conceptions about random behaviour are challenged.

20. Tristram's Hardware (MM1/2)

(KP:10.1)

As a marketing gimmick, Tristram's Hardware Store is sending the 'scratch' card below to all its valued customers (500 households in total). The card assures them that if the sum of the three numbers equals or exceeds 15, they will receive a \$10 voucher for goods from the store. Each of the three squares are randomly assigned the values 1 to 9 inclusive.



- Write a program which simulates this marketing gimmick, and calculates the proportion of 'winning' scratch-cards. Run the program 10 times, and record the value of this proportion in each trial.

Program Details

Program Instructions	Explanation of command	Key sequence to access commands
0 † W ·	(initialise 'win' and 'ticket no.')	→ - STO
0 † N ·		
Lbl 1·	(insert a label for goto statement)	Lbl - SHIFT•PRGM•F3•F1
1 + I nt (9 x Ran #) † A ·	(integer b/w 1 and 9 in square a of scratchie)	Int - OPTN•F6•F4•F2
1 + I nt (9 x Ran #) † B ·	(integer b/w 1 and 9 in square b of scratchie)	Ran# - OPTN•F6•F3•F4
1 + I nt (9 x Ran #) † C ·	(integer b/w 1 and 9 in square c of scratchie)	⇒ - SHIFT•PRGM•F3•F3
A + B + C 15 † W + 1 † W ·	(if sum 15 then increment 'wins')	≥ - SHIFT•PRGM•F6•F3•F5
N + 1 † N ·	(increment 'ticket no.')	< - SHIFT•PRGM•F6•F3•F4
N < 500 † Goto 1·	(if ticket no. <500, go to Label 1)	Goto - SHIFT•PRGM•F3•F2
" PROP. WINS "·	(calculate and print p, the prop. of wins (if complete))	" - ALPHA•F2
W † N † P ·		

- Tristram believes that he will need to budget no more than \$2000 for winning tickets. Is this a reasonable estimate of the prize costs? Explain your thinking.

Note: This task illustrates how writing a small program can extend the functionality of the graphics calculator. In the example given, program looping is used to simulate the process.

21. Quality Control with Balloons (MM3/4)

(KP:2.2;5.1;5.3;7.1)

A balloon manufacturer knows that he will lose customers if there are more than two defective balloons in his packet of 20. Assume packets are filled at random with good and defective balloons. Let x be the probability that a balloon is defective when it is manufactured.



1. Write an expression in terms of x for the Probability, $P(x)$, that in a batch of twenty balloons, less than 3 balloons are defective (that is, either 0 or 1 or 2 balloons are defective).
2. Enter and plot this function on your graphics calculator. Use the fact that x and $P(x)$ are probabilities to help set an appropriate viewing window.
3. Find $P(0.1)$ and state its meaning.
4. What percentage of balloons must be good if the probability of a customer getting less than 3 defective balloons in a bag of 20 balloons is at least 90%? (Using the 'jump-to' feature may be useful here.)
5. How would your answer to questions 3 and 4 change if
 - (i) at most 1 balloon in a bag of 20 was allowed to be defective;
 - (ii) no balloons in a bag of 20 were allowed to be defective?
6. What might be the implications of these changes for the balloon producers?

Note: Real problems involving probability can often be quite difficult to tackle analytically. The task here, involving the binomial distribution, uses the 'jump-to' feature to solve an otherwise difficult problem.

22. Integral Areas and Volumes (SM3/4)

(KP:2.2;4.2;7.4;7.5)

1. Shade the area bounded by the graphs whose equations are $y = \cos x$, $y = \cos 2x + 2$ and the lines $x = 0$ and $x = \pi/4$. Then calculate the area of the bounded region.
2. (a) Calculate the volume of the sphere generated when the circle with equation $x^2 + y^2 = 9$ is rotated about the x-axis. Check your answer by applying the known formula $V = 4/3 \pi r^3$ for the volume of a sphere.

(b) Using the rule for a circle of radius r centred on the origin work out analytically how this volume formula has been derived.

Note: This task uses the numeric integration feature to tackle standard applications of integral calculus such as the area between two curves and solids of revolution. Also illustrated is the advantage of the shading feature in providing a visual cue.

23. The Normal Curve (MM3/4)

(KP:2.2;4.3;7.4;7.5)

The standard normal curve can be constructed by performing transformations of the function with rule $y = e^x$.

1.
 - (a) Enter the rule $Y1 = e^x$
 - (b) Enter the rule $Y2 = Y1^2$
 - (c) Enter the rule $Y3 = \sqrt{Y2}$
 - (d) Enter the rule $Y4 = \frac{1}{Y3}$
 - (e) Enter the rule $Y5 = \frac{Y4}{\sqrt{2p}}$. Then plot the graphs of functions Y1 to Y5
2. Use algebra to find a rule for the function Y5 in terms of x.
3. The function in Y5 represents the rule for the standard normal probability distribution function. Disable all other plots and then plot only Y5.
 - (a) Show that the area under the graph between $x = -1$ and $x = 1$ is approximately 0.68. What special meaning does this area represent?
 - (b) Show that the area under the graph between $x = -2$ and $x = 2$ is approximately 0.95. What special meaning does this area represent?
 - (c) Show that the area under the graph between $x = -3$ and $x = 3$ is approximately 0.997. What special meaning does this area represent?

Note: The standard normal curve can be built up from the exponential curve by a series of transformations. This task follows one possible sequence including a consideration of the appropriate viewing window dimensions in which to work with the graph. Linking the area under the curve to some standard normal probabilities is also considered.

24. The Cooling Law Problem(MM&SM3/4)

(KP:9.1-5)

A temperature probe is placed in a saucepan filled with hot water, and the temperature is recorded at 30 second intervals for 5 minutes. The ambient room temperature is 18.5° . Plot the data and try to fit a rule that will successfully model the temperature of the water after 30 minutes, and justify your choice.



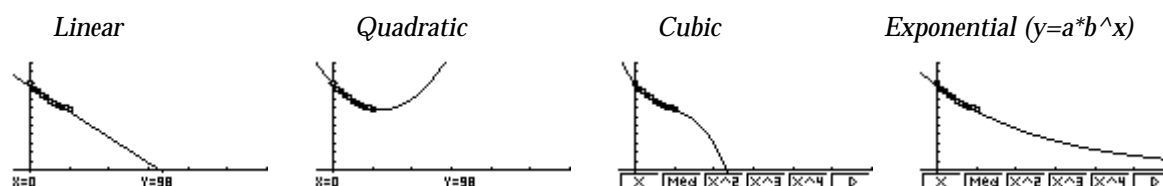
Time (t)	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Temperature (T)	98	91.5	87.5	84	80.5	78	75.5	73	71	69.5	68

- Enter the data above into the graphics calculator, and try to fit a linear rule to the data. How well does the line fit the data?
- Look at the difference between successive temperature readings in the table above. Why is a linear rule not likely to fit the data well?
- Try to fit a quadratic rule of the form $T = a(t - h)^2 + k$ to the data by eye. Use the point (5,68) as the vertex of the parabola. Experiment with different functions until you have one which fits the data well.
- What does your quadratic model predict as the temperature after 30 minutes? Comment on the usefulness of the quadratic model in light of this finding.
- Another possible model is an exponential one. If such a relationship exists, the ratio between successive terms should be approximately constant. Calculate these ratios and place them in the table below. Does this support the use of an exponential model?

Time (t)	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Temperature (T)	98	91.5	87.5	84	80.5	78	75.5	73	71	69.5	68
Temp(t+0.5)/Temp(t)											

- Apply the standard exponential regression model ($y = a \cdot b^x$). What temperature does this predict after 30 minutes? What is worrying about this prediction?

This raises a worrying aspect of fitting regression models to data. An exponential relationship seems plausible, and yet the suggested one has limited predictive power. Indeed other regression models fit the data well, but are not helpful as predictors for the situation (see below).



Now try to fit a model by creating a new list of 'excess' temperatures ($T - 18.5$), and doing an exponential regression on ($T - 18.5$) versus t .

- Enter the 'excess' temperature values into List 3, and then reconfigure the scatter plot so that you are plotting List 3 (excess temperature) against List 1 (time).
- Now reapply the exponential regression model using the excess temperatures. What temperature does this predict after 30 minutes? How does this compare with the earlier models?

Note: This task involves collecting real data, and attempting to fit a functional model to this data. When the model is to be used for prediction purposes, however, an understanding of the situation is essential in determining the most appropriate model. In the above task, any regression fit will be good for the data given, but only one of the models shown allows sensible extrapolation.

25. Was it a six? (SM3/4)

(KP:4.8;5.3)

In an celebrity match, Zoe Goss smashes a wayward ball from Shane Warne high in the air towards the boundary with an initial speed (u) of 32 m/s at an angle of 56° from the horizontal. The nearest boundary fence is 98.5 metres from the point of impact, and is 1.2 metres high. The parametric rules which model its flight are:

- $x(t) = ut \cos \alpha$
- $y(t) = 1 + ut \sin \alpha - \frac{1}{2} gt^2$

1. Enter appropriate viewing window parameters to examine the flight of the ball after it leaves the bat, and then re-plot the graph.

Tmin = _____ Tmax = _____ Tpitch = _____

Xmin = _____ Xmax = _____ Xscl = _____ Ymin = _____ Ymax = _____ Yscl = _____

2. Was it a six, that is, does the shot clear the boundary fence?

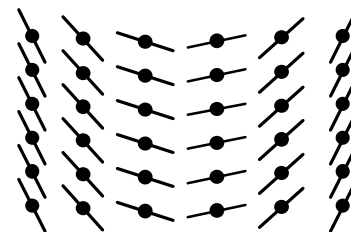
3. If it does clear the fence, by how much does it clear it. If it does not clear the fence, suggest what adjustments Zoe might have made that would have resulted in the ball 'just clearing' the fence.

Note: Some problem solving situations lend themselves to the use of parametric rules. In the example above, the ability of the graphics calculator to simulate the position of a cricket ball whose coordinates are defined parametrically is exploited.

26. Direction Fields (MM1-4)

(KP:10.1)

Imagine that a function is defined so that $dy/dx = x$. This can be simulated in the class, by positioning students in rows to simulate the cartesian plane, and then have them hold their arms outstretched (horizontally) at a gradient approximately equal to the x-coordinate at which they have been assigned (see illustration right). Then have a student, holding there arms outstretched, attempt to walk a path from left to right through the plane.



The program listing below will illustrate a similar principle by creating 'direction fields'. In the example screens below, note that the program

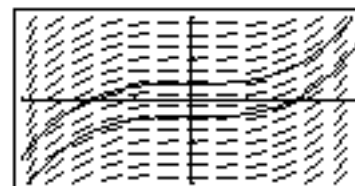
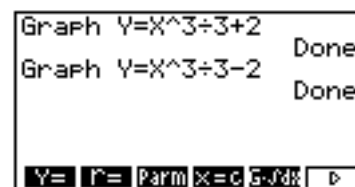
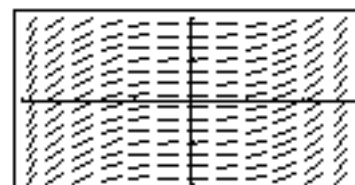
- assumes you have entered a suitable gradient function in Y1 (say $Y1 = x^2$) and created an appropriate viewing window (some experimentation may be necessary to achieve the desired effect);
- constructs a direction field.

Once constructed, students should be asked to draw integral functions using that conform to the contours of the direction field. From RUN mode, use the keys **SHIFT F4 (SKTCH)** **F5 (GRAPH)** **F1 (Y=)** to draw the graphs.

```

ClrText·
Cls·
AxesOff: GridOff: FuncOff: Label Off·
Text 1, 1, "DIRECTION FIELDS"·
Text 8, 1, "===== "·
Text 22, 1, "CONSTRUCTS A DIRECTION FIELD"·
Text 29, 1, "IN THE CURRENT VIEWING WINDOW"·
Text 36, 1, "(ASSUMES THAT A SUITABLE"·
Text 43, 1, "GRADIENT FUNCTION HAS"·
Text 50, 1, "BEEN ENTERED IN Y1) "»
FuncOn: AxesOn·
ClrText·
Locate 1, 2, "CONTINUE?"·
Locate 1, 3, "----- "·
Locate 1, 4, "(1) NO"·
Locate 1, 5, "(2) YES"·
Locate 1, 6, "----- "·
Locate 6, 7, "(CHOOSE 1, 2)": ?↑V·
If V=1: Then Goto 5: IfEnd·
If V=2: Then Goto 6: IfEnd·
If (V-1) Or (V-2): Then Goto 3: IfEnd·
Lbl 6·
14↑L·
18↑W·
(Ymax- Ymi n) ÷L↑V·
(Xmax- Xmi n) ÷W↑H·
Cls·
FuncOff·
0↑R·
Ymi n+V÷2↑Y·
Lbl 1·
R+1↑R·
0↑C·
Xmi n+H÷2↑X·

Lbl 2·
C+1↑C·
P1↑M·
•MEH÷2+Y↑S·
MEH÷2+Y↑T·
X- H÷2↑P·
X+H÷2↑Q·
If Abs (T- S) > V·
Then Goto 3·
IfEnd·
Lbl 4·
Y↑F·
Plot P, S: Plot Q, T: Li ne·
F↑Y·
X+H↑X·
If C<W·
Then Goto 2·
IfEnd·
Y+V↑Y·
If R<L·
Then Goto 1·
IfEnd·
ClrText·
Stop·
Lbl 3·
Y+V÷2↑T·
Y- V÷2↑S·
(T- Y) ÷M+X↑Q·
(S- Y) ÷M+X↑P·
Goto 4·
·
Lbl 5·
ClrText·
Stop
    
```



Note: Direction fields provide a wonderful way to illustrate that there are a family of integral curves for any given gradient function. The significance of the 'constant of integration' can be clearly see27. Home Screen Calculations (GM/FM)

27. Home Screen Calculations

Many simple expressions are built into the graphics calculator and can be accessed directly from the keypad or from the appropriate menu. The size of the display area gives a graphics calculator a distinct advantage over a scientific calculator for arithmetic calculations.

Arithmetic calculations

1. Evaluate $2/3 + 1/4$ and write your answer as a fraction.
2. Calculate ${}^5\sqrt{5.7}$

Trigonometric calculations

3. Evaluate $\sin 30^\circ$, and then convert it to a fraction.
4. Solve $\sin x = 4/5$ and give your answer in degrees, minutes, and seconds.

Permutations and combinations

5. Evaluate ${}^{10}P_4$ and ${}^{10}C_4$, and verify that ${}^{10}P_4 = 4! \times {}^{10}C_4$.

For more complex expressions, such as a particular binomial probability, the concatenation feature and variable storage feature of the graphics calculator can be used to advantage.

Binomial probabilities

To answer the question 'What is the probability of three sixes in 25 rolls of a fair die?', enter the following expression:

$$25 \rightarrow N: 3 \rightarrow R: 1/6 \rightarrow P: (1-P) \rightarrow Q: N C R * (P^R) * (Q^{(N-R)})$$

Modify this expression to answer the following questions.

6. (a) What is the probability of 10 heads in 25 tosses of a fair coin?
(b) What is the probability of five colour-blind males in a teaching staff of 50 if 10% of all males are colour-blind?

Recursion

- Enter your age.
- Enter **1 + 1/Ans**.
- Press the **EXE** key 10 or more times.
- Compare your final result with a neighbour. Can you explain what has happened?

Note: The purpose of these items is to familiarise the user with a range of options for using home screen calculations. Also shown are some tasks that might make use of such features.

28. Storing Results in Lists (GM/FM)

The use of lists, containing patterned or random data, is an important feature. Storing a series of results in a list is a very useful tool for future analysis.

Entering patterned data into lists

To enter, for example, the squares of the first ten whole numbers into a list, there are at least four distinct methods.

Method 1. *The tedious way:*

- Enter the list and type in the numbers

Method 2. *The half-tedious way:*

- Enter the following expressions from the Home Screen (RUN mode):

$0 \rightarrow N$

$N+1 \rightarrow N: N^2 \rightarrow \text{List 1}(N)$

- Press the EXE key 10 times and check the contents of List 1

Method 3. *The sequence way:*

- Enter the rule as a sequence directly into the list (RUN mode):

$\text{List 1} = \text{Seq}(N^2, N, 1, 10, 1)$

- Check the contents of List 1

Method 4. *The simplest way:*

- Enter {1, 2, 3, ...} into List 1 and set List 2 as a function of List 1 in RUN mode:

$\text{List 2} = \text{List 1}^2$

- Check the contents of List 2

- (a) Using Methods 2 and 3, enter the reciprocals of the first ten whole numbers into List 1.
(b) Repeat (a) for the sequence $-9, -7, \dots, 9$.
- (a) Using method 4, enter the sequence $-9, -7, -5, \dots$ into List 2.
(b) Repeat (a) for the sequence $1 \times 2, 2 \times 3, 3 \times 4, \dots$

Guessing patterns

3. Find a rule for the n^{th} term of each sequence:

(a)	1	2	3	4	5	...	n
	4	7	10	13	16	...	?
(b)	1	2	3	4	5	...	n
	0	2	6	12	20	...	?

Note: The purpose of these items is to familiarise the user with a range of options for generating and storing useful results (including the use of list variables). Also shown are some tasks that might make use of such features.

29. Simulating a Random Process (GM)

(KP:10.1)

A random number can be generated on a spreadsheet using the **Ran#** function. The **Ran#** function will return a random value between 0 and 1. The following formulas can be used to produce more useful random numbers. The **Int** function converts a decimal value to the next lowest integer. **Int** and **Ran#** together can be used to produce random integer values.

- **Ran#** random number between 0 and 0.999999999
- **6*Ran#** random number between 0 and 5.999999999
- **6*Ran#+1** random number between 1 and 6.999999999
- **Int(6*Ran#+1)** random integer between 1 and 6 inclusive
- **Int((B-A+1)*Ran#+A)** random integer between A and B inclusive

Long Term Frequency

1. (a) Use Method 2 of Activity 2 (page 2) with the above technique to generate ten random integers between 1 and 6 (inclusive) and store them in successive elements of list variable List 1.
- (b) Investigate the contents of List 1

Sampling for Attributes

Relatively simple programs can be entered which will simulate random processes. The following program will simulate random samples from a population which contains 30% defectives.

<pre>=====SAMPLE ===== (10×Ran#+1)→X Int X→N N≤3→"DEF" N>3→"OK"</pre>	<pre>DEF DEF OK DEF OK OK OK OK</pre>
--	---------------------------------------

Program Details (for TI-82)

Program Instructions	Explanation of command	Key sequence to access particular commands
(10×Ran#+1) →X	(store number b/w 1 and 10.9999999 in X)	Ran# - OPTN•F6•PRB•F4
Int X→N	(store integer part of X in N)	Int - OPTN•F6•NUM•F2
N≤3: "DEF"	(if N≤3 then Display "DEF")	® - SHIFT•PROG•REL•F6 (F3)
N>3: "OK"	(if N>3 then Display "OK")	î - SHIFT•PROG•JUMP

Note: This task introduces simple methods using simulation. In particular, it illustrates how writing a small program can extend the functionality of the graphics calculator. In the example given, 'if' statements are used to decide if a sampled item is reported as defective or otherwise..

30. Compound Interest (GM)

(KP:2.1;2.2;3.1;5.1;5.3;7.1;11.1)

After how many years will he have doubled his money? (To the nearest year)

Approach 1 - Repeated calculation

```
1000×1.06      1060
Ans×1.06
                1123.6
                1191.016
                1262.47696
                1338.225578
```

```
1418.519112
1503.630259
1593.848075
1689.478959
1790.847697
1898.298558
2012.196472
```

Approach 2- Guess, check, & improve

```
1000×1.06^8    1593.848075
1000×1.06^15   2396.558193
1000×1.06^12   2012.196472
```

```
log 2 ÷ log 1.06
                11.89566105
```

When does $A = 2P = PR^n$ $n \rightarrow$

Establish that $n = \frac{\log 2}{\log 1.06}$ and then round up.

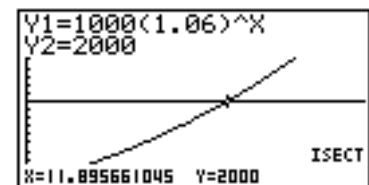
Set start value at 0 and pitch value at 1, and calculate which year.

Approach 5 - Function Rule & Graph

x and $Y2 = 2000$

Trace or Jump-to feature.

```
View Window
Xmin : 0
max : 20
scale : 1
Ymin : 1000
max : 3000
scale : 100
[INIT TRIG STD STO RCL]
```



Extension: Doubling Time

$$\frac{70}{\text{interest}} = \frac{1}{r}$$

interest rate expressed as a percentage. Complete the table to verify the accuracy of this rule of thumb for various usual range of interest rates?

Interest Rate (r)		8	12		20	25
Doubling Time (from Graph or Table)						
Doubling Time (from $d = 70/r$)						

Why does the rule of thumb work? Approach 3 shows that doubling time is $\frac{\log 2}{\log(1 + \frac{r}{100})}$. Now $\log 2 \approx 0.6931$

or roughly 0.7. So the doubling time is roughly $0.7 / \log(1 + \frac{r}{100})$; and the rule of thumb can be written $0.7 / (\frac{r}{100})$.

Graph $y = \frac{r}{100}$ and $y = \log(1 + \frac{r}{100})$ for $0 < r < 30$ on the same set of axes to see how close they are.

Note: The purpose of this task is to illustrate a number of methods by which a standard application involving compound interest can be approached. The method used may depend on the entry level of the students, but each method has intrinsic merit. This task also showcases some key calculator features.

31. Smoking and Birth Weights (GM/FM)

(KP:8.1-8.5)

A nurse wonders whether mothers who smoke give birth to babies with lower birth weights. He asks all the women with children whom he knows whether or not they smoke and for the birth weights of their babies.

The table below shows the results.



<i>Smoker</i>	3.1	2.87	3.41	2.73	2.27	2.67	3.21	3.55	2.9	2.76	2.87	3.15	2.73	2.93	3.01
<i>Non-Smoker</i>	3.38	3.61	3.07	3.49	3.18	3.47	3.52	3.64	3.69	3.13	3.44	4.46	4.06	3.47	

Birth weights of babies in kilograms according to whether or not the mother smokes

1. Comment on the selection of the sample. Is it likely to be representative of the local population?
2. Which variables are not controlled?
3. For the babies in the 'smokers' group, calculate the sample mean, standard deviation, range, median and interquartile range for the birth weights.
4. Obtain the same statistics for the babies from the 'non-smokers' group, and then plot histograms and box plots for each group. What do they show?
5. Do you think that this study proves anything?

Note: Comparing independent samples is a key technique in statistical analysis. This task provides a context for such an analysis in which basic statistics and graphs are used to determine the differences between the two population types. In addition, the difference between paired and non-paired data is highlighted.

32. From Two Points to a Line (GM)

(KP:2.1;4.1;9.1;9.3-9.5)

'Find the equation of the line that passes through the points A (-1,3) and B (4,7)'

Approach 1 - Home Screen

- The two known points A (-1,3) and B (4,7) .
- The gradient must be $m = \frac{7 - 3}{4 - (-1)} = \frac{4}{5} = 0.8$
- Now using the $c = y - mx$, and one of the points (4,7), calculate $c = 7 - 0.8(4) = 3.8$

Approach 2 - Fitting a line by eye

- Enter the two known points A (-1,3) and B (4,7) and produce a scatter plot.
- Try to enter a rule of the form $y = mx + c$ that passes directly through both points.

Approach 3 - Fitting by applying a linear regression model

- Enter the two known points A (-1,3) and B (4,7) and produce a scatter plot.
- Use the linear regression model ($y = ax + b$) to obtain values for the gradient and y-intercept

Approach 4 - Matrix Method

The line with equation $y = mx + c$ that passes through both points A and B implies that at A: $3 = m(-1) + c$ and at B: $7 = m(4) + c$. The equation of the line can then be obtained by using matrix algebra to find the values of m and c .

$$\begin{bmatrix} -1 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} m \\ c \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix} \rightarrow \begin{bmatrix} m \\ c \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 4 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 7 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 3.8 \end{bmatrix}$$

Note: The purpose of this task is to illustrate a number of methods for determining the equation of a straight line passing through two points. The method used may depend on the entry level of the students, but each method has intrinsic merit.

33. Celsius and Fahrenheit (GM)

(KP:2.1-2.4;5.1-5.5;11.1)

John and David have recently been on a trip to the United States of America, where they measure temperature on the 'Fahrenheit' scale rather than the 'Celsius' scale. They were told that when converting from Celsius to Fahrenheit to apply the rough rule 'double and add thirty'.



1. On the day that they left Melbourne, the reported daily maximum was 20°C . In order to prepare for temperature readings in the USA, John and David applied the rough rule to estimate the corresponding temperature in degrees Fahrenheit. What was this temperature?
2. On the first day of their trip, they arrived in San Diego, where the predicted daily maximum temperature was reported to be 81°F . Using the rough rule, to what temperature in degrees Celsius does this correspond?
3. The actual conversion rule is $\text{Fahrenheit}^{\circ} = \frac{9}{5} \text{Celsius}^{\circ} + 32$. What is the difference between the rough rule and the actual rule of conversion for a temperature of 20°C ?
4. On a hot day, the temperature is 30°C or more.
 - (a) What would be an equivalent 'hot day' temperature for the Fahrenheit scale?
 - (b) How does the difference between the rough rule and the actual rule change as the days get hotter?
5. For what temperature in degrees Celsius will the rough rule give the same converted value as the actual rule?
6. The rough rule is considered to be good enough if it is in error by no more than 5°F . For what range of temperatures (degrees Celsius) would the rough rule be considered 'good enough'?
7. There is a temperature at which the numeric value is the same in degrees Celsius and degrees Fahrenheit. What is this temperature? How far out, in degrees Fahrenheit, is the rough rule in this case?

Note: This task involves an accessible application involving linear functions which exploits features such as 'tracing' and the 'jump-to' feature to find points of intersection.

34. LPG vs. Petrol Costs (GM)

(KP:2.2;2.3;5.1-5.5;6.1-6.4;7.1)

A client asks her financial consultant whether it is better to install an LPG tank on her recently acquired car or to stick with petrol. The consultant researches the costs of each option and notes the following facts:



	Petrol	LPG
<i>Installation Cost (\$)</i>	0	2000
<i>Average Annual Distance Travelled (km)</i>	20,000	20,000
<i>Fuel Economy (litres/100 km)</i>	10	12.5
<i>Fuel Cost (\$/litre)</i>	0.79	0.32

Construct a function for the cumulative cost of each option over a number of years. Note that we are ignoring other costs such as depreciation and maintenance.

1. Use your graphics calculator to enter and plot both functions.
2. After how many years and months does the cumulative cost of the LPG option become cheaper?
3. If she travels 10,000 km year, does this mean that the LPG option takes twice as long to become the cheaper option? Give reasons for your answer.
4. If the fuel costs for LPG doubled, does this mean that the LPG option takes twice as long to become the cheaper option? Give reasons for your answer.
5. If the installation costs for LPG doubled, does this mean that the LPG option takes twice as long to become the cheaper option? Give reasons for your answer.

Note: This task involves an application involving linear functions which exploits features such as the use of parameters to compare models as conditions change.

35. Mobile Phone Charges (GM)

(KP 2.2-2.4;5.1-5.5;6.1-6.9;7.1)

OZ-Mobile is a newly launched mobile phone network provider that will offer competitive charging rates to attract customers from its competitors. Customers can choose from three different charging schemes.



OZ-Mobile Scheme A - \$10 rental per month and a call charge of \$1.20 per minute.

OZ-Mobile Scheme B - \$20 rental per month and a call charge of \$0.80 per minute.

OZ-Mobile Scheme C - \$35 rental per month and a call charge of \$0.40 per minute.

To make the problem simpler, if only part of a minute is used, only that part of a minute is charged to the user. Let $A(m)$, $B(m)$, and $C(m)$ be the monthly cost functions in dollars for each of the schemes where m represents the number of minutes that the caller has been on-line during that month.

Call Time	Cost for Scheme A	Cost for Scheme B	Cost for Scheme C	Most Expensive Scheme	Least Expensive Scheme
20 Minutes	\$34.00	\$36.00	\$43.00	C	A
30 minutes					
40 minutes					
50 minutes					

- Copy and complete the table above;
- Write down the rules for $A(m)$, $B(m)$, and $C(m)$.
- On your graphics calculator, enter and plot each of the three cost functions on the same set of axes.
 - Adjust the viewing window to show all three lines and their points of intersection. Draw a quick sketch graph of what you see in the viewing window. Label the axes and lines appropriately.
 - Find the coordinates of the points at which the graphs intersect. Add these to your sketch. What do these points represent?
- Write a brief report explaining which scheme will be best for different customers, based upon the number of minutes that they might be on-line in a month. Your report should indicate when scheme A is best, when scheme B is best, and when scheme C is best.

Problem Reference - 'Graphic Algebra - Explorations with a graphics calculator' by Asp, Dowsey, Stacey, Tynan (Key Curriculum Press)

Note: This task involves an application involving linear functions which exploits features such as tables to find points of intersection. The problem also challenges students to locate a suitable viewing window in which to view the relevant points of intersection. Students are also encouraged to write a brief report that summarises their understanding of the problem solution.

37. Cricket Problem (GM/FM)

(KP:9.1-9.5)

In test cricket, each player has two chances to bat, the first innings and the second innings. In a test match, the scores of Australian and English players were as follows:

Australia

Player Number	1	2	3	4	5	6	7	8	9	10	11
1st Innings	67	90	0	39	19	102	74	7	0	0	5
2nd Innings	5	13	4	24	0	12	51	2	24	0	0

England

Player Number	1	2	3	4	5	6	7	8	9	10	11
1st Innings	47	80	117	26	28	6	10	21	7	0	0
2nd Innings	34	14	0	83	71	2	7	88	5	10	0

- Using your graphics calculator, enter the data for 1st innings and 2nd innings scores and then draw a scatter plot.
- Comment on the features of this data as shown on the scatter plot.
- Calculate the median scores for each innings, and draw a vertical line through the median score for the first innings and a horizontal line through the median score for the second innings. Now attempt to obtain the q correlation coefficient for this data. Does this coefficient confirm the features you noticed in question 2?
- Suppose you wish to see whether for a batsman 'in form', the first innings score is a good predictor of a second innings score. As a measure of 'form', let us use the condition that a batsman must score 20 runs or greater in the first innings. Using this condition, delete the data pairs for any players who do not satisfy it.
- Repeat question 3 for the players whose scores have satisfied the condition. What does this tell you about this set of players?
- Obtain an r -correlation for the players whose scores have satisfied the condition. How does this value support or contradict the q -correlation coefficient you obtained in question 5?
- Fit a straight line by eye for the players whose scores have satisfied the condition, and obtain the equation of this line. Plot this line over the data.
- Use your equation from question 7 to predict the second innings score of a player who scored 35 in the first innings. How useful do you think this prediction might be? Give some reasons for this answer.
- If figures were available for all matches in the series, how would you expect the scatter plot and the correlation coefficient to change? That is, what would be the effect of more data on the scatter plot?

Extension

- Collect data from a recent Test cricket series and investigate what the data tells you.

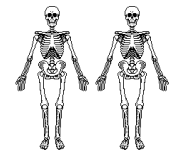
Reference: We are grateful to Neil Roberts for providing this problem context and some of the associated questions.

Note: Fitting a straight line by eye is an important skill. The cricket data provide a realistic context in which to exploit the ability of the graphics calculator to plot a line over a scatterplot and assess its goodness of fit by eye. The activity also highlights how calculating q -correlations might be approached via the graph screen.

38. Forensic Fun (GM/FM)

(KP:2.1;5.3;9.1-9.5)

A human skull and part of an arm have been found by a bushwalker in a national park. Although forensic pathologists can tell that the remains come from an adolescent, they would like to be able to predict the height of this person. From the remains, three measurements they can obtain are head circumference, forearm length and middle finger length. To help them to determine the best predictor, your task is to analyse some relevant data.



The table below lists the heights, head circumference, forearm length and middle finger length of 20 students.

Height (cm)	163	156	171	185	150	169	150	172	175	169	166	188	179	162	170	163	161	165	179	165
Head circumf. (cm)	57	55	56	56	54	58	57	57	59	59	57	61	58	59	57	56	56	56	58	58
Forearm length (cm)	25	25	25	26	22	27	25	27	26	23	24	28	27	23	24	26	23	24	26	26
Finger length (mm)	80	69	80	90	75	89	80	82	79	80	78	85	95	74	85	70	78	75	82	79

- By drawing appropriate scatter plots, and calculating correlations, decide which of the head circumference, forearm length or finger length is the best predictor of a person's height.

Once you have decided which measure is the best predictor of height, try to fit a linear rule by eye of the form $y = ax + b$, where y is the height and x is the measure you have chosen as the best predictor.

- Plot your rule over the data, and answer the following questions.
 - Are your points 'close to the line'?
 - How many of your points lie on each side of the line?
 - Do the points above the line seem to be spread out like the points below the line?

Adjust your rule if necessary to give a better fit.

Now that you have developed a linear rule for linking the two variables y and x , it would be useful to test how well your rule fits your sample measurements of those variables. First copy the x values into the table below.

- For each value of x , find the height, y , given by your rule. This is called the 'predicted' height. It may be different from the measured height. Enter the predicted heights in the third row of the table below.
 - In the fourth row, enter the difference between the measured heights and the predicted heights. Use the size of the difference only, not whether the difference is positive or negative.

Height (y cm)	163	156	171	185	150	169	150	172	175	169	166	188	179	162	170	163	161	165	179	165
x																				
Predicted Height																				
Diff. b/w Ht & Pred. Ht.																				

As a rough guide to how well your rule fits, an *Average Difference* can be calculated.

- Add the differences (last row) from the table that you have created. This will give you the Total Difference.
 - Calculate the Average Difference by dividing the Total Difference by the number of students in the sample (20 in this case).
- Gather data for a new sample of 20 students. Plot this data and the rule you have chosen previously using your graphics calculator.
 - Judging by eye, does the line appear to fit the data well? Give reasons.
 - Calculate the predicted heights, and also differences between the heights and predicted heights for this new sample.
 - Calculate the Average Difference for this new sample. Compare the Average Difference for this new sample with the Average Difference for the original sample. If they are different, can you think of why this is the case?
- Now that you have established a good predictor, use it to help the forensic pathologists, given that the measurements they obtained were: head circumference = 56 cm; forearm length = 25 cm; and middle finger length = 83 mm.

Note: Deciding on the best of a number of possible predictors is an important skill. Also highlighted in this task are the possible opportunities of gathering real data which involve the students.

39. Least Squares Regression (FM)

(KP 10.1)

This activity uses the program 'LSREGR' to present a first principle approach to least squares regression.

Program Listing for 'LSREGR'

```

Filename: LSREGR
AxesOff.
FuncOff.
BG-None.
Goto 0.
.
Lbl A.
ClrText.
Locate 1,2,"ASSUMES DATA".
Locate 1,3,"IS IN LIST 1,2".
Locate 1,5,"(OR WILL GIVE)".
Locate 1,6,"DIM ERROR)"»
S-Gph1 DrawOn, Scatter, List 1, List 2, 1, Squ
are.
S-Gph2 DrawOff.
DrawStat»
Goto 0.
.
Lbl B.
ClrText.
LinearReg List 1, List 2.
€€List 1+*List 6.
Sum ((List 6-List 2)â)†Q.
"TRY TO FIT".
"Y=MX+C".
"M=".
?†M
"C=".
?†C.
M€List 1+C†List 3.
S-Gph2 DrawOn, xyLine, List 1, List 3, 1, Dot.
Int ((Max(List 1)-Min(List 1))÷10)†Xscl.
Int ((Max(List 2)-Min(List 2))÷10)†Yscl.
DrawStat.
1†N.
While N@Dim List 1.
Plot List 1[N], List 2[N]:Plot List 1[N], L
ist 3[N]:Line.
N+1†N.
WhileEnd.
List 3-List 2†List 4.
List 4â†List 5.
Sum List 5†S.
G SelOn 1.
Fix 3.
Text 1,5,"YOUR SUM DIFF=".
Text 1,66,S.
Text 8,5,"MIN SUM DIFF=".
Text 8,66,Q»
Goto 0.
.
Lbl C.
ClrList.
ClrText.
"PRESS MENU-STAT".
"TO ENTER NEW DATA".
" ".
"X IN List 1".
"Y IN List 2".
" ".
"THEN RE-RUN PROGRAM".
Stop.
.
Lbl D.
ClrText.
"[List 1] [List 2]".
1†N.
For 1†I To Dim List 1.
If Frac (I÷6)-0.
Then N+1†N.
Locate 1,N, List 1[I].
Locate 11,N, List 2[I].
IfEnd.
If Seq(Int (6€Ran#), X, 1, 250, 1)†List 6.
ClrText.
"[List 1] [List 2]".
Locate 1,2, List 1[I].
Locate 11,2, List 2[I].
2†N.
IfEnd.
Next.
Seq(Int (6€Ran#), X, 1, 250, 1)†List 6.
ClrText.
Goto 0.
.
Lbl E.
ClrText.
LinearReg List 1, List 2.
€€List 1+*List 6.
Sum ((List 6-List 2)â)†Q.
Fix 2.
"SMALLEST SUM OF".
"DIFFERENCES".
"POSS. FOR L1, L2=".
Locate 16,3,Q.
Locate 1,4," ".
Locate 1,5,"WHEN M=".
Locate 8,5,€.
Locate 1,6,"AND C=".
Locate 8,6,*»
ClrText.
AxesOn.
ClrGraph.
Norm.
Stop.
.
Lbl 0.
AxesOn.
GridOff.
ClrText.
Locate 1,2,"LS-REGRESSION MENU".
Locate 1,3,"(1) GRAPH DATA".
Locate 1,4,"(2) TRY A FIT".
Locate 1,5,"(3) ENTER NEW DATA".
Locate 1,6,"(4) VIEW DATA".
Locate 1,7,"(NO. [5 TO QUIT])?":?†V.
V=1†Goto A.
V=2†Goto B.
V=3†Goto C.
V=4†Goto D.
V=5†Goto E.

```

Sample Screens

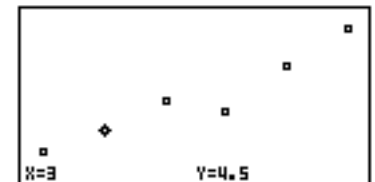
```

?
LS-REGRESSION MENU
(1) GRAPH DATA
(2) TRY A FIT
(3) ENTER NEW DATA
(4) VIEW DATA
(NO. [5 TO QUIT])?

```

	List 1	List 2	List 3	List 4
1	2	3.1		
2	3	4.5		
3	4	6.7		
4	5	6		
5	6	9.2		

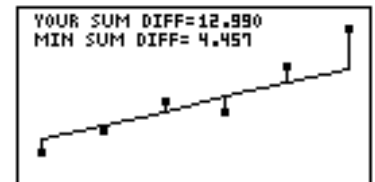
SRTA SRTD DEL DEL INS D



```

Y=MX+C
M=
C=

```



```

SMALLEST SUM OF
DIFFERENCES
POSS. FOR L1, L2=4.46
- Disp -
WHEN M=1.65
AND C=-0.53

```

Note: This program illustrates a first principles approach to least squares regression, whereby the student is challenged to enter the gradient and y-intercept for a straight line which will minimise the 'sum of the squared differences'. It uses both a graphic and numeric (with list variables) approach to the concept of least squares regression equations.

40. The Falling Stone (GM/FM)

(KP:2.1;4.1;9.1;9.3;9.4;9.5;11.1)

A stone is dropped from a high bridge into the water. A camera takes a photo every 0.4 seconds showing how far the stone has fallen. The distances are shown in the table below for the first two seconds.

Time Elapsed, t (seconds)	0	0.4	0.8	1.2	1.6	2.0
Distance Fallen, d (metres)	0	0.78	3.14	7.04	12.56	19.6

1. Use your graphics calculator to enter the data and draw a scatter plot. We will try to find a function whose graph is a good fit to the data.
2. Try to fit a rule of the form $d = at$ to the plotted data. Can you find a value for a which is a good fit?

Now we will try to fit a rule of the form $d = kt^2$ to the data. To do this we will plot d against t^2 .

3. Calculate the values for t^2 , and enter them into the table below.

t^2						
d	0	0.78	3.14	7.04	12.56	19.6

4. The t^2 values are your new x values, and the d values are still your y values. Draw a scatter plot of this data. Try to find a straight line which fits this data of the form $y = kx$. Can you find a value for k which is a good fit? Once you have a good fit, write your rule in the form $d = kt^2$.
5. Compare your rule to that obtained by doing a quadratic regression.

Clear all functions and data, and then enter your function rule $d = kt^2$ from question 4. Now you are ready to answer some questions about the falling stone.

6. (a) Use your graph to predict how far the stone will have fallen after 3 seconds.
(b) The bridge is 100 metres above the water. About how long will it take for the stone to hit the water?
7. (a) Find how far the stone fell in the last one hundredth of a second before hitting the water.
(b) Use the information from (a) to find approximately how fast the stone was travelling when it hit the water.
(c) Find how long it took for the stone to fall the last metre before hitting the water.
(d) Use the information from (c) to find approximately how fast the stone was travelling when it hit the water.
(e) Compare your answer for (d) with your answer for (b). Which is the most accurate? Why?

Problem Reference - 'Graphic Algebra - Explorations with a graphics calculator' by Asp, Dowsey, Stacey, Tynan (Key Curriculum Press)

Note: This task investigates a real situation in which the initial aim is to decide which of two possible functional models is appropriate. The use of list variables to linearise the data is a particular feature. Once the most appropriate model has been obtained, it is then used to interpolate and extrapolate sensibly.

41. My New Car (FM)

I was really keen to buy a new Ozmobile and when I went down payment. They were able to offer a loan for the full amount with an interest rate of 10% per year. I hoped to



1. Would payments of \$450 per month pay off the loan in 5 years? If not, how much is still owed?
2. Repeat question 1 for a payment of \$500 per month.
3. Try three other payments as in questions 1 and 2.
4. Record the 5 payment amounts in the questions above in List 1 and the corresponding amounts still owed in List 2. Draw a scatter plot showing the amount still owed as a function of the payment per month.
5. Determine the equation for the amount still owed after 5 years as a function of the payment per month. Hence determine the payment needed to pay off the loan in 5 years.
6. Suppose \$575 per month was the most I could afford.
 - (a) How long would it take to pay off the loan?
 - (b) What size loan can I really afford if I want to pay it off over 5 years (at \$575 per month)?

Note: This task involves a first principles approach to paying off a loan. Features include the use of recurrence relations using the sequence mode, list variables, and using a linear model for predictive purposes.

42. The Old Swamp (FM)

(KP:12.1)

An endangered species of bird, the 'Yellow-Crested Farnarkler', is found to live only in and around a certain swamp known as the Old Swamp. The birds were counted in 1986 and the population was found to be 125. A year later the population had increased to 150.



1. Suppose that the populations in successive years formed an arithmetic sequence. What would be the 1996 population in this case?
2. Now suppose that the populations in successive years followed a geometric sequence. What would be the 1996 population in this case?

Eventually the number of Farnarklers in the Old Swamp builds up to 350. Scientists who are monitoring the situation then consider it safe to resettle another swamp by transporting 30 Farnarklers per year from the Old Swamp to 'New Swamp'. The initial Farnarkler populations of the Old and New Swamps are thus 320 and 30 respectively, and it is assumed that each population increases by 15 percent per year.

3. Find the Farnarkler population of each swamp after one year, just after the second set of birds has been transported.
4. The size of the Farnarkler population at the Old Swamp after n years, P_n , just after the birds have been transported for that year, is given by the difference equation
$$P_n = 1.15P_{n-1} - 30, n = 1, 2, \dots, P_0 = 320$$
Write down a similar difference equation for the size of the Farnarkler population at the New Swamp after n years, Q_n .
5. (a) Predict the Farnarkler populations of both the Old Swamp and the New Swamp after five years.
(b) Predict the number of years, to the nearest whole number, that pass before the Farnarkler populations of the two swamps are most nearly equal.

Note: Arithmetic and geometric sequences can be investigated on a graphics calculator using sequence mode. In this task, adapted from an early VCE CAT question, students explore the properties of such sequences and their usefulness for prediction in this problem context.

43. Fiddling with Fibonacci (FM)

(KP:12.1)

The Italian monk Leonardo de Pisa (late 12th and early 13th centuries), who was also known as Fibonacci, is famous for the sequence named after him. The Fibonacci sequence is defined as follows: let the first two terms be 1 and 1 and take each subsequent term to be the sum of the two preceding terms.

1. Write down a difference equation for the n^{th} Fibonacci number f_n in terms of f_{n-1} and f_{n-2} .
2. Enter the Fibonacci sequence into your graphics calculator and find f_{20} both graphically and via a table of values.
3. Explore, for a number of values of n , the expression $f_n f_{n-2} - f_{n-1}^2$. What does this suggest? Verify the result for the first 20 Fibonacci numbers.
4. Explore, for a number of values of n , the expression f_n / f_{n-1} . What does this suggest? Verify the result using a graphical and tabular method.
5. The French mathematician Lucas (late 19th century) looked at a Fibonacci-like sequence in which the generating rule is the same but the first two terms are 1 and 3. Let l_n be the n^{th} Lucas number.
 - (a) Enter the Lucas sequence into your graphics calculator and find l_{20} . How does it compare with f_{20} ?
 - (b) Determine whether the Lucas numbers possess the same properties as the Fibonacci numbers that you found in Questions 3 and 4.

Note: This activity involves an investigation into Fibonacci-like sequences and exploits the sequence mode and list variable features of the graphics calculator.

44. Building Houses (GM)

A building contractor specialises in the construction of garages, brick homes and timber frame homes. In a particular 2 month period, she has orders to build 7 garages, 3 brick homes and 9 timber frames. The orders can be represented by the row matrix $[7 \ 3 \ 9]$.

The major raw materials required by these 3 constructions are steel, wood, glass, paint, and labour. The table below shows how many units of each of these raw materials are required to build the three construction types.



	<i>Steel</i>	<i>Wood</i>	<i>Glass</i>	<i>Paint</i>	<i>Labour</i>
Garages	5	9	3	6	18
Brick Homes	8	13	15	13	25
Timber Homes	4	25	17	19	20

- Using a matrix method, calculate how much of each material is needed to fulfil the building commitments in the next two months?
- An estimate of the unit material costs are given below. The values represent the cost (in dollars) per unit.

$$\text{Steel} = 17 \quad \text{Wood} = 19 \quad \text{Glass} = 20 \quad \text{Paint} = 15 \quad \text{Labour} = 26$$

- Find the cost of each type of construction.
- Find the total cost of raw materials for all constructions.

Note: Graphics calculators can perform simple matrix arithmetic. This task provides a standard context for such matrix manipulation.

45. Looking at Polar Graphs (GM)

(KP:4.9)

1. Plot the polar curves (in an appropriate viewing window):
 - (a) $r = 3 \cos \theta$
 - (b) $r = 1 + 3 \cos \theta$
 - (c) $r = \cos 3\theta$
 - (d) $r = 1 + \cos 3\theta$
2. Explore the polar curves $r = a + \cos b\theta$ for various values of a and b and comment on the effects of these two parameters on the shape of the curve.
3. Explore the spirals defined by the rules:
 - (a) $r = \frac{\theta}{c}$ where c is a constant;
 - (b) $r = d^{\theta/3}$ where d is a constant greater than 1.

[Note: the basic spiral of shells is the logarithmic or equiangular spiral, since it is always at the same angle to its radius.]

Note: Graphics calculators can draw graphs of relationships expressed functionally, parametrically, in sequence form or in polar form. This activity provides an opportunity for exploration of polar graphs.